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Statistical methods in process management — Capability and performance

Part 7: Capability of measurement processes

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National foreword

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Statistical methods in process management — Capability and performance —

Part 7:
Capability of measurement processes

Méthodes statistiques dans la gestion de processus — Aptitude et performance —

Partie 7: Aptitude des processus de mesure

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 22514-7 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

ISO 22514 consists of the following parts, under the general title *Statistical methods in process management — Capability and performance*:

— *Part 1: General principles and concepts*

— *Part 2: Process capability and performance of time-dependent process models*

— *Part 3: Machine performance studies for measured data on discrete parts*

— *Part 4: Process capability estimates and performance measures*

— *Part 6: Process capability statistics for characteristics following a multivariate normal distribution*

— *Part 7: Capability of measurement processes*

A future Part 5 on process capability and performance for attributive characteristics is planned. A future Part 8 on the machine performance of a multi-state production process is under preparation.

Introduction

The purpose of a measurement process is to produce measurement results obtained from defined characteristics on parts or processes. The capability of a measurement process is derived from the statistical properties of measurements from a measurement process that is operating in a predictable manner.

Calculations of capability and performance indices are based on measurement results. The uncertainty of the measurement process used to generate capability and performance indices must be estimated before the indices can be meaningful. The actual measurement uncertainty needs to be adequately small.

If the measurement process is used to judge whether a characteristic of a product conforms to a specification or not, the uncertainty of the measurement process must be compared to the specification itself. If the measurement process is used for process control of a characteristic, the uncertainty needs to be compared with the process variation. Limits of acceptability should be stated for both cases.

The quality of measurement results is given by the uncertainty of the measurement process. This is defined by the statistical properties of multiple measurements, or estimates of properties, based on the knowledge of the measurement process.

The methods described in this part of ISO 22514 only address the implementation uncertainty. (For more information on implementation uncertainty, see ISO 17450-2.) Therefore, they are only useful if it is known that the method uncertainty and the specification uncertainty are small compared to the implementation uncertainty. This part of ISO 22514 describes methods to define and calculate capability indices for measurement processes based on estimated uncertainties. The approach given in ISO/IEC Guide 98-3, *Guide to the expression of uncertainty in measurements (GUM)* is the basis of this approach.

Statistical methods in process management — Capability and performance —

Part 7: Capability of measurement processes

1 Scope

This part of ISO 22514 defines a procedure to validate measuring systems and a measurement process in order to state whether a given measurement process can satisfy the requirements for a specific measurement task with a recommendation of acceptance criteria. The acceptance criteria are defined as a capability figure (C_{MS}) or a capability ratio (Q_{MS}).

NOTE 1 This part of ISO 22514 follows the approach taken in ISO/IEC Guide 98-3, *Guide to the expression of the uncertainty in measurement (GUM)*, and establishes a basic, simplified procedure for stating and combining uncertainty components used to estimate a capability index for an actual measurement process.

NOTE 2 This part of ISO 22514 is primarily developed to be used for simple one-dimensional measurement processes, where it is known that the method uncertainty and the specification uncertainty are small compared to the implementation uncertainty. It can also be used in similar cases, where measurements are used to estimate process capability or process performance. It is not suitable for complex geometrical measurement processes, such as surface texture, form, orientation and position measurements that rely on several measurement points or simultaneous measurements in several directions.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1:2006, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2:2006, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 5725-1, *Accuracy (trueness and precision) of measurement methods and results — Part 1: General principles and definitions*

ISO 5725-2, *Accuracy (trueness and precision) of measurement methods and results — Part 2: Basic method for the determination of repeatability and reproducibility of a standard measurement method*

ISO 5725-3, *Accuracy (trueness and precision) of measurement methods and result — Part 3: Intermediate measures of the precision of a standard measurement method*

ISO 5725-4, *Accuracy (trueness and precision) of measurement methods and results — Part 4: Basic methods for the determination of the trueness of a standard measurement method*

ISO 5725-5, *Accuracy (trueness and precision) of measurement methods and results — Part 5: Alternative methods for the determination of the precision of a standard measurement method*

ISO 5725-6, *Accuracy (trueness and precision) of measurement methods and results — Part 6: Use in practice of accuracy values*

ISO 7870-1, *Control charts – Part 1: General guidelines*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-2 and ISO 5725 (all parts), and the following apply.

3.1
maximum permissible measurement error
maximum permissible error
limit of error
MPE
extreme value of measurement error, with respect to a known reference quantity value, permitted by specifications or regulations for a given measurement, measuring instrument, or measuring system

NOTE 1 Usually, the term "maximum permissible errors" or "limits of error" is used where there are two extreme values.

NOTE 2 The term "tolerance" should not be used to designate 'maximum permissible error'.

[ISO/IEC Guide 99:2007, 4.26]

3.2
measurand
quantity intended to be measured

NOTE 1 The specification of a measurand requires knowledge of the kind of quantity, description of the state of the phenomenon, body, or substance carrying the quantity, including any relevant component, and the chemical entities involved.

NOTE 2 In the second edition of the VIM and in IEC 60050-300:2001, the measurand is defined as the 'quantity subject to measurement'.

NOTE 3 The measurement, including the measuring system and the conditions under which the measurement is carried out, might change the phenomenon, body, or substance such that the quantity being measured may differ from the measurand as defined. In this case, adequate correction is necessary.

EXAMPLE 1 The potential difference between the terminals of a battery may decrease when using a voltmeter with a significant internal conductance to perform the measurement. The open-circuit potential difference can be calculated from the internal resistances of the battery and the voltmeter.

EXAMPLE 2 The length of a steel rod in equilibrium with the ambient Celsius temperature of 23 °C will be different from the length at the specified temperature of 20 °C, which is the measurand. In this case, a correction is necessary.

NOTE 4 In chemistry, "analyte", or the name of a substance or compound, are terms sometimes used for 'measurand'. This usage is erroneous because these terms do not refer to quantities.

[ISO/IEC Guide 99:2007, 2.3]

3.3
measurement uncertainty
uncertainty of measurement
uncertainty
non-negative parameter characterizing the dispersion of the quantity values being attributed to a **measurand** (3.2), based on the information used

NOTE 1 Measurement uncertainty includes components arising from systematic effects, such as components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty. Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.

NOTE 2 The parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.

NOTE 3 Measurement uncertainty comprises, in general, many components. Some of these may be evaluated by Type A evaluation of measurement uncertainty from the statistical distribution of the quantity values from series of measurements and can be characterized by standard deviations. The other components, which may be evaluated by Type B evaluation of measurement uncertainty, can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information.

NOTE 4 In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to the **measurand** (3.2). A modification of this value results in a modification of the associated uncertainty.

[ISO/IEC Guide 99:2007, 2.26]

3.4
Type A evaluation of measurement uncertainty
Type A evaluation
evaluation of a component of **measurement uncertainty** (3.3) by statistical analysis of measurement quantity values obtained under defined measurement conditions

NOTE 1 For various types of measurement conditions, see repeatability condition of measurement, intermediate precision condition of measurement, and reproducibility condition of measurement.

NOTE 2 For information about statistical analysis, see e.g. ISO/IEC Guide 98-3.

NOTE 3 See also ISO/IEC Guide 98-3:2008, 2.3.2, ISO 5725, ISO 13528, ISO/TS 21748, ISO 21749.

[ISO/IEC Guide 99:2007, 2.28]

3.5
Type B evaluation of measurement uncertainty
Type B evaluation
evaluation of a component of **measurement uncertainty** (3.3) determined by means other than a **Type A evaluation of measurement uncertainty** (3.4)

EXAMPLES Evaluation based on information

— associated with authoritative published quantity values,

— associated with the quantity value of a certified reference material,

— obtained from a calibration certificate,

— about drift,

— obtained from the accuracy class of a verified measuring instrument,

— obtained from limits deduced through personal experience.

NOTE See also ISO/IEC Guide 98-3:2008, 2.3.3.

[ISO/IEC Guide 99:2007, 2.29]

3.6
standard uncertainty of measurement
standard uncertainty of measurement
standard uncertainty
measurement uncertainty (3.3) expressed as a standard deviation

[ISO/IEC Guide 99:2007, 2.30]

3.7
combined standard measurement uncertainty
combined standard uncertainty
standard measurement uncertainty (3.6) that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model

NOTE In case of correlations of input quantities in a measurement model, covariances must also be taken into account when calculating the **combined** standard measurement uncertainty; see also ISO/IEC Guide 98-3:2008, 2.3.4.

[ISO/IEC Guide 99:2007, 2.31]

3.8
expanded measurement uncertainty
expanded uncertainty
product of a **combined standard measurement uncertainty** (3.7) and a factor larger than the number one

NOTE 1 The factor depends upon the type of probability distribution of the output quantity in a measurement model and on the selected coverage probability.

NOTE 2 The term “factor” in this definition refers to a coverage factor.

NOTE 3 Expanded measurement uncertainty is termed “overall uncertainty” in paragraph 5 of Recommendation INC-1 (1980) (see the GUM) and simply “uncertainty” in IEC documents.

[ISO/IEC Guide 99:2007, 2.35]

3.9
measurement bias
bias
estimate of a systematic measurement error

[ISO/IEC Guide 99:2007, 2.18]

3.10
measurement result
set of quantity values being attributed to a **measurand** (3.2) together with any other available relevant information

NOTE 1 A measurement result generally contains “relevant information” about the set of quantity values, such that some may be more representative of the measurand than others. This may be expressed in the form of a probability density function (PDF).

NOTE 2 A measurement result is generally expressed as a single measured quantity value and a measurement uncertainty. If the measurement uncertainty is considered to be negligible for some purpose, the measurement result may be expressed as a single measured quantity value. In many fields, this is the common way of expressing a measurement result.

NOTE 3 In the traditional literature and in the previous edition of the VIM, measurement result was defined as a value attributed to a measurand and explained to mean an indication, or an uncorrected result, or a corrected result, according to the context.

[ISO/IEC Guide 99:2007, 2.9]

3.11
measurement model
model of measurement
model
mathematical relation among all quantities known to be involved in a measurement

NOTE 1 A general form of a measurement model is the equation $h(Y, X_1, \ldots, X_n) = 0$, where Y, the output quantity in the measurement model, is the **measurand** (3.2), the quantity value of which is to be inferred from information about input quantities in the measurement model $X_1, \ldots, X_n$.

NOTE 2 In more complex cases, where there are two or more output quantities in a measurement model, the measurement model consists of more than one equation.

[ISO/IEC Guide 99:2007, 2.48]

3.12
measurement task
quantification of a **measurand** (3.2) according to its definition

NOTE 1 The measurement task is synonymous with the purpose of applying the measurement procedure.

NOTE 2 The measurement task can be used, e.g.:

— to compare the measurement results with one or two specification limits in order to state whether the value of the measurand is an admissible value.

— to state whether the measurand characterizing a manufacturing process is within the specifications given.

— to obtain a confidence interval of given average length for the difference between two values of the same measurand.

3.13
measurement process
set of operations to determine the value of a quantity

[ISO 9000:2005, 3.10.2]

3.14
resolution
smallest change in a quantity being measured that causes a perceptible change in the corresponding indication provided by a measuring equipment

NOTE 1 Resolution can depend on, for example, noise (internal or external) or friction. It may also depend on the value of a quantity being measured.

[ISO/IEC Guide 99:2007, 4.14]

NOTE 2 For a digital displaying device, the resolution is equal to the digital step.

NOTE 3 Resolution not necessarily linear.

3.15
reference quantity value
reference value
quantity value used as a basis for comparison with values of quantities of the same kind

NOTE 1 A reference quantity value can be a true quantity value of a measurand, in which case it is unknown, or a conventional quantity value, in which case it is known.

NOTE 2 A reference quantity value with associated measurement uncertainty is usually provided with reference to:

a) a material, e.g. a certified reference material,

b) a device, e.g. a stabilized laser,

c) a reference measurement procedure,

d) a comparison of measurement standards.

[ISO/IEC Guide 99:2007, 5.18]

3.16
measurement repeatability
repeatability
measurement precision under repeatability conditions of measurement

[ISO/IEC Guide 99:2007, 2.21]

3.17
measurement reproducibility
reproducibility
measurement precision under reproducibility conditions of measurement

[ISO/IEC Guide 99:2007, 2.25]

3.18
stability of a measurement process
property of a measurement process, whereby its properties remain constant in time

3.19
item
entity
object
anything that can be described and considered separately

4 Symbols and abbreviated terms

4.1 Symbols

a	half width of a distribution of possible values of input quantity
a_{OBJ}	maximal form deviation
α	significance level
B_{i}	bias
β_0	intercept of the calibration function
$\hat{\beta}_0$	estimated intercept of the calibration function
β_1	slope of the calibration function
$\hat{\beta}_1$	estimated slope of the calibration function
C_{MP}	measurement process capability index
C_{MS}	measuring system capability index
C_{p}	process capability index
C_{pk}	minimum process capability index
$C_{\mathrm{p,obs}}$	observed process capability index
$C_{\mathrm{p,p}}$	real process capability index
d_{LR}	interval from the last reference value, for which all operators have assessed the result as unsatisfied to the first reference value, for which all operators have the result as approved

d_{UR}	from the last reference value, for which all operators have assessed the result as approved to the first reference value, for which all operators have the result as unsatisfied
d	average interval
k	coverage factor
K	total number of replicate measurements on one reference. The reference can be a reference standard or a reference workpiece
k_{CAL}	coverage factor from the calibration certificate
l	measured length
L	lower specification limit
M_{PE}	maximum permissible error (of the measuring system) (MPE-value)
m_{ji}	frequencies in the Bowker-test
N	number of standards
n	number of measurements
P	probability
P_{p}	process performance index
$P_{\mathrm{p,\ obs}}$	observed process performance index
$P_{\mathrm{p,\ p}}$	real process performance index
Q_{attr}	attributive measurement process capability ratio
Q_{MS}	measuring system capability ratio
Q_{MP}	measurement process capability ratio
R_{E}	resolution of measuring system
s	sample standard deviation (for the measuring system repeatability)
T	temperature
$t_{1-(\alpha/2)}$	the two-sided critical value of Student's t distribution
U	upper specification limit
u_{α}	standard uncertainty on the coefficient of expansion
u_{AV}	standard uncertainty from the operator's repeatability
u_{BI}	standard uncertainty from the measurement bias
u_{CAL}	calibration standard uncertainty on a standard
u_{MP}	combined standard uncertainty on measurement process
u_{EV}	standard uncertainty from maximum value of repeatability or resolution
u_{EVR}	standard uncertainty from repeatability on standards
u_{EVO}	standard uncertainty from repeatability on test parts
u_{GV}	standard uncertainty from reproducibility of the measuring system

u_{IAi}	standard uncertainty from interactions
u_{LIN}	standard uncertainty from linearity of the measuring system
u_{MP}	combined standard uncertainty on measurement process
u_{MPE}	standard uncertainty calculated based on maximum permissible error
u_{MS}	combined standard uncertainty on measuring system
$u_{\mathrm{MS\text{-}REST}}$	standard uncertainty from other influence components not included in the analysis of the measuring system
u_{OBJ}	standard uncertainty from test part inhomogeneity
u_{RE}	standard uncertainty from resolution of measuring system
u_{REST}	standard uncertainty from other influence components not included in the analysis of the measurement process
u_{STAB}	standard uncertainty from the stability of measuring system
u_{T}	standard uncertainty from temperature
u_{TA}	standard uncertainty from expansion coefficients
u_{TD}	standard uncertainty from temperature difference between workpiece and measuring system
U_{attr}	uncertainty on an attributive measurement
U_{CAL}	uncertainty on the calibration of a standard
U_{MS}	uncertainty of the measuring system
U_{MP}	uncertainty of the measurement process
y_j	j th measurement value
$\bar{y}$	average of all measurements
$\bar{x}_g$	arithmetic mean of all the sample values
x_i	i th measurement input quantity
x_{m}	reference quantity value

4.2 Abbreviated terms

ANOVA	analysis of variance
DOE	design of experiments
GPS	geometrical product specifications
R&R	repeatability and reproducibility
GUM	guide to the expression of the uncertainty of measurement
MPE	maximum permissible error
SPC	statistical process control
VIM	international vocabulary of metrology

5 Basic principles

5.1 General

The method described in this part of ISO 22514 covers a large part of the estimation of measurement uncertainty that occurs in practice. In some cases, where the preconditions set out for this method (no correlation between influence components, no sensitivity factors, simple linear model present) are not present, the user must utilize the general current method for determining the measurement uncertainty that is described in ISO/IEC Guide 98-3: 2008.

The following method addresses the implementation uncertainty (see also ISO 17450-2). Therefore, it shall be determined before the method is applied that the method uncertainty and the specification uncertainty are small compared to the implementation uncertainty. Further, the method is not suitable and shall not be used for complex geometrical measurement processes, such as surface texture, form, orientation and location measurements that rely on several measurement points or simultaneous measurements in several directions, or both.

The ISO/IEC Guide 98-3 (GUM) permits the evaluation of standard uncertainties by any appropriate means. It distinguishes the evaluation by the statistical treatment of repeated observations as a Type A evaluation of uncertainty, and the evaluation by any other means as a Type B evaluation of uncertainty. In evaluating the combined standard uncertainty, both types of evaluation are to be characterized by squared standard uncertainties and treated in the same way. The standard uncertainties can be aggregated to obtain the (combined) standard measurement uncertainty. This evaluation of uncertainty is carried out, according to ISO/IEC Guide 98-3, using the law of propagation of uncertainty. Full details of this procedure and the additional assumptions on which it is based are given in ISO/IEC Guide 98-3.

To assess a measuring system or a measurement process, the capability ratio Q_{MS} or Q_{MP} or the capability index C_{MP} or C_{MS} can be calculated based on the combined standard measurement uncertainty and the specification.

The combined expanded uncertainty should be substantially smaller than the specification of the characteristic being measured.

If the uncertainty components estimated from an experiment (Type A evaluation) do not correspond to the expected spread of these components in the actual measurement process, then these components may not be estimated experimentally. Instead, they should be derived through the use of a mathematical model (Type B evaluation; e.g. constant temperature in a measuring laboratory when conducting a study and the normal temperature variations of the place of the future application). The practitioner needs to fully understand the model to be used.

Figure 1 describes the step by step approach of the method. Linearity, repeatability and bias can be found using a reference standard as shown in the flowchart. Alternatively, bias can be found based on the MPE-value (maximum permissible error).

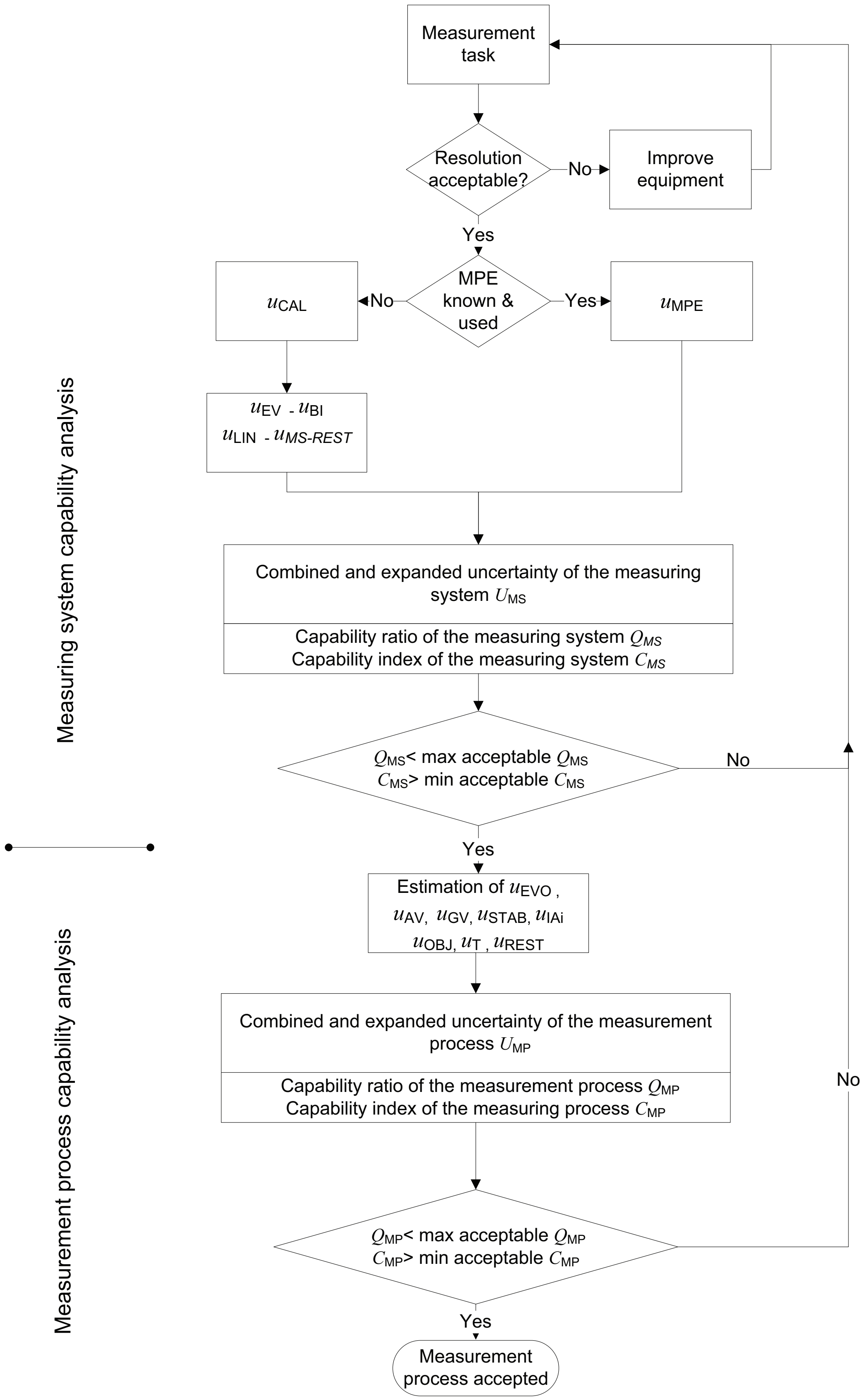


Figure 1 — Measurement process capability analysis

5.2 Resolution

The resolution is one of the contributors to the measurement uncertainty. It shall never be lower than the resolution effect. If the expanded uncertainty calculated based on the actual resolution is bigger than the requirement to the measurement process, then the measuring system should be improved.

By default, to use a measuring system to define the conformance to a bilateral specification, without a specific rule established between the supplier and purchaser, the resolution shall be lower than 1/20 of the specification interval.

By default, to use a measuring system to control a manufacturing process by using the SPC tools in accordance with a bilateral specification, without a specific rule established between the supplier and purchaser, the resolution shall be lower than 1/5 of the process variation.

5.3 MPE known and used

If a standard measuring system is used, then a maximum permissible error (MPE), or more often a number hereof, should be defined for the actual system. The calibration system is used to document the compliance with the requirement to the defined metrological characteristic(s) given as one or more maximum permissible errors.

In this case, the MPE value or, if more than one metrological characteristic influences the measuring task, the combined result of the actual MPE values can be used to calculate the capability of the measuring system instead of the experimental method. If a population of different equipment should be used as measuring system, then the method using MPE may be recommended. If only one defined measuring system can be used in connection with the measurement process, then the experimental method is preferable because the combined uncertainty will normally be smaller.

5.4 Capability and performance limits for measuring system and measurement process

If the measuring system is to be classified to a specific measurement process, it is important to set a limit on measurement uncertainty. In this way, the selection of a measuring system is simplified for upcoming measurement tasks.

If there is no requirement for a maximum Q_{MP} or a minimum C_{MS} , then proceed and calculate Q_{MS}.

The following method is based on the precondition that some uncertainty components associated with the measurement process, such as non-homogeneity of the measured object, resolution and temperature should be modelled mathematically.

6 Implementation

6.1 General

As for other processes, the measurement process is under the influence of both random and systematic sources of variation. In order to estimate and control the variation of the measurement process, it is necessary to identify all important sources of the variation, and if possible, to monitor them. In general, uncertainty components that are less than 10 % of the largest uncertainty component are considered to be unimportant.

6.2 Factors that influence the measurement process

6.2.1 General

In industrial practice, the reported uncertainty of the measurement process is usually limited to the uncertainty derived from repeatability of the measurement process on a reference standard, or an item typical of that to be produced, known as a workpiece. The uncertainties arising from any linearity deviation will either intentionally be set to zero or acquired from the manufacturer’s specification, e.g. in terms of adopted error limit (M_{PE} values).

The use of the commonly known repeatability experiment on a reference standard to estimate the repeatability and bias of the measurement process is recommended. Based on this experiment, one can then estimate a measurement capability index. This method may be extended by the use of more than one reference standard, located near or inside the specification limits. In both cases, the measuring system can be corrected by use of the identified systematic error(s).

If the linearity of the measuring system has to be determined, it can be done by means of a linearity study based on at least three reference standards. The result of this investigation (the regression function) can then be used for correction of the measurement result. Hereby, the uncertainty caused by the linearity deviation will be reduced.

6.2.2 Uncertainty components that belong to the measuring system

6.2.2.1 Types

The uncertainty components related to the measuring system are either

— maximum permissible error,

or

— the combination of

  — calibration uncertainty,

  — repeatability and/or resolution,

  — bias,

  — linearity, and

  — other uncertainty components.

6.2.2.2 Estimation of uncertainty using MPE value

When a measuring equipment or measuring standard is known to conform to stated MPE values for each of the metrological characteristics, these MPE values should be used to estimate the uncertainty component as shown in Table 1.

Table 1 — MPE uncertainty

Uncertainty components	Symbol	Test/model
MPE value	u_{MPE}	Standard uncertainty due to maximum permissible error. $u_{\mathrm{MPE}} = \frac{M_{\mathrm{PE}}}{\sqrt{3}}$ where a rectangular distribution is assumed. In cases where more than one MPE value influences the measurement process, the combined standard uncertainty can be calculated from: $u_{\mathrm{MPE}} = \sqrt{\frac{M_{\mathrm{PE1}}^2}{3} + \frac{M_{\mathrm{PE2}}^2}{3} ....}$

6.2.2.3 Measuring system resolution

The actual proposed measuring system should have a high enough resolution so that the expanded uncertainty calculated from the standard uncertainty of the resolution is much lower (common practice is 5 %) than the specification interval for the characteristic to be measured (measurand).

The resolution of the measuring system, or the step in the last digit of a digital display, or rounded measured value, will always cause an uncertainty component. When the repeatability uncertainty component is derived from experimental data, the effect from resolution, etc., is included if the repeatability uncertainty component (u_{EVR}) is greater than the component based on resolution.

If the uncertainty of the repeatability component is greater than that of the resolution component, then the resolution component is included in the repeatability component. If not, then the component u_{RE} should be added to the model as shown in Table 2.

Table 2 — Uncertainty from resolution

Uncertainty components	Symbol	Test/model
Resolution of the measuring system	u_{RE}	$u_{RE} = \frac{1}{\sqrt{3}} \cdot \frac{R_E}{2} = \frac{R_E}{\sqrt{12}}$ where R_E is the resolution and is assumed to follow a rectangular distribution. If analogue scales are used, the actual distribution can be another e.g. normal distribution.

6.2.2.4 Calculation of repeatability, bias and linearity using reference standards or calibrated workpieces

The used reference standards or workpieces should be traceable to stated references, usually national or international standards or so-called consensus standards (standards agreed by both customer and supplier). The present uncertainty during this calibration should be determined.

Table 3 — Uncertainty of standard calibration

Uncertainty components	Symbol	Test/model
Calibration	u_{CAL}	Standard deviation of uncertainty due to calibration (from certificate). In cases where the uncertainty in protocol is given as expanded uncertainty, it should be divided by the corresponding coverage factor: $u_{CAL} = U_{CAL} / k_{CAL}$

Linearity analyses must be made sufficiently often such that the estimated value for M_{PE} is not exceeded between two linearity analyses.

6.2.2.5 Experimental method (using regression analysis)

The experimental method considers how a relationship $Y = A + BX$ (describing how the dependent variable Y

varies as a function of the independent variable X) can be determined from measurement data. The measurement data arise when a measuring system specified by (unknown) values A and B of the calibration function parameters is “stimulated” by standards with calibrated values of X_i, given in standard units, and the corresponding “responses”, or indications Y_i, of the instrument are recorded.

Table 4 — Uncertainty from measuring system

Uncertainty components	Symbol	Test/model
Uncertainty arising from linearity	u_{LIN}	Instance 1: $u_{\mathrm{LIN}} = 0$ Instance 2: $u_{\mathrm{LIN}} = \frac{a}{\sqrt{3}}$ where a is half width of the range of a uniform distribution or the known MPE-value. Instance 3: u_{LIN} is determined experimentally together with u_{EVR} (see instance 2 below) Instance 4: u_{LIN} is determined based on the results from the calibration certificate
Uncertainty arising from bias	u_{BI}	From the measurements on a reference standard, u_{BI} can be calculated based on the distance between the standard and the average of the measured values. $u_{\mathrm{BI}} = \frac{\lvert \overline{x}_g - x_m \rvert}{\sqrt{3}}$
Repeatability using reference standards	u_{EVR}	Instance 1: minimum 30 repeated measurements on a reference standard, whereby u_{EVR} can be estimated Instance 2: K repeated measurements on each of the N (≥ 2) different reference standards with $N{*}K \geq 30$. Estimate from the linear regression function Estimate both u_{EVR} and u_{LIN} by the ANOVA method.
Other uncertainty components not included in the above	$u_{\mathrm{MS\text{-}REST}}$	E.g. scale shift (use of different measuring faces)

6.2.3 Additional uncertainty components belonging to the measurement process

6.2.3.1 General

In an analysis of a defined measurement process under real conditions, an identification and determination of additional uncertainty components of the process should be carried out together with the above described uncertainty components coming from the measuring system.

6.2.3.2 Determination of uncertainty components from experiments (Type A)

Table 5 — Uncertainty from repeatability and reproducibility of the measurement process

<table>
<tr><th>Uncertainty components</th><th>Symbol</th><th>Test/model</th></tr>
<tr><td>Repeatability using workpieces</td><td>u_{EVO}</td><td rowspan="5">Always use a minimum of 5 workpieces
— measured by a minimum of 2 operators or
— measured by a minimum of 2 different measuring systems (if relevant).
Minimum sample size: 30
Estimation of uncertainty components by the ANOVA method.
[VIM, GUM, ISO 5725, ISO 13528, ISO/TS 21748, ISO 21749]
If no operator influence is present, the number of workpieces should be increased.</td></tr>
<tr><td>Effect of operators changing in reproducibility conditions of measurement</td><td>u_{AV}</td></tr>
<tr><td>Reproducibility of the measuring system
(Place of measurement)</td><td>u_{GV}</td></tr>
<tr><td>Effect of changing over the times in reproducibility conditions of measurement</td><td>u_{STAB}</td></tr>
<tr><td>Interactions</td><td>u_{IAi}</td></tr>
</table>

NOTE 1 In special circumstances (e.g. high cost of test), two repetitions can be acceptable.

NOTE 2 If the number of samples is smaller than 30, the Student's t-test can be used to expand the extended uncertainty. See Clause 8.

6.2.3.3 Determination of uncertainty components not included in the experiments (Type B)

In addition to the estimated uncertainty components of the measuring system (6.2.2), and the estimated uncertainty components of the measurement process (6.2.3.2), the following additional uncertainty components should be determined using mathematical models.

Table 6 — Other uncertainty on the measurement process

Uncertainty components	Symbol	Test/model
Non-homogeneity of the part	u_{OBJ}	$u_{\mathrm{OBJ}} = \frac{a_{\mathrm{OBJ}}}{\sqrt{3}}$ where a_{OBJ} is the maximum permitted or expected error due to the object (e.g. form deviation).
Temperature	u_{T}	The influence from temperature can be calculated using the formula: $u_{\mathrm{T}} = \sqrt{u_{\mathrm{TD}}^2 + u_{\mathrm{TA}}^2}$ The uncertainty from temperature differences u_{TD} could e.g. be estimated in compliance with ISO 14253-2. $u_{\mathrm{TD}} = \frac{\Delta T \cdot \alpha \cdot l}{\sqrt{3}}$ where α is the expansion coefficient; ΔT is the difference in temperatures; and a rectangular distribution is assumed. The uncertainty on expansion coefficients could be estimated in compliance with ISO 15530-3. $u_{\mathrm{TA}} = \frac{\lvert T - 20°C \rvert \cdot u_\alpha \cdot l}{\sqrt{3}}$ where T is the average temperature during the measurement; u_α is the uncertainty on the coefficient of expansion; l is the observed value for length measurement.

NOTE 1 T is temperature in the formula above. It should not be confused with specification interval or target value used elsewhere in this part of ISO 22514.

NOTE 2 In the case that a compensation for temperature difference is not made, a contribution for this difference should be included in the estimation in the formula above.

NOTE 3 The part is the object to be measured, including object measured by embedded devices in production.

6.2.3.4 Impact of the deviation of workpiece on the measurement result

In many measurement processes, the surface of the workpiece is in contact with the measuring system during the measurement. Depending on the surface texture, form deviation and geometrical deviations from the nominal geometry, the contact between the measuring system and the workpiece will result in an uncertainty component. Depending on the measurand and the repartition of the measuring on the workpiece, the impact of the form deviation does not have the same level (if the measurand corresponds to the maximum value, and we take only one measure, then the form deviation impacts directly, but if we turn the workpiece and take the maximum observed value, the form deviation is integrated in the evaluation, and does not impact in measurement uncertainty).

The component a_{OBJ} can be found from requirements on the drawing or by experiments suitable to find the maximum form deviation or similar non-homogeneities.

Add the component u_{OBJ} to the model, as shown in Table 10.

6.2.3.5 Resolution

If the repeatability component using workpieces (u_{EVO}) is greater than that of the resolution component, then the resolution component is included in the repeatability component. If not, then the component u_{RE} should be added to the model as shown in Table 1.

6.2.3.6 Temperature influence

6.2.3.6.1 Uncertainty calculation

The uncertainty from temperature influence u_{T} should be calculated based on the uncertainty component caused by temperature difference and uncertainty from unknown expansion coefficients.

$$u_{\mathrm{T}} = \sqrt{u_{\mathrm{TD}}^2 + u_{\mathrm{TA}}^2}$$

6.2.3.6.2 Uncertainty component caused by temperature differences and expansion

The standard reference temperature for geometrical product specifications (GPS) and GPS measurements is 20 °C (see ISO 1). There may be reference temperatures for applications other than geometrical (e.g. electrical influences from temperature) that may be caused by absolute temperature as well as time and spatial temperature gradients result in linear expansion, bending, etc., of the measuring system. The measurement setup and the object being measured cause an uncertainty component u_{TD}.

The transformation from temperature to length is given by the linear expansion equation:

$$\Delta L = \Delta T \cdot \alpha \cdot l$$

where

ΔT is the relevant temperature difference;

α is the temperature expansion coefficient of the material;

l is the effective length under consideration.

A known deviation in temperature from the reference temperature can be corrected as a systematic error component if appropriate.

The uncertainty u_{TD} can, for example, be estimated in accordance with ISO 14253-2.

6.2.3.6.3 Uncertainty on the coefficient of expansion

An uncertainty contribution from the variation of the expansion coefficient of the measured workpieces will often be present. In this case, the uncertainty u_{TA} is calculated by:

$$u_{\mathrm{TA}} = \frac{|T - 20\ ^\circ\mathrm{C}| \cdot u_\alpha \cdot l}{\sqrt{3}}$$

where u_α is the standard uncertainty of the expansion coefficient of the workpieces.

Alternatively, the uncertainty u_{TA} can be estimated in accordance with ISO 15530-3.

7 Studies for calculating the uncertainty components

7.1 Measuring system

7.1.1 General

In order for a study to provide meaningful information, it is a prerequisite that the resolution of the measuring system be determined and adequate for the actual measurement process.

It should be confirmed that the standard uncertainty from repeatability is not smaller than the standard uncertainty from the resolution. Otherwise the uncertainty from the resolution should be used instead of the repeatability (max{u_{EVR},u_{EVO},u_{RE}}).

The method applied is based on knowledge to the linearity of actual measuring system. If the linearity is to be regarded as known, the repeatability and bias can be found using one (or more) standard(s).

7.1.2 Repeatability and bias based on one reference standard

7.1.2.1 General

If the uncertainty component u_{LIN} is equal to zero or estimated from the maximum permissible error (M_{PE}), the component u_{EVR} should be determined experimentally. The determination of the uncertainty u_{EVR} comes from the repeatability estimated from measurements on a reference standard or workpiece. It should be based on the spread of a minimum of 30 repeated measurements, to estimate the combined effect of bias and repeatability. In this case, the bias and the variation will be used together as two different uncertainty components u_{BI} and u_{EVR}.

7.1.2.2 Preconditions

— The reference quantity value of the reference standard or workpiece should have a quantity value close to the target value. The maximum deviation of the reference standard from target value depends on the characteristics of the measuring system.

— The reference quantity value x_m of the reference standard or workpiece should be determined (normally by calibration).

— The reference standard or workpiece shall be removed and replaced between each measurement.

— In the case of physically one-sided tolerance (“natural limit”), the reference quantity value of the reference standard or workpiece should have a quantity value close to the specification value.

7.1.2.3 Procedure

Take at least 30 measurements on the reference standard or calibrated workpiece.

Based on the actual values, the measurement bias (B_i), the standard uncertainty of repeatability from reference standard and the standard uncertainty of the bias are estimated from:

$$u_{EVR} = s = \sqrt{\frac{1}{K-1} \cdot \sum_{i=1}^{K} (x_i - \bar{x}_g)^2} \quad \text{and} \quad B_i = \left| \bar{x}_g - x_m \right|$$

where

K is the number of repeated measurements;

x_i is the single value of the ith measurement;

$\overline{x}_g$ is the arithmetic mean of all the sample values.

$$u_{\mathrm{BI}} = \frac{B_{\mathrm{i}}}{\sqrt{3}} = \frac{\left|\overline{x}_g - x_{\mathrm{m}}\right|}{\sqrt{3}}$$

This formula can only be used in cases, where we cannot distinguish between systematic and random errors.

As long as a zero setting of the measuring equipment can cause extra variation, it is important to set zero on the measuring system using the defined standard or workpiece between each attempt.

If more than one standard is used in the experiment to determine the repeatability, the largest mean deviation from the respective standard should be used as the bias value. If the variance is assumed to be constant, the average variance should be used.

7.1.3 Linearity analysis based on a minimum of three reference standards

7.1.3.1 Calculations if linearity deviations are present

In 6.2.2.3, the following experiment (see ISO 11095) is used to determine the uncertainty from deviations from linearity of the measuring system. If linearity deviations are present, estimates of the uncertainty components u_{LIN} (linearity uncertainty) and u_{EVR} (repeatability on a standard) should be calculated based on the following method.

1) On at least three reference standards, perform at least three repeated measurements. The minimum sample size is 30.
2) Perform a regression analysis. Observe that the residual standard deviation is constant over the spread of measurement. The residual standard deviation is later used in the estimation of the uncertainty.
3) Perform an analysis of variance (ANOVA).
4) Estimate the uncertainty components u_{EVR} and u_{LIN} based on the results of the ANOVA in item 3 above.
5) Correct the measurement results on future measurements according to the calculated linearity, where appropriate.

7.1.3.2 Preconditions

Generally, the following preconditions apply.

— The residual standard deviation (standard deviation from repeated measurements on the standards) is always constant (see Table 9).

— The regression function is linear (regression line).

— The uncertainties about the “true” values of the reference standards are small compared to the size of the deviations of the measurements of the standard.

— The measurements are representative of the future use of the measuring system regarding the environment and other conditions.

— The repeated measurements of the reference standards are independent from each other and are normally distributed.

— The values of the standards are approximately equidistantly placed throughout the relevant measurement range.

7.1.3.3 Conditions

The conditions of the method are described explicitly below.

A regression line is displayed based on the measured values using the graphical display as shown in Figure 2. This gives the first impression of:

1) whether the measurement process is under control during the experiment,

2) the appropriateness of the preconditions (e.g. linearity, residual standard deviation constant),

3) the measurement values compared to the conventional “true” value, and

4) the presence of outliers and temporal trends that need further investigation.

7.1.3.4 Example of linearity analysis

The formula for the regression line is:

$$y_{ij} = \beta_0 + \beta_1 \cdot x_i + \varepsilon_{ij}$$

Table 7 — Measured data

	Observations on standards														
i	x_m	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	$\bar{y}_i$	s
1	2,0	2,7	2,5	2,4	2,5	2,7	2,3	2,5	2,5	2,4	2,4	2,6	2,4	2,49	0,12
2	4,0	5,1	3,9	4,2	5,0	3,8	3,9	3,9	3,9	3,9	4,0	4,1	3,8	4,13	0,45
3	6,0	5,8	5,7	5,9	5,9	6,0	6,1	6,0	6,1	6,4	6,3	6,0	6,1	6,03	0,20
4	8,0	7,6	7,7	7,8	7,7	7,8	7,8	7,8	7,7	7,8	7,5	7,6	7,7	7,71	0,10
5	10,0	9,1	9,3	9,5	9,3	9,4	9,5	9,5	9,5	9,6	9,2	9,3	9,4	9,38	0,15

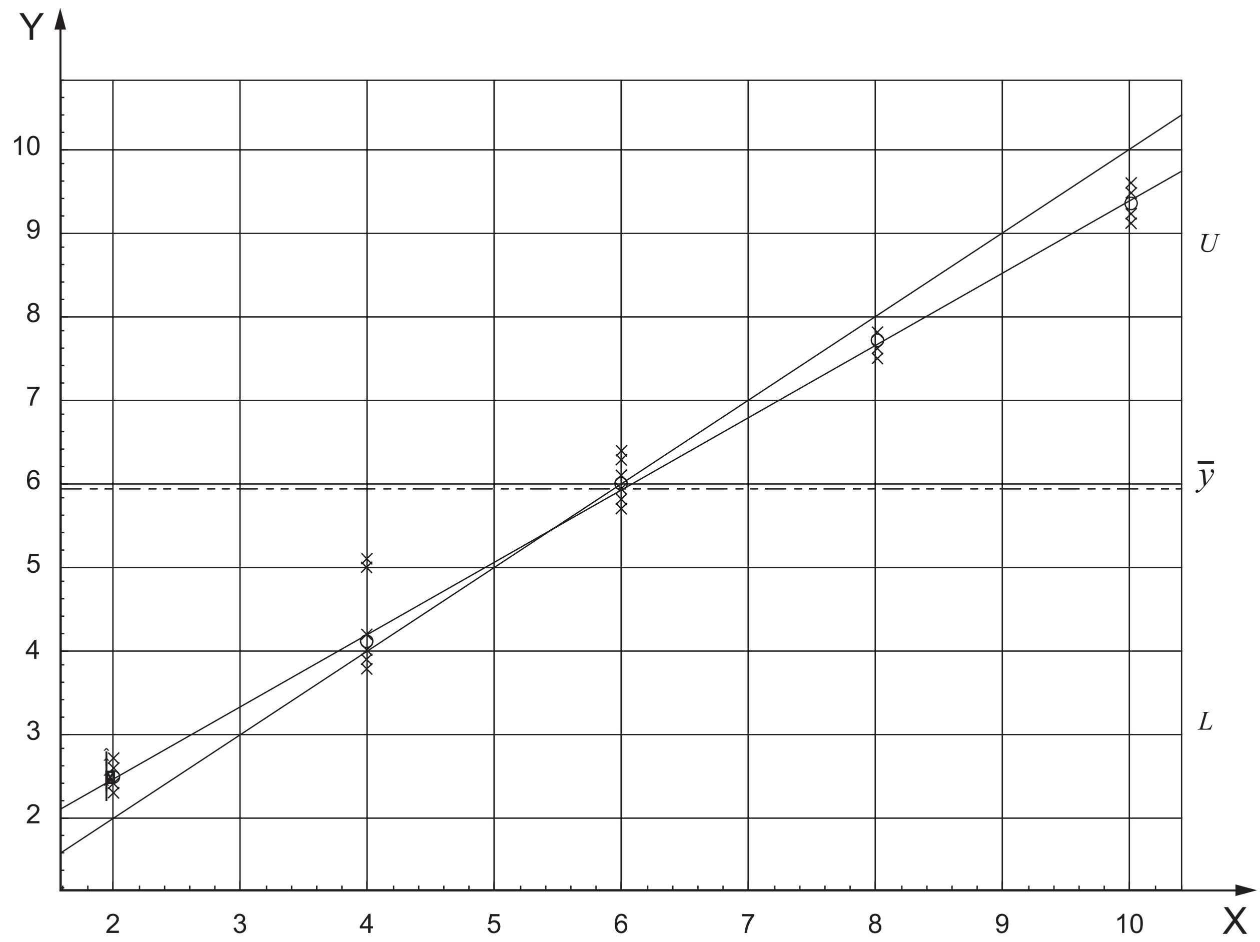
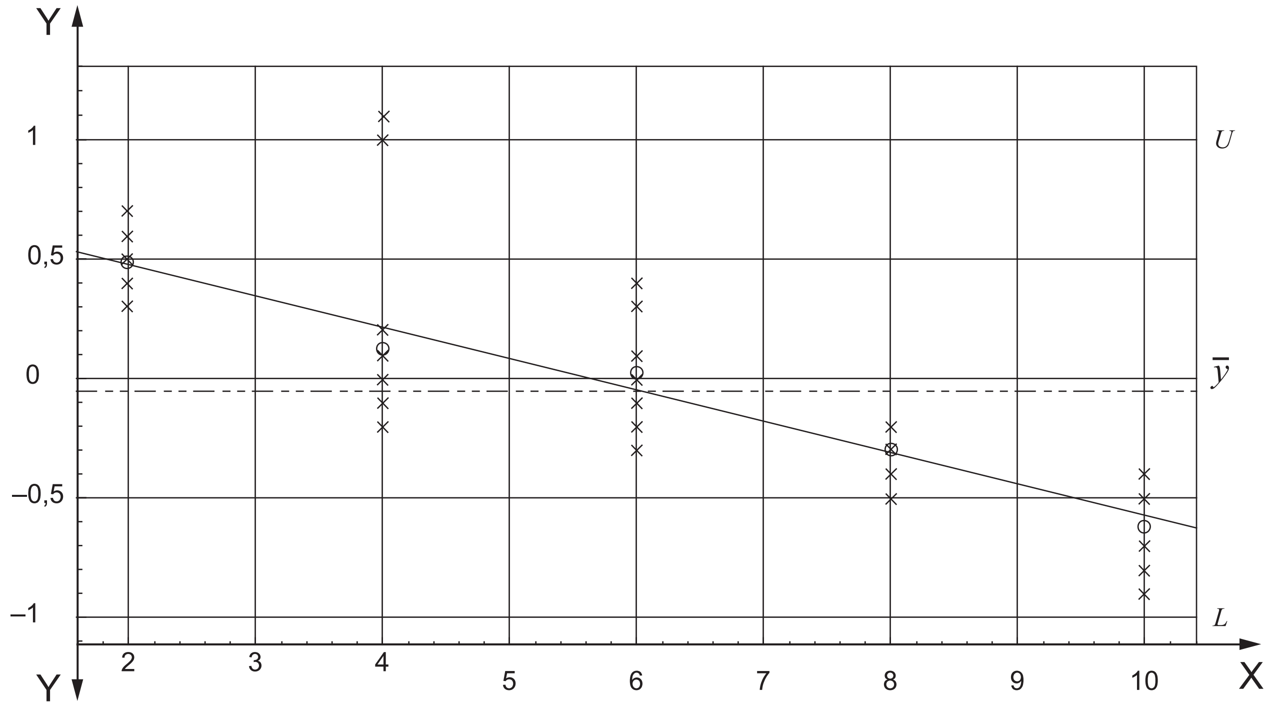


Figure 2 — Graphical display of linearity



Key

X diameter (reference)

Y bias (value-reference)

Figure 3 — The identity line equal to $\bar{y}$ minus regression line

If one or more outliers are present, the experiment should be repeated after the elimination of detected outliers.

The user may consult ISO 16269-4 for advice on how to detect outliers. Failure to detect the presence of outliers will result in an incorrect correction. Information on how to calculate uncertainty components and the regression function can be found in Table A.3.

7.1.4 Estimation on the uncertainty components

The calculation of estimates from the uncertainties due to the lack of fit of the regression function, u_{LIN} and the repeatability of the measurement on the standards (pure error) u_{EVR} should be found in the analysis of variance Table A.5.

Table 8 — Uncertainty from linearity

Uncertainty components	Symbol	Test/model
Linearity	u_{LIN}	$y_{ij} = \beta_0 + \beta_1 \cdot x_i + \varepsilon_{ij}$ $\hat{y} = 0{,}736\,7 - 0{,}131\,7x$ Linearity = 0,58 mm (at the upper specification limit x = 10 mm) $u_{\mathrm{LIN}} = \dfrac{0{,}58}{\sqrt{3}}$

7.2 Uncertainty components of the measurement process

7.2.1 General

Components of the measurement process carried out under real conditions should be added to the estimated uncertainty components of the measuring system, calculated in 6.2.2.

In 6.2.3.2, a standard experiment (precision experiment) to estimate the uncertainty components u_{EVO}, u_{AV}, u_{GV}, and u_{IAi} are defined.

7.2.2 Uncertainty components from analysis of variance

The repeatability and reproducibility analysis provides independent estimates of the repeatability and reproducibility of the measurement process.

The analysis should be based on a minimum of 5 workpieces, with either

1) a minimum of 3 operators with a minimum of 2 repeated measurements, or

2) a minimum of 2 operators with a minimum of 3 repeated measurements.

Alternatively, if in cases where only one operator is using different measuring systems, item 2 should be replaced by using a minimum of two different measuring systems so that an estimate of the reproducibility of the system can be made.

In total, there should be a minimum sample size of 30 measurements.

Decompose the variances and estimates of the uncertainty components including the interaction between these and the variances. When estimating the uncertainty components there should be a distinction between several different situations. See the decomposition in Table A.4.

Other uncertainty components (e.g. stability u_{STAB}) can be added to an extended ANOVA model. In this case, the experiment should be extended appropriately, provided that certain interactions can be excluded from the experiment, an appropriate experimental plan to limit the experimental effort can be used.

Further examples on the analysis can be found in ISO/TR 12888.

8 Calculation of combined uncertainty

8.1 General

The combined uncertainty of the measuring system and the measurement process is to be calculated as given in Table 9. The calculation can only be carried out in the given way if there is no correlation between the components. Further information about the calculation can be found in ISO/IEC Guide 98-3:2008 (Clause 5).

Table 9 — Calculation of the uncertainty

Uncertainty components	Symbol	Combined measurement uncertainty
Calibration of the standard or workpiece	u_{CAL}	$u_{MS} = \sqrt{u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EV}^2 + u_{MS-REST}^2}$ where $u_{EV} = \max\{u_{EVR}, u_{RE}\}$
Deviations from linearity	u_{LIN}	
Bias	u_{BI}	
Repeatability on standards	u_{EVR}	
Resolution	u_{RE}	
Other uncertainty components (measuring system)	$u_{MS-REST}$	
Repeatability on workpiece	u_{EVO}	$u_{MP} = \left\{ \begin{array}{l} u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EV}^2 + u_{MS-REST}^2 + u_{AV}^2 + u_{GV}^2 \\ + u_{STAB}^2 + u_{OBJ}^2 + u_T^2 + u_{REST}^2 + \sum_i u_{IAi}^2 \end{array} \right\}^{\wedge 0,5}$ where $u_{EV} = \max\{u_{EVR}, u_{EVO}, u_{RE}\}$
Reproducibility of operator	u_{AV}	
Reproducibility of the measuring system (Different locations of the measurement process)	u_{GV}	
Reproducibility over time	u_{STAB}	
Interactions	u_{IAi}	
Inhomogeneity of measurand	u_{OBJ}	
Temperature	u_T	
Other uncertainty components (measurement process)	u_{REST}	

The combined standard uncertainty of the measuring system can be estimated using the formula:

$$u_{MS} = \sqrt{u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EV}^2 + u_{MS-REST}^2}$$

where

$$u_{EV} = \max\{u_{EVR}, u_{RE}\}$$

In a similar way, the combined standard uncertainty of the measurement process can be estimated using the formula:

$$u_{MP} = \sqrt{u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EV}^2 + u_{MS-REST}^2 + u_{AV}^2 + u_{GV}^2 + u_{STAB}^2 + u_{OBJ}^2 + u_T^2 + u_{REST}^2 + \sum_i u_{IAi}^2}$$

where

$$u_{EV} = \max\{u_{EVR}, u_{EVO}, u_{RE}\}$$

8.2 Calculation of expanded uncertainty

We can find the expanded U_{MS} from the standard uncertainty u_{MS} by multiplying the uncertainty by the coverage factor k.

$$U_{MS} = k \cdot u_{MS}$$

The same method is used to find the expanded U_{MP} from the standard uncertainty u_{MP}

$$U_{MP} = k \cdot u_{MP}$$

Calculation of the expanded uncertainty is based on an approximate 95 % confidence interval; therefore, the coverage factor $k = 2$ is used.

NOTE If the sample size n is smaller than the preferred 30, it is necessary to use Student's t distribution instead of the standard normal distribution to estimate the uncertainty components. This will then result in the expanded measurement uncertainty:

$$U = t_{1-(\alpha/2)}(v) \cdot u$$

The number of degrees of freedom v is obtained from the product of the number of workpieces, the number of operators, the number of gauges and the reduction in the value 1 of the number of repeatability measurements minus 1 ($n \cdot p \cdot (k-1)$).

EXAMPLE 1 3 workpieces, 2 operators, 2 gauges and 3 repeated measurements:

For $v = 3 \cdot 2 \cdot 2 \cdot (3-1) = 24$, one will find $t_{1-(\alpha/2)}(24) = 2{,}11$

EXAMPLE 2 3 workpieces, 2 operators, 2 gauges and 2 repeated measurements:

For $v = 3 \cdot 2 \cdot 2 \cdot (2-1) = 12$, one will find $t_{1-(\alpha/2)}(12) = 2{,}23$

9 Capability

9.1 Performance ratios

9.1.1 General

The capability of a measurement process can be calculated either as a performance ratio or a capability index. Calculating of indices is preferred.

To assess the measuring system or the measurement process, the performance ratio (Q_{MS} or Q_{MP}) is to be calculated based on the measurement uncertainties given in Clause 8. According to Clause 8, a distinction is made between the performance ratios for the measuring system (Q_{MS}) and the measurement process (Q_{MP}).

It is recommended that Q_{MS} does not exceed 15 % and Q_{MP} does not exceed 30 % (by common practice).

The 95 % confidence interval should be calculated for the uncertainty of the calculated ratios.

The process spread (99,73 % interval of the production process) can be used as an alternative reference figure, when the measurement process is used as a part of SPC (statistical process control) system.

9.1.2 Performance ratio of the measuring system

$$Q_{MS} = \frac{2 \cdot U_{MS}}{U - L} \cdot 100 \text{ (\%)}$$

The formula is based on the specification as reference.

9.1.3 Performance ratio of the measurement process

$$Q_{\text{MP}} = \frac{2 \cdot U_{\text{MP}}}{U - L} \cdot 100\ (\%)$$

The formula is based on the specification as reference.

9.2 Capability indices

The two capability indices (for system and process) can be calculated based on the general definition of a capability index which can be found in ISO 3534-2:2006, 2.7.

The capability of a measuring system can be expressed as a capability index called C_{MS}.

$$C_{\text{MS}} = \frac{0{,}3 \cdot (U - L)}{6\hat{u}_{\text{MS}}}$$

The capability of a measurement process can be expressed as a capability index called C_{MP}.

$$C_{\text{MP}} = \frac{0{,}3 \cdot (U - L)}{3\hat{u}_{\text{MP}}}$$

It is recommended that C_{MS} and C_{MP} exceed 1,33.

10 Capability of the measurement process compared to the capability of the production process

10.1 Relation between observed process capability and measurement capability ratio

There is the following connection between an observed process capability or process performance ($C_{\text{p; obs}}$ $P_{\text{p; obs}}$), the actual real process capability or performance ($C_{\text{p;p,}}$ $P_{\text{p;p}}$) and the capability ratio (Q_{MP}) of the measurement process:

$$C_{\text{p;p}} = \left(\frac{1}{C_{\text{P;obs}}^2} - 2{,}25 \cdot Q_{\text{MP}}^2 \right)^{-0{,}5}$$

Details of the derivation of this formula are given in B.4.

The formula is based on the following assumptions.

— Measurements of the manufactured characteristic are normally distributed.

— The production process is normally distributed and in a state of statistical control.

— The calculation of the C_{p} index is based on 99,73 % reference value estimated by 6 standard deviations.

— The observed, empirical standard deviation is:

$$s_{\text{obs}}^2 \sim (\sigma_{\text{P}}^2 + \sigma_{\text{MP}}^2)\chi^2(\nu)$$

where

σ_{P} denotes the standard deviation of the production process;

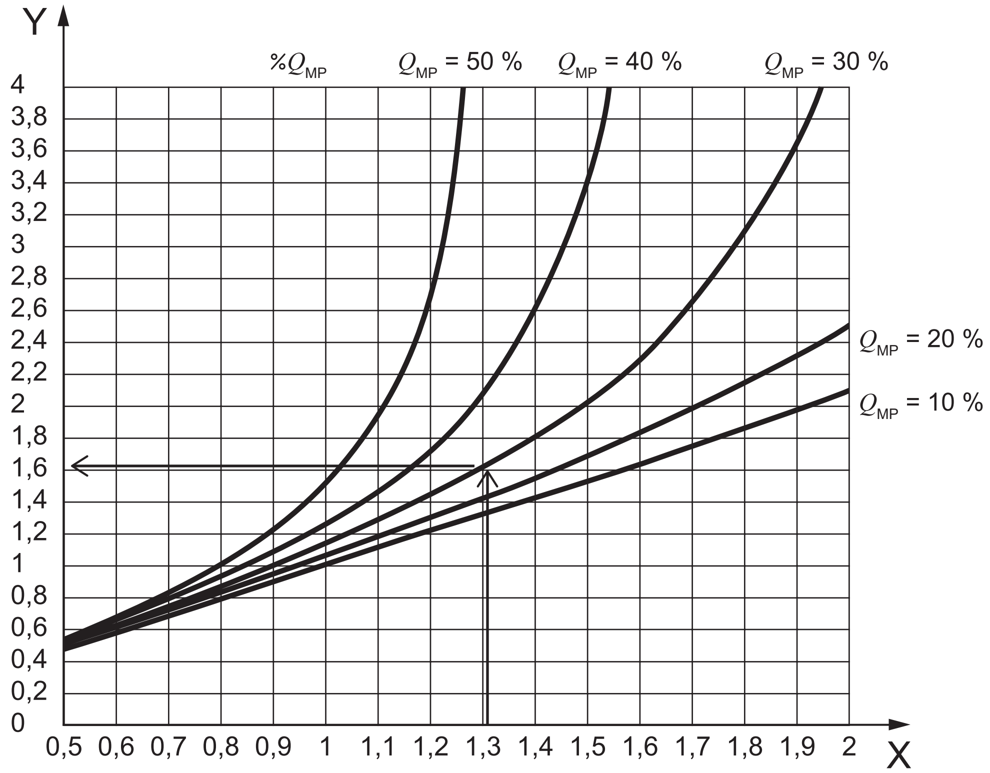
σ_{MP} denotes the standard deviation of the measurement process.

The area of uncertainty regarding the specification limits is symmetrical.

The coverage factor used to calculate the combined uncertainty is 2.

EXAMPLE The formula above, Figure 5 and Tables 10 and 11 below show that a real capability index of 2,21 from an actual production process when the measurement capability figure Q_{MP} = 40 % results in a observed capability index of 1,33.

NOTE This example is about theoretical capability indices. The estimated capabilities are random variables subject to error and an estimated observed capability index in this situation will vary around 1,33 with a variation that depends on the sample size.



Key

X observed C-value

Y real C-value

Figure 4 — Capability index of the process alone as a function of the observed capability index for a range of capability fractions of the measurement process

Table 10 — Observed and real indices

Observed C - value	**Real C – value for the process with…**				
	Q_{MP} = 10 %	Q_{MP} = 20 %	Q_{MP} = 30 %	Q_{MP} = 40 %	Q_{MP} = 50 %
0,67	0,67	0,68	0,70	0,73	0,77
1,00	1,01	1,05	1,12	1,25	1,51
1,33	1,36	1,45	1,66	2,21	18,82
1,67	1,72	1,93	2,53	na	na
2,00	2,10	2,50	4,59	na	na

EXAMPLE A capability value $C_p = 1{,}00$ is calculated based on measurements from a production process and the measurement process has a $Q_{MP} = 30$ %.

$$C_{p;p} = \left(\frac{1}{C_{p;obs}^2} - 2{,}25 \cdot Q_{MP}^2 \right)^{-0{,}5} = \left(\frac{1}{1^2} - 2{,}25 \cdot 0{,}3^2 \right)^{-0{,}5} = 1{,}118\,5$$

10.2 Relation between observed process capability and measurement capability

The relation between the process and measurement capability can also be calculated. The following connection exists between an observed process capability or process performance ($C_{p;\,obs}\ P_{p;\,obs}$), the actual real process capability or performance ($C_{p;p,}\ P_{p;p}$) and the capability index (C_{MP}) of the measurement process:

$$C_{p;p} \frac{1}{\sqrt{1 + (\sigma_{MP} / \sigma_P)^2}}$$

Table 11 — Observed and real indices

Observed C - value	**Real C - value for the process when…**				
	C_{MP} = 2	C_{MP} = 1,66	C_{MP} = 1,33	C_{MP} = 1	C_{MP} = 0,5
0,67	0,67	0,67	0,68	0,68	0,73
1,00	1,01	1,02	1,03	1,05	1,25
1,33	1,36	1,37	1,39	1,45	2,21
1,67	1,72	1,75	1,79	1,93	59
2,00	2,10	2,14	2,24	2,5	Na

11 Ongoing review of the measurement process stability

11.1 Ongoing review of the stability

The short-term as well as the long-term stability has to be taken into account when the capability of the measurement process is calculated. However, a change in bias caused by drift, unintentional damage or new additional uncertainty components, which were not known by the time of calculation of the capability, can change the bias in the measurement process over time. A control chart should be used to determine those possible significant changes in the measurement process.

The following sequence is recommended.

Step 1:

Select an appropriate reference standard or calibrated workpiece with a known value for the test characteristic.

Step 2:

Carry out regular measurement of the reference standard (workpiece).

Step 3:

Plot the measured values on a control chart. The action limits are calculated in accordance with known methods of quality control charting techniques (see ISO 7870-1).

Step 4:

Check for out-of-control. If no out-of-control signal is detected, it is assumed that the measurement process has not changed significantly. If an out-of-control signal is detected, the measurement process is assumed to have changed and shall be reviewed. With this approach, the measurement process is continuously monitored and significant changes can be detected.

NOTE It is important to determine the qualification interval to be taken into account by the calibration of the measuring system.

11.2 Monitoring linearity

If there was doubt about the linearity of the measuring system during the calculation and if a regression function has been experimentally determined, the method given here can be used for the ongoing review of the linearity of the measuring system.

This method can be used for the continuous monitoring of the measurement with a suitable quality control chart (SPC chart). A chart gives a signal when the regression function needs to be updated.

Step 1:

Calculate control limits with statistical figures found in 7.1.3.

Upper/lower control limits:

$$U_{\mathrm{CL}} = \frac{\hat{\sigma}}{\hat{\beta}_1} t_{(1-\varepsilon/2K)}(nK-2)$$
$$L_{\mathrm{CL}} = -\frac{\hat{\sigma}}{\hat{\beta}_1} t_{(1-\varepsilon/2K)}(nK-2)$$

Step 2:

Select the K reference standards. The reference standards (minimum 2) must be chosen in a way that their nominal values cover the range of observations that occur during the actual production conditions.

Step 3:

Repeat measurements on the reference standards. For example, the reference standards should be measured every day in a working week.

Step 4:

Transform the p measurement values on the K standards. Transform the p values of the K standards with the help of the regression function:

$$x = \frac{y - \beta_0}{\beta_1}$$

Then, calculate each of the differences between the “true” and the transformed values.

Step 5:

Plot the differences on a control chart.

Step 6:

Decide the validity of the regression function. This decision will depend on whether all the differences of all standards are within the control limits. Apply all of the appropriate SPC rules as described in ISO 7870-1.

12 Capability of attribute measurement processes

12.1 General

Because of the nature of attribute measurements, it is only possible to obtain an outcome that is either conforming or not. To establish the capability of the measurement process, a large number of measurements are required.

A suitable approach for calculating the capability of attribute measurement processes must take into account that the probability of a particular test result is dependent on the characteristic type. For example, the probability of a correct test result is nearly 100 % of actual measured values that lie outside the uncertainty limits. For information about the specification limits, see Figure 5. On the other hand, the probability is approximately 50 % if the measurement results lie in the middle of the uncertainty range ("Decision by pure chance"). The uncertainty zone should, as a rule of thumb, not exceed 20 %.

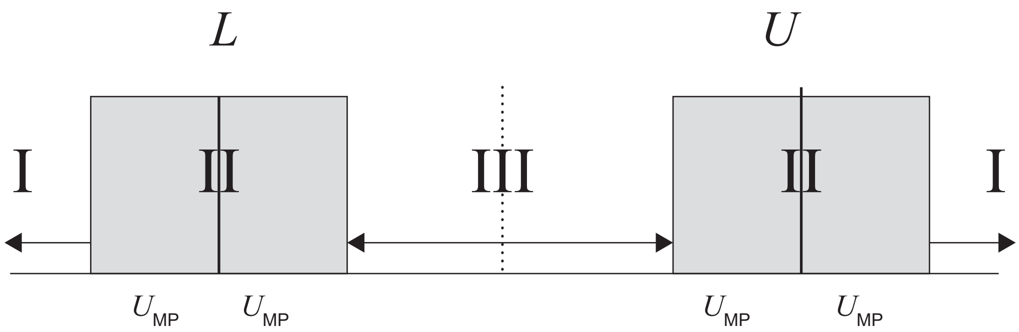


Figure 5 — Uncertainty range (II)

In principle, the proposed approach makes a distinction between calculation of measurement capability (cross-tab method), without or with reference values (signal detection approach). If reference values are available, a two-step approach is proposed.

12.2 Capability calculations without using reference values

When a calculation of measurement capability has to be done without using reference values, only a test of whether or not there are significant differences between operators can be made. However, an assessment of whether or not the test has led to the correct result cannot be taken. This fact must always be considered when no reference values are present.

The choice of test parts may have a decisive influence on the outcome of this test method. It cannot be taken into account in this case. At least a proportion (e.g. 40 %) of the test parts should be in the uncertainty range (zone II in Figure 5).

The following standard experiment is proposed.

At least 40 different test parts should be tested 3 times by 2 different operators, called A and B. Each of the 120 different measurement results on the 40 parts, which the operator A or operator B has achieved, is assigned to one of the following three classes.

— Class 1: all three test results on the same part gave the result "good".

— Class 2: the three test results on the same part gave different results.

— Class 3: all three test results on the same part gave the result "bad".

An example of a test result is summarized in Table 12.

Table 12 — Test result from an attribute measurement process

Frequency n_{ij}		Operator B		
		Class 1 Result “+++”	Class 2 Different results	Class 3 Result “- - -”
Operator A	Class 1 Result “+++”	7	3	1
	Class 2 Different results	10	4	7
	Class 3 Result “- - -”	2	1	5

The two operators in Table 12 can now be tested using a Bowker-Test of symmetry. If there are no significant differences between operators, the resulting frequencies in Table 12 will be sufficiently symmetrical with respect to main diagonal.

The hypothesis H_0: $mij = mji$ ($i, j = 1, \ldots, 3$ with $i \neq j$) says that the frequencies mij and mji which lies symmetrical with respect to the main diagonal are identical.

Test statistic $\S^{2} = \sum_{i>ju} \frac{(n_{ij} - n_{ji})^2}{n_{ij} + n_{ji}} = \frac{(10-3)^2}{10+3} + \frac{(2-1)^2}{2+1} + \frac{(1-7)^2}{1+7} = 8{,}603$ is compared to $1-\alpha$ fractile in the χ^2-distribution with 3 degrees of freedom.

The null hypothesis test states that changes from one category to another are random in nature. The hypothesis on symmetry is rejected on the level if the test value is greater than the $1-\alpha$ fractile in the χ^2-distribution with 3 degrees of freedom. In this case, the hypothesis is rejected because the calculated value 8,603 is greater than the value 7,815 which is the 95 % fractile of the χ^2 (3) distribution.

In principle, this method is also to be used with more than two operators. In such cases, each operator makes three tests on the measured object and subsequently, all combinations of two combinations of operators should be tested individually.

NOTE In this case, the significance level is changed for the overall statements by these multiple tests.

12.3 Capability calculations using reference values

12.3.1 Calculation of the uncertainty range

This method is based on signal detections and therefore requires workpieces with known reference values. To address the area of risk around the specification limits, about 25 % of the workpieces should be at or close to the lower specification limit and 25 % of the workpieces at the upper specification limit.

The purpose of this method is to determine the uncertainty range, in which an operator is unable to make an unambiguous decision. Figure 6 illustrates the test results of an attribute measurement process obtained from a set of reference values.

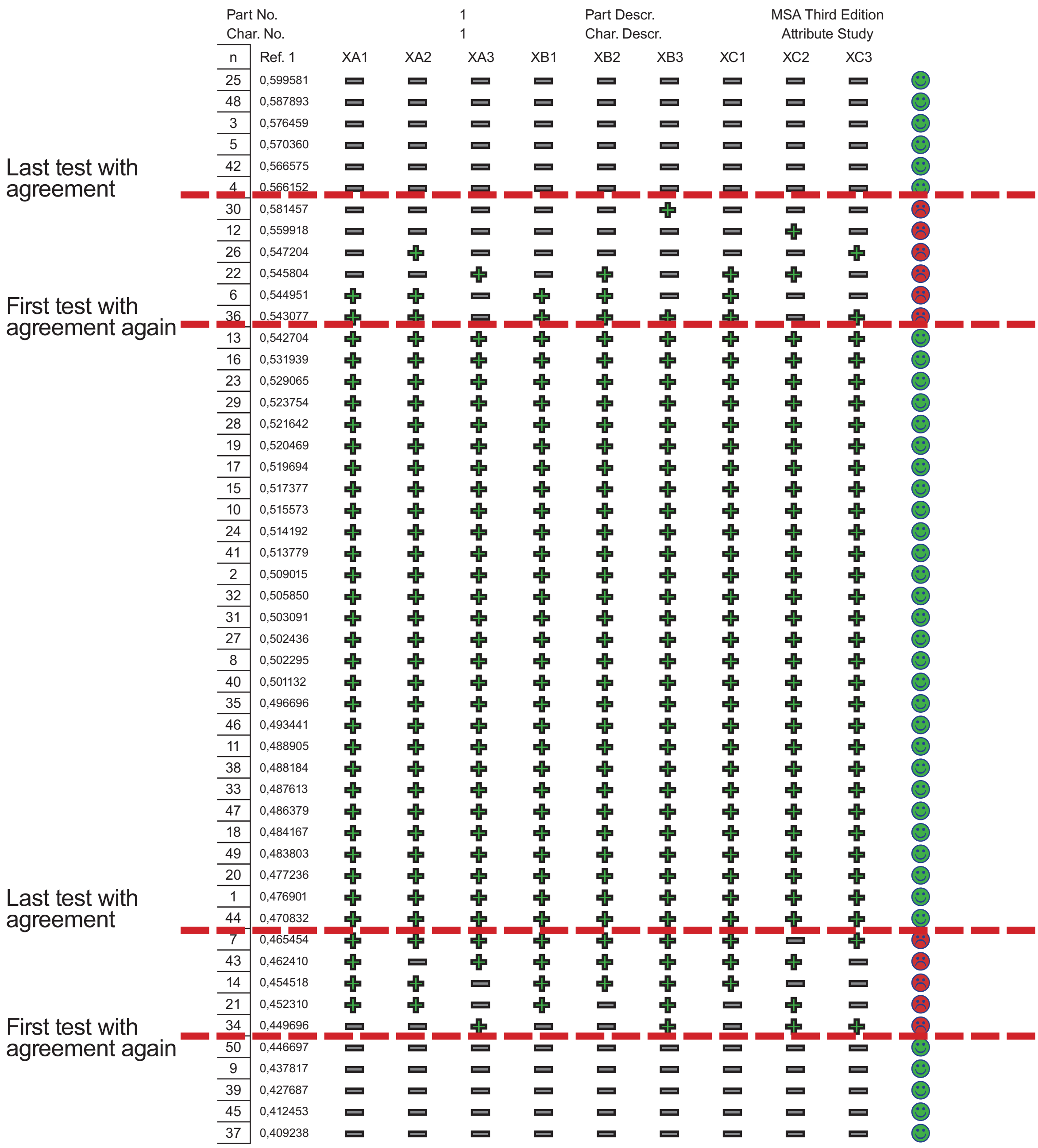


Figure 6 — Test result of an attribute measurement process

12.3.2 Symbols

In Figure 6, the reference measurement values are introduced in the form of a code. A green plus sign means that the operator has indicated the result from the test piece as approved. A grey minus sign means that the operator has indicated the result from the test piece as not approved.

A green smiley means that all three operators have indicated the result from the test piece as approved or rejected in all three tests, and that this assessment is consistent with the reference value.

A red smiley indicates a case where at least one of the operators has come to a test result which is not consistent with the reference value.

12.3.3 Working steps for determining the uncertainty range

Step 1:

Sort the table according to the measured reference size. In Figure 6, a sorting in descending order is made - from the highest reference value descending to the lowest reference value.

Step 2:

Select the last reference value for which all operators have assessed all the results as being unsatisfactory (not approved). This is the transition from symbol "–" to symbol "+".

0,566 152	–
0,561 457	X

Step 3:

Select the first reference value for which all operators the first time assessed all results being approved. This is the transition from symbol "X" to the symbol "+".

0,543 077	X
0,542 704	+

Step 4:

Select the last reference value for which all operators last time assessed all the results as being approved. This is the transition from the "+" symbol to the symbol "X".

0,470 832	+
0,465 454	X

Step 5:

Select the first reference value for which every operator has again first assessed all the results as unsatisfactory (not approved). This is the transition from symbol "X" to the symbol "–".

0,449 696	X
0,446 697	–

Step 6:

Calculate the d_{UR} interval from the last reference value, for which all operators have assessed the result as unsatisfied (not approved) to the first reference value, for which all operators have the result as approved.

$$d_{\mathrm{UR}} = 0{,}566\ 152 - 0{,}542\ 704 = 0{,}023\ 448$$

Step 7:

Calculate the d_{LR} interval from the last reference value, for which all operators have assessed the result as approved to the first reference value, and for which all operators have the result as unsatisfied (not approved).

$$d_{\mathrm{LR}} = 0{,}470\ 832 - 0{,}446\ 697 = 0{,}024\ 135$$

Step 8:

Calculate the average "d" of the two intervals:

$d = (d_{\mathrm{UR}} + d_{\mathrm{LR}})/2 = 0{,}023\ 448 + 0{,}024\ 135 = 0{,}023\ 791\ 5$

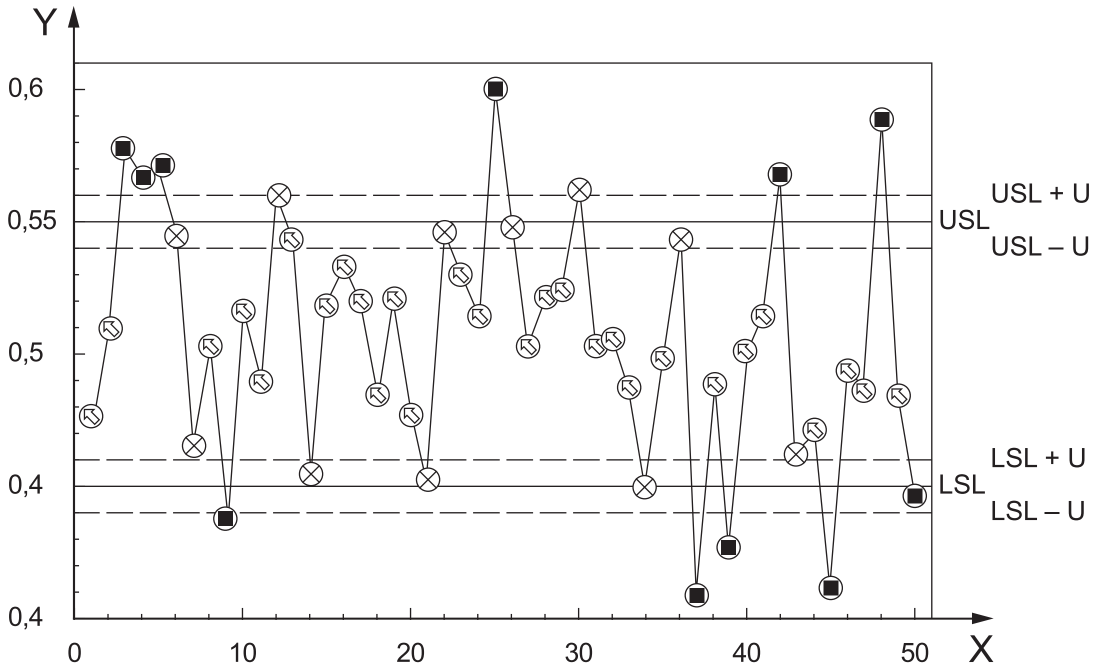
Step 9:

Calculate the uncertainty range:

$U_{\mathrm{attr}} = d/2 = (0{,}023\ 791\ 5)/2$ and

$Q_{\mathrm{attr}} = 2 \cdot U_{\mathrm{attr}}/(U - L) = 2 \cdot [(0{,}023\ 791\ 5)/2]/0{,}1 = 0{,}24$ where $U - L = 0{,}1$ mm

Thus, we find $Q_{\mathrm{attr}} = 24$ %.



Key

X reference number

Y attribute study (mm)

Figure 7 —Value chart

Figure 7 shows another way of representing the measurement capability of all test results, all the reference values and the uncertainty range. Some practitioners may prefer this display.

NOTE The effort for this method is considerable as, in this example, in addition to the 50 reference measurements, at least 450 other test measurements have to be made and documented.

For the selection of workpieces, it must be presumed that the uncertainty region will be covered (see Figure 6).

12.4 Ongoing review

Because of the fact that the measuring system can change e.g. caused by wear, it is necessary to periodically conduct a review of the system.

For ongoing monitoring of the measurement process, at least one operator should measure at least three workpieces all with defined reference values. The workpieces should be selected in a way that the reference values are located outside the uncertainty ranges so that a clear result can be expected (all tests are consistent with the reference value; see Figure 8, e.g. a workpiece in zone I (lower), a workpiece in zone III and a workpiece in zone I (upper).

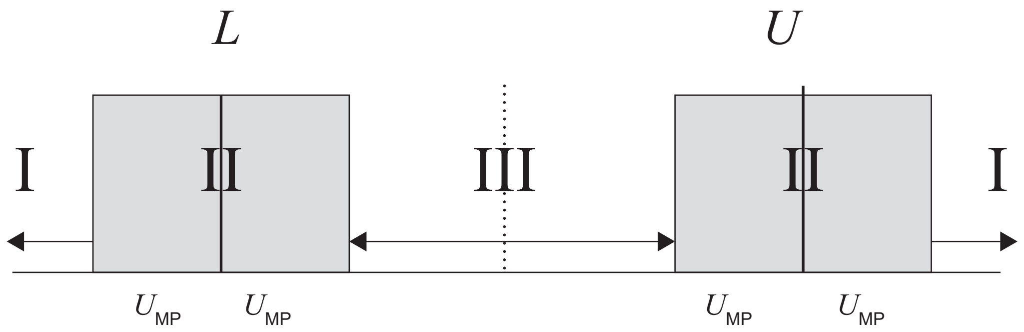


Figure 8 — Uncertainty range

The test is accepted if all three test results are consistent with the reference value. If this is not the case, the measuring system should not be used until it has been corrected or changed.

The size of the uncertainty range can either be determined experimentally (see Clause 11), or derived from the actual defined requirements for an appropriate measurement process (Q).

$$U_{\mathrm{MP;max}} = Q_{\mathrm{MP}} \cdot (U - L)/2$$

Take into account that the extended uncertainty is usually given to be the 95 % level. In this test, it is not calculated.

Use binomial distribution to calculate the confidence interval.

Annex A
(informative)

Examples

A.1 Example of linearity study with at least three standards

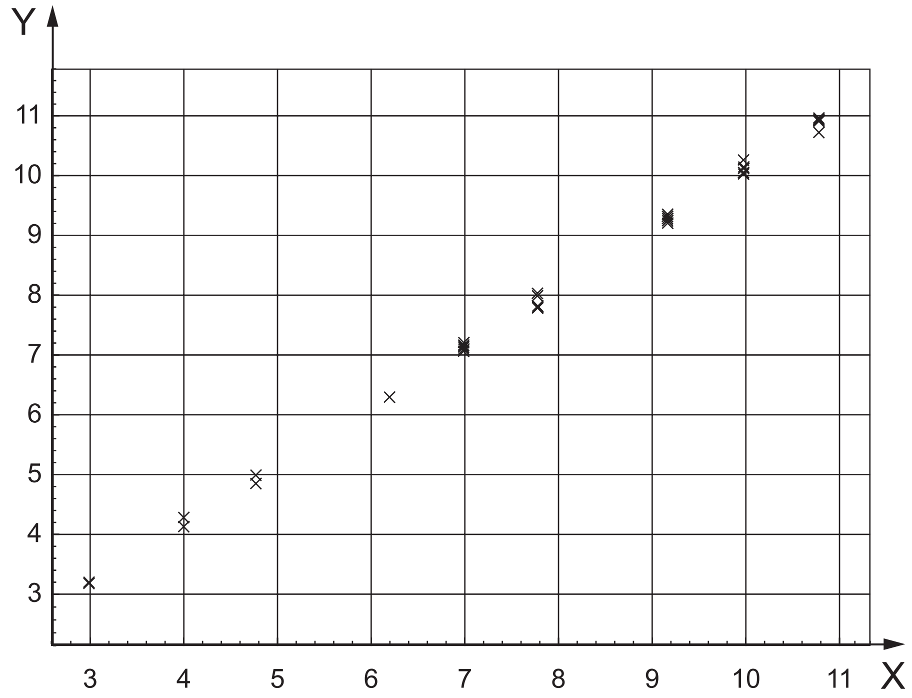
A.1.1 General

This example has been taken from ISO 11095. The example describes an experiment carried out on an imaging system (an optical microscope with a measuring device). The data are measured values and true values of intervals in the range of 0,5 microns to 12 microns. It is assumed according to the calibration certificate that the calibration uncertainty u_{CAL} is 0,005 µm.

Table A.1 — Values from repeated measurements on reference materials

Conventional true values x_n of the 10 reference materials	Values y_{nj} from K = 4 repeatability measurements on N = 10 reference materials			
	y_{n1}	y_{n2}	y_{n3}	y_{n4}
6,19	6,31	6,27	6,31	6,28
9,17	9,27	9,21	9,34	9,23
1,99	2,21	2,19	2,22	2,20
7,77	8,00	7,81	7,95	7,84
4,00	4,27	4,15	4,15	4,15
10,77	10,93	10,73	10,92	10,89
4,78	4,95	4,87	5,00	5,00
2,99	3,24	3,17	3,21	3,21
6,98	7,14	7,07	7,18	7,20
9,98	10,23	10,02	10,07	10,17

Data in Table A.1 are plotted in Figure A.1.



Key

X reference (mm)

Y measured value (mm)

Figure A.1 – Plot of measured and true values

A.1.2 Estimations of regression function parameters:

Given values:

$N = 10$ Number of standards

$K = 4$ Number of repeatability measurements

Calculated values:

$\bar{x} = 6{,}462$ Arithmetic mean of true values

$\bar{y} = 6{,}614$ Arithmetic mean of measured values

Estimated parameters:

$\hat{\beta}_0 = 0{,}235\ 8$ y-axis intercept

$\hat{\beta}_1 = 0{,}987\ 0$ Slope

Regression function $\hat{y}_n = 0{,}235\ 8 + 0{,}987 x_n$

Residual $e_{nj} = y_{nj} - \hat{y}_n = y_{nj} - (0{,}235\ 8 + 0{,}987 x_n)$

Table A.2 — Calculation of residuals

True values x_n of the 10 reference materials	Estimated values $\hat{y}_n$	Residuals			
		e_{n1}	e_{n2}	e_{n3}	e_{n4}
6,19	6,345 5	–0,035 5	–0,075 5	–0,035 5	–0,065 5
9,17	9,286 9	–0,016 9	–0,076 9	0,053 1	–0,056 9
1,99	2,200 0	0,010 0	–0,010 0	0,020 0	0,000 0
7,77	7,905 0	0,095 0	–0,095 0	0,045 0	–0,065 0
4,00	4,183 9	0,086 1	–0,033 9	–0,033 9	–0,033 9
10,77	10,866 2	0,063 8	–0,136 2	0,053 8	0,023 8
4,78	4,953 8	–0,003 8	–0,083 8	0,046 2	0,046 2
2,99	3,187 0	0,053 0	–0,017 0	0,023 0	0,023 0
6,98	7,125 3	0,014 7	–0,055 3	0,054 7	0,074 7
9,98	10,086 4	0,143 6	–0,066 4	–0,016 4	0,083 6

A.1.3 Estimation of the uncertainty components

Calculation of estimates of the uncertainties due to the lack of adaptation of regression function (lack-of-fit) u_{LIN}, (Table A.3) and from the repeatability of the standards (pure error) u_{EVR}:

Table A.3 — Calculation of variance

Uncertainty component	Degrees of freedom v	Sum of squares S_S	Estimated variance $\hat{\sigma}_i^2$	$u_i = +\sqrt{\hat{\sigma}^2}$	Test statistic F	Critical value F_0
Lack of fit	8	$S_{S\,\mathrm{LIN}}$	0,002 8	0,053 3	0,691 8	2,266 1
Repeatability on standard	30	$S_{S\,\mathrm{EVR}}$	0,004 1	0,064 1		
$S_{S\,\mathrm{E}}$= 0,146 2 (0,146 222 631 4)						
$S_{S\,\mathrm{EVR}}$=0,123 4 (0,123 450 000 0)						
$S_{S\,\mathrm{LIN}}$= 0,022 8 (0,022 772 631 4)						
$F_{0,95}(8,30)$ = 2,266 1						

A.2 Experimental determination of the measurement process uncertainty

In addition to the estimated uncertainty components from the measuring system found in A.1, some additional uncertainty components (u_{EVO}, u_{AV}, u_{IAi}) from the measurement process should be determined by the evaluation of the results from this process under the real conditions. In Table A.4, the following data are collected:

Table A.4 — Results from three operator's measurements on 10 parts

Operator	Part no.	Measurement 1	Measurement 2	Measurement 3
1	1	8,120	8,435	8,480
1	2	7,445	6,815	7,490
1	3	9,965	10,010	9,560
1	4	6,140	5,960	6,365
1	5	5,690	5,600	5,780
1	6	2,855	2,450	2,585
1	7	10,685	10,595	10,775
1	8	6,725	6,275	6,545
1	9	4,970	5,105	5,510
1	10	9,875	10,100	9,875
2	1	8,200	8,290	8,245
2	2	7,300	7,120	7,075
2	3	9,660	9,340	9,250
2	4	6,095	6,185	6,185
2	5	5,080	5,340	5,440
2	6	2,315	2,585	2,315
2	7	10,450	10,840	11,050
2	8	6,240	6,120	6,300
2	9	5,015	5,285	5,150
2	10	10,080	9,800	9,970
3	1	8,525	8,435	8,345
3	2	7,535	7,355	7,085
3	3	9,830	9,695	9,515
3	4	6,140	6,140	6,050
3	5	5,780	5,735	5,555
3	6	2,630	2,360	2,585
3	7	10,865	11,000	11,180
3	8	6,590	6,500	6,725
3	9	5,060	5,195	5,105
3	10	10,190	9,785	9,965

From the measurements in Table A.4, the following analysis of variance table (Table A.5) can be calculated.

Table A.5 — Analysis of variance table

Uncertainty component	Degrees of freedom v	Sum of squares S_S	Mean square M_S	Estimated variance $\hat{\sigma}_i^2$	$u_i = +\sqrt{\hat{\sigma}^2}$	Test statistic F	Critical value F_0 α = 5 %
Operator	2	0,519	0,260	0,007 38	0,085 91	6,810	3,150
Part to part	9	526,9	58,54	6,500	na	1536	2,040
Interaction between operator and part	18	0,686	0,038 1	0,002 05	0,045 28	1,193	1,778
Reproducibility	60	1,917	0,0320	0,032 0	0,178 9	—	—

Since the interaction between operator and part is not significant ($F < F_0$), pooling is used. One can develop a modified variance table such as in Table B.2

Table A.6 — Modified analysis of variance table

Uncertainty component	Degrees of freedom v	Sum of squares S_S	Mean square M_S	Estimated variance $\hat{\sigma}_i^2$	$u_i = +\sqrt{\hat{\sigma}^2}$	Test statistic F	Critical value F_0 α = 5 %
Operator	2	0,519	0,260	0,007 54	0,086 83	7,776	3,150
Part to part	9	526,9	58,54	6,501	na	1754	2,002
Reproducibility	78	2,603	0,033 4	0,033 4	0,182 7	—	—

The uncertainty components of the measurement process are then found:

$u_{AV} = 0{,}086\ 83$

$u_{EVO} = 0{,}182\ 7$

A.3 Determination of the uncertainty components not taken into account by experiments

Determination of the uncertainty components of Type B not included in the experiments in A.1 and A.2.

Uncertainty component caused by resolution u_{R}:

$$u_{RE} = \frac{5 \cdot 10^{-3}}{\sqrt{12}} = 0{,}001\ 44$$

The uncertainty component u_{RE} is smaller than u_{EVR}. Therefore, the component u_{RE} will not be used.

The components

u_{OBJ}

u_{T}

u_{STAB}

u_{REST}

are all set to 0.

A.4 Determination of the combined and expanded uncertainty

The combined uncertainty of the measuring system: $u_{MS} = 0{,}083\ 6$

and the expanded uncertainty: $U_{MS} = 0{,}167\ 2$

The combined uncertainty of the measurement process: $u_{MP} = 0{,}209\ 3$

and the expanded uncertainty: $U_{MP} = 0{,}418\ 5$

A.5 Assessing the capability of the measuring system and measurement process

If the specification is given, $U - L = 11 - 2 = 9$

It will lead to the following capability ratios:

$$\%Q_{\text{MS}} = 3{,}7\ \%$$

$$\%Q_{\text{MP}} = 9{,}3\ \%$$

It will also lead to the following capability indices.

$$C_{\text{MS}} = \frac{0{,}3 \cdot (U - L)}{6u_{\text{MS}}} = \frac{0{,}3 \cdot (11 - 2)}{6 \cdot 0{,}083\,6} = 5{,}38$$

$$C_{\text{MP}} = \frac{0{,}3 \cdot (U - L)}{3u_{\text{MP}}} = \frac{0{,}3 \cdot (11 - 2)}{3 \cdot 0{,}209\,3} = 4{,}30$$

Annex B
(informative)

Statistical methods used

B.1 F-test

In a comparison of two variances, the hypothesis of equality of two variances is rejected if

$$\frac{s_1^2}{s_2^2} < \frac{1}{F_{1-\alpha/2/(\nu_2,\nu_1)}} \text{ or } \frac{s_1^2}{s_2^2} > F_{1-\alpha/2/(\nu_1,\nu_2)}$$

NOTE More information about tests can be found in ISO 2854.

B.2 Estimation of the regression function

The regression model specifies that observations y_y satisfy

$$y_{ij} = \beta_0 + \beta_1 \cdot x_i + \varepsilon_{ij}$$

where

y_{ij}	is the jth of K measurements on the ith of N standards;
x_i	is the conventional true value for the ith standard;
ε_{ij}	are the $N(0,\sigma_r^2)$ distributed deviations of y_{ij} from the expected value;
$\beta_0 + \beta_1 \cdot x_i$	is the mean of the ith standard;
ε	is the residual.

Formulas to estimate the unknown parameter: β_0 and β_1

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N} (x_i - \overline{x}) \cdot (y_i - \overline{y})}{\sum_{i=1}^{N} (x_i - \overline{x})^2}$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \cdot \overline{x}$$

The residuals ε_{ij} can be evaluated based on the estimates on y_{ij}.

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \sum_{j=1}^{k} (e_{ij})^2}{n \cdot k - 2} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{k} (y_{ij} - \hat{y}_i)^2}{n \cdot k - 2}$$

where $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \cdot x_n$

To verify the independence of the measurements, ε_{ij} can be plotted over time and to verify the normality ε_{ij} can be considered in a probability plot.

B.3 ANOVA tables

ANOVA tables to be used for the calculations in Tables A.3, A.5 and A.6.

Table B.1 — Analysis of variance table for Table A.3

Uncertainty component	Degrees of freedom v	Sum of squares S_S	Estimated variance $\hat{\sigma}_i^2$	$u_i = +\sqrt{\hat{\sigma}^2}$	Test statistic F	Critical value F_0
Lack of fit	$n - 2$	$S_{S\,\text{LIN}}$	$\hat{\sigma}_{\text{LIN}}^2 = \dfrac{S_{S\,\text{LIN}}}{n-2}$	$u_{\text{LIN}} = +\sqrt{\sigma_{\text{LIN}}^2}$	$\dfrac{\hat{\sigma}_{\text{LIN}}^2}{\hat{\sigma}_{\text{EVR}}^2}$	$F_{1-\alpha(v_1,v_2)}$
Repeatability on standard	$nk - n$	$S_{S\,\text{EVR}}$	$\hat{\sigma}_{\text{EVR}}^2 = \dfrac{S_{S\,\text{EVR}}}{nk-n}$	$u_{\text{EVR}} = +\sqrt{\sigma_{\text{EVR}}^2}$		

$y_{n\bullet} = \dfrac{1}{n}\sum_{n} y_{ij}$

$S_{SE} = \sum_{i}\sum_{j}(y_{ij} - \hat{y}_n)^2$

$S_{S\,\text{EVR}} = \sum_{i}\sum_{j}(y_{ij} - \hat{y}_{n\bullet})^2$

$S_{S\,\text{LIN}} = S_{S\,\text{E}} - S_{S\,\text{EVR}}$

$v_1 = n - 2$

$v_2 = nk - n$

Table B.2 — Analysis of variance table for Table A.5

Uncertainty component	Degrees of freedom v	Sum of squares S_S	Mean square M_S	Estimated variance $\hat{\sigma}_i^2$	$u_i = +\sqrt{\hat{\sigma}^2}$	Test statistic F
Operator	$N_A - 1$	$S_{S\,Av}$	$M_{S\,AV} = \frac{S_{S\,AV}}{N_A - 1}$	$\frac{M_{S\,AV} - M_{S\,IA}}{N_A N_R}$	$u_{AV} = +\sqrt{\hat{\sigma}^2_{AV}}$	$\frac{M_{S\,AV}}{M_{S\,IA}}$
Part to part	$N_P - 1$	$S_{S\,pv}$	$M_{S\,PV} = \frac{S_{S\,PV}}{N_P - 1}$	$\frac{M_{S\,PV} - M_{S\,IA}}{N_A N_R}$	na	$\frac{M_{S\,PV}}{M_{S\,IA}}$
Interaction	$(N_A - 1)$ $(N_p - 1)$	$S_{S\,IA}$	$M_{S\,IA} = \frac{S_{S\,IA}}{(N_A - 1)(N_P - 1)}$	$\frac{M_{S\,IA} - M_{S\,EVO}}{N_R}$	$u_{IA} = +\sqrt{\hat{\sigma}^2_{IA}}$	$\frac{M_{S\,IA}}{M_{S\,EVO}}$

The critical value operator F_0 α =5 % is $F_{1-\alpha}\left[(N_A - 1,(N_A - 1)(N_P - 1)\right]$

The critical value part to part F_0 α =5 % is $F_{1-\alpha}\left[(N_P - 1,(N_A - 1)(N_P - 1)\right]$

The critical value interaction F_0 α =5 % is $F_{1-\alpha}\left[(N_P - 1),(N_P N_R - 1)\right]$

$$S_{S\,AV} = N_R N_P \sum_i (\bar{y}_{j\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet})^2$$

$$S_{S\,PV} = N_R N_A \sum_j (\bar{y}_{\bullet j \bullet} - \bar{y}_{\bullet\bullet\bullet})^2$$

$$S_{S\,IA} = N_R \sum_i \sum_j (\bar{y}_{ij\bullet} - \bar{y}_{j\bullet\bullet} - \bar{y}_{\bullet j\bullet} + \bar{y}_{\bullet\bullet\bullet})^2$$

$$S_{S\,EVO} = \sum_i \sum_j \sum_m (y_{ijm} - \bar{y}_{ij"})^2$$

$y_{\bullet\bullet\bullet} = \sum_i \sum_j \sum_m y_{ijm}$	$\bar{y}_{\bullet\bullet\bullet} = \frac{y_{\bullet\bullet\bullet}}{N_R N_A N_P}$
$y_{j\bullet\bullet} = \sum_j \sum_m y_{ijm}$	$\bar{y}_{j\bullet\bullet} = \frac{y_{j\bullet\bullet}}{N_R N_P}$
$y_{\bullet j\bullet} = \sum_i \sum_m y_{ijm}$	$\bar{y}_{\bullet j\bullet} = \frac{y_{\bullet j\bullet}}{N_R N_A}$
$y_{ij\bullet} = \sum_m y_{ijm}$	$\bar{y}_{ij\bullet} = \frac{y_{ij\bullet}}{N_R}$

If the interaction between operator and part is not significant, i.e. if $F < F_0$, repeatability and interaction should be combined to a single component (pooled).

Then:

$$S_{S\,Pool} = S_{S\,EV} + S_{S\,IA}$$

$$M_{S\,Pool} = \frac{S_{S\,Pool}}{N_A N_P (N_R - 1) + (N_A - 1)(N_P - 1)}$$

$M_{S\,Pool}$ replaces $M_{S\,IA}$ in the first two rows of variance table.

For the variance, the components are:

$$u_{EVO} = +\sqrt{M_{S\,Pool}}$$

$$u_{\mathrm{AV}} = +\sqrt{\frac{M_{S\,\mathrm{AV}} - M_{S\,\mathrm{Pool}}}{N_{\mathrm{P}} \cdot N_{\mathrm{R}}}}$$

Table B.3 — Analysis of variance table for Table A.6

Uncertainty component	Degrees of freedom (ν)	Sum of squares (S_S)	Mean square (M_S)	Estimated variance $\hat{\sigma}_i^2$	$u_i = +\sqrt{\hat{\sigma}^2}$	Test statistic F
Part to part	$N_{\mathrm{p}}-1$	$S_{S\,\mathrm{pv}}$	$M_{S\,\mathrm{PV}} = \frac{S_{S\,\mathrm{PV}}}{N_{\mathrm{P}}-1}$	$\frac{M_{S\,\mathrm{PV}} - M_{S\,\mathrm{EVO}}}{N_{\mathrm{R}}}$	na	$\frac{M_{S\,\mathrm{PV}}}{M_{S\,\mathrm{EVO}}}$
Reproduci-bility	$N_{\mathrm{p}} \cdot (N_{\mathrm{R}}-1)$	$S_{S\,\mathrm{EVO}}$	$M_{S\,\mathrm{EVO}} = \frac{S_{S\,\mathrm{EVO}}}{N_{\mathrm{P}}(N_{\mathrm{R}}-1)}$	$M_{S\,\mathrm{EVO}}$	$u_{\mathrm{EVO}} = +\sqrt{\hat{\sigma}_{\mathrm{EVO}}^2}$	—
The critical value part to part F_0 $\hat{\beta}_0$ α = 5 % is $F_{1-\alpha}\left[(N_{\mathrm{P}}-1),(N_{\mathrm{P}} \cdot N_{\mathrm{R}}-1)\right]$ $S_{S\,\mathrm{PV}} = N_{\mathrm{R}} \sum_j (\bar{y}_{j\bullet} - \bar{y}_{\bullet\bullet})^2$ $S_{S\,\mathrm{EVO}} = \sum_j \sum_m (y_{jm} - \bar{y}_{j\bullet})^2$						
$y_{\bullet\bullet} = \sum_j \sum_m y_{jm}$			$\bar{y}_{\bullet\bullet} = \frac{y_{\bullet\bullet}}{N_{\mathrm{R}} N_{\mathrm{P}}}$			
$y_{j\bullet} = \sum_m y_{jm}$			$\bar{y}_{j\bullet} = \frac{y_{j\bullet}}{N_{\mathrm{R}}}$			

If the analysis is about the measuring uncertainty components repeatability and interaction between measuring system and part, it is analogous to Table A.4 with replacement of operator with measuring system.

B.4 Relation between capability of the measurement process an capability of the production process

The following calculations assume the presence of a normal distributed process.

1. For the product characteristic with $N(\mu;\sigma^2)$,

$$\hat{C}_{\mathrm{P}} = \frac{U - L}{6s}$$

2. Let the observed capability index be denoted by $C_{\mathrm{p;obs}}$.

$C_{\mathrm{p;obs}}$ is the capability under the influence of both the variation in the production process and the variation in the measurement process.

$$C_{\mathrm{P;obs}} = \frac{U - L}{6\sqrt{\sigma_{\mathrm{P}}^2 + \sigma_{\mathrm{MP}}^2}},$$

where

σ_{P} denotes the standard deviation from the production process;

σ_{MP} denotes the standard deviation from the measurement process.

It is found that:

$$C_{\mathrm{p;obs}} = \frac{U-L}{6\sqrt{\sigma_{\mathrm{P}}^2 + \sigma_{\mathrm{MP}}^2}}$$
$$= \frac{U-L}{6\sigma_{\mathrm{P}}\sqrt{1+\sigma_{\mathrm{MP}}^2 / \sigma_{\mathrm{P}}^2}}$$
$$= \frac{U-L}{6\sigma_{\mathrm{P}}\sqrt{1+(\sigma_{\mathrm{MP}} / \sigma_{\mathrm{P}})^2}}$$
$$= C_{\mathrm{p;p}} \frac{1}{\sqrt{1+(\sigma_{\mathrm{MP}} / \sigma_{\mathrm{P}})^2}}$$

3. Relation between capability ration and the real capability index denoted $C_{\mathrm{p;p}}$

From the formulae in Clause 9:

$$Q_{\mathrm{MP}} = \frac{2U_{\mathrm{MP}}}{U-L} \cdot \quad \Rightarrow Q_{\mathrm{MP}} = \frac{4\sigma_{\mathrm{MP}}}{U-L},$$

4. From the formula in point 2 above:

$$6^2 \cdot \sigma_{\mathrm{P}}^2 + 6^2 \cdot \sigma_{\mathrm{MP}}^2 = \frac{(U-L)^2}{C_{\mathrm{p;obs}}^2}$$

5. From the formulae in points 1, 3 and 4 above:

$$6^2 \cdot \frac{(U-L)^2}{6^2 \cdot C_{\mathrm{p;p}}^2} + \frac{6^2}{4^2} \cdot Q_{\mathrm{MP}}(U-L)^2 = \frac{(U-L)^2}{C_{\mathrm{p;obs}}^2},$$

where $C_{\mathrm{p;p}}$ denotes the capability index of the process and it is calculated using only the standard deviation σ_{P} of the process.

$C_{\mathrm{p;p}}$ is larger than $C_{\mathrm{p;obs}}$ and the two are only identical under the unrealistic assumption that the variance of the measurement process is 0.

Derived from Clause 5:

$$\frac{1}{C_{\mathrm{p;p}}^2} + \frac{9}{4}\frac{Q_{\mathrm{MP}}^2}{1} = \frac{1}{C_{\mathrm{p;obs}}^2},$$

and rearranged to give $C_{\mathrm{p;p}}$ as a function of the observed capability $C_{\mathrm{p;obs}}$ and the capability fraction of the measurement process.

$$C_{\mathrm{p;p}} = \left(\frac{1}{C_{\mathrm{p;obs}}^2} - \frac{9}{4} \cdot Q_{\mathrm{MP}}^2 \right)^{-1/2}$$

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