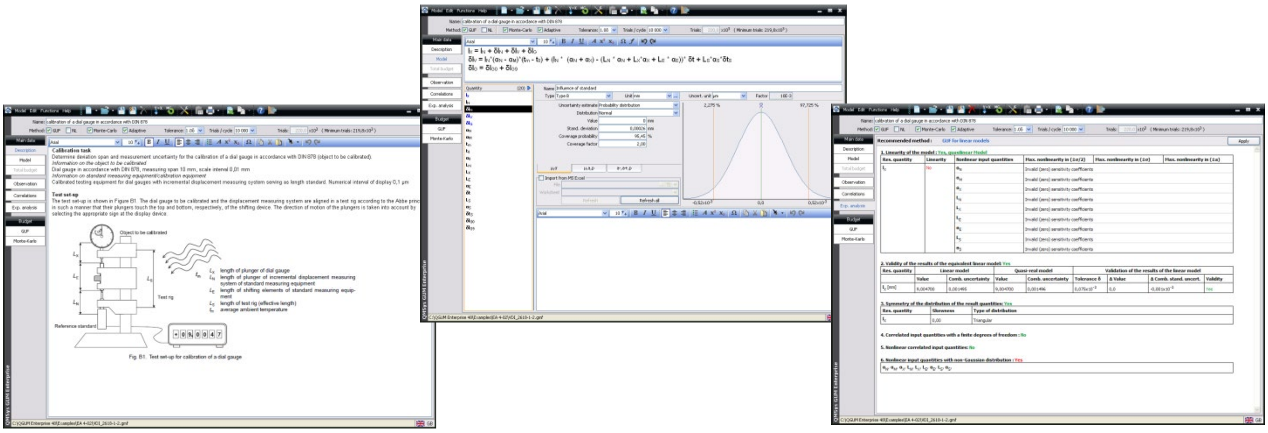
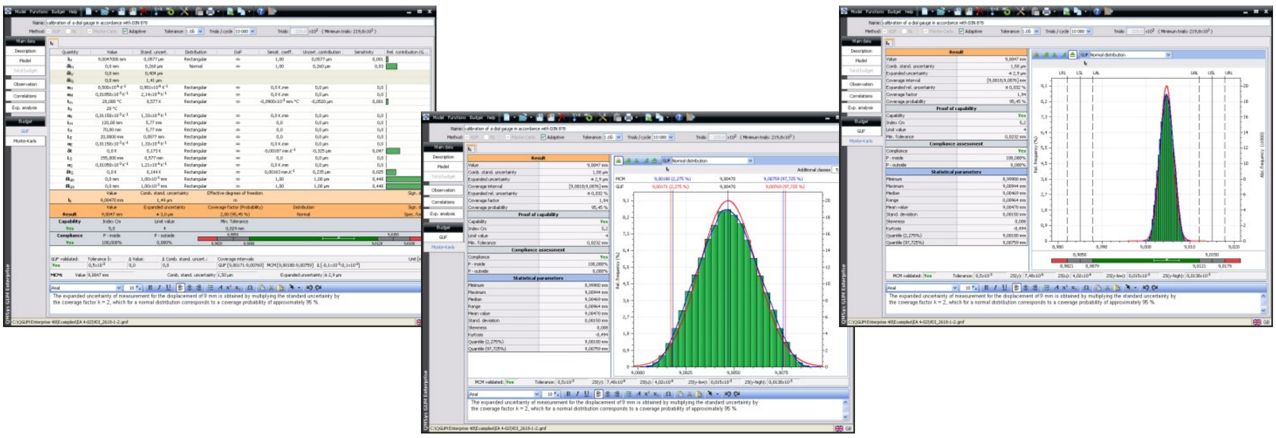
QMSys Metrology and measurement software



QMSys GUM Software

Tools for Measurement Uncertainty Analysis



Software editions:

- QMSys GUM Enterprise 5.1
- QMSys GUM Professional 5.1
- QMSys GUM Calculator 5.1
- QMSys GUM Excel Add-In 5.1

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1. Introduction

1.1. General

QMSys GUM Software products are comprehensive tools for analysis of the measurement uncertainty of physical measurements, chemical analyses, and calibrations. Whether you are a scientist, metrologist, design engineer, production engineer, test engineer or anyone dealing with measurement accuracy, you need to know only the information that falls within your technical specialty. Our products are the ultimate assistant to your practice, refining it with proven professionalism and reliability. *QMSys GUM Software* furnishes the statistical analysis, while you furnish the technical knowledge.

The software complies with numerous international guidelines and standards. Some of the most recognized ones are:

- **ISO/IEC Guide 98-3:2008 (GUM:1995)** Guide to the expression of uncertainty in measurement
- **ISO/IEC Guide 98-3:2008/Suppl. 1:2008** Supplement 1 to the "GUM" - Propagation of distributions using a Monte Carlo method
- **ANSI/NCSL Z540.2** U.S. Guide to the Expression of Uncertainty in Measurement
- **EA-4/02** Expression of the Uncertainty of Measurement in Calibration
- **DAkkS-DKD-3** Expression of the Uncertainty of Measurement in Calibration
- **UKAS M3003** Expression of Uncertainty and Confidence in Measurement
- **EURACHEM/CITAC Guide CG 4** Quantifying Uncertainty in Analytical Measurement
- **VDA Band 5** Measuring Process Suitability
- **ASME PTC 19.1-2005** Test Uncertainty
- **ILAC-G8:09/2019** Guidelines on Decision Rules and Statements of Conformity
- **ISO/IEC/EN 17025:2005** Concerning the evaluation of the measurement uncertainty.

The software uses three different methods to calculate the measurement uncertainty:

- **GUF Method for linear models** - this method is applied to linear and quasi-linear models and corresponds to GUM Uncertainty Framework. The software calculates the partial derivatives (the first term of a Taylor series) to determine the sensitivity coefficients of the equivalent linear model and then calculates the combined standard uncertainty in accordance with the Gaussian error propagation law.
- **GUF Method for nonlinear models** - this method is provided for nonlinear models with symmetric distribution of the result quantities. In this method, a series of numerical methods are used – e.g. nonlinear sensitivity analysis, second and third order sensitivity indices, quasi-Monte Carlo with Sobol sequences. The additional influences, such as non-linear relationships, correlations, distribution type or interaction of the input quantities, are also considered when calculating the uncertainty contributions. The results obtained with this method coincide with the analytical method remarkably closely.
- **Monte Carlo Method** - this method is described in the first supplement to GUM, and it is the only appropriate method for many calculations of the uncertainty since the equations of the model are often not linear. In the Monte Carlo technique, a suitable distribution is attributed to each input quantity. From these distributions, a "random value" for each is simulated and a value of the target quantity is calculated from this set of input data. This procedure is repeated many times, so that a set of data are obtained for the result quantity, which represents a random sample from the "potential" values of the result quantity as a function of variations in the input quantities according to their distribution. The mean value and the standard deviation of this random sample are estimates for the value of the result quantity and its standard uncertainty. To achieve reliable estimates, a high number of replicates are necessary - usually from $2x10^5$ up to 10^6. The Monte Carlo technique, however, provides far more than an estimate for the result quantity and its standard uncertainty, namely: an estimated distribution of the result quantity and a realistic coverage interval.

Using these methods, the *QMSys GUM Software* offers plausible and accurate calculation of the measurement uncertainty for virtually all types of measurements:

- linear and nonlinear models
- symmetric and asymmetric distributions of the result quantities
- correlated input variables with arbitrary probability distribution

The software is applicable even in cases that are not described in GUM, GUM Suppl.1:

- correlated non-normally (non-Gaussian) distributed input quantities
- correlated input quantities with finite degrees of freedom
- nonlinear models with more than two correlated input quantities
- non-linear models with non-normally distributed input quantities

A special algorithm for generation of exact correlated values, while maintaining the specified probability distributions, is developed, and implemented in the program to ensure the accuracy and validity of the results in these cases.

The program supports the systematic procedure in building an uncertainty analysis, as requested in the corresponding standards and guides. This process consists of the following basic steps:

- Creation of mathematical model, which describes the relationship between the quantities in the respective measurement
- Analysis of the required information as the standard measurement uncertainty or the distribution of values of input quantities
- Entry of the observations
- Determination of the correlation coefficient between the input quantities
- Analysis of the model and selecting the appropriate method for calculating the measurement uncertainty
- Calculation of the measurement uncertainty and preparation of the measurement uncertainty budget
- Validation of the results – estimate, combined uncertainty, and coverage interval (expanded uncertainty)

The computation examples in the documents ***GUM, GUM Supplement 1, EA-4/02, DAkkS-DKD-3 and EURACHEM/CITAC Guide CG 4*** are added to the software package as example models that can be analyzed with the program.

The result of the evaluation is a clearly structured measurement uncertainty budget in a table form. This table holds all used quantities with their quantity names and values, the associated standard uncertainty and effective degrees of freedom, the sensitivity coefficient automatically derived from the model equation and the contribution to the standard uncertainty of the result of the measurement. Finally, the complete result of the examination is presented as a value with associated expanded uncertainty and automatically or manually selected coverage factor.

The Monte Carlo method displays a histogram, statistical parameters of the estimated distribution of the result quantities and validation of the results. For result quantities with asymmetric distribution, the program estimates the shortest coverage interval, the asymmetric expanded measurement uncertainty and asymmetric coverage factor.

The summary budget offers the following additional analysis:

- Regression analysis and calculation of the equation of the expanded measurement uncertainty for a certain measurement range
- Diagrams of the expanded measurement uncertainty for a certain measurement range
- Correlation analysis of the result quantities.

The result of the uncertainty analysis together with all input data can be printed with the help of configurable templates as a report. All input texts are part of the printout and are used for documentation purposes.

Each analysis can be completely saved in a file with a selectable name. In this way, the examination is available at any time for a later review or editing. Each saved analysis can be used as a starting point for new uncertainty analyses using the same model, but with new and changed data.

1.2 Validation of the QMSys GUM Software

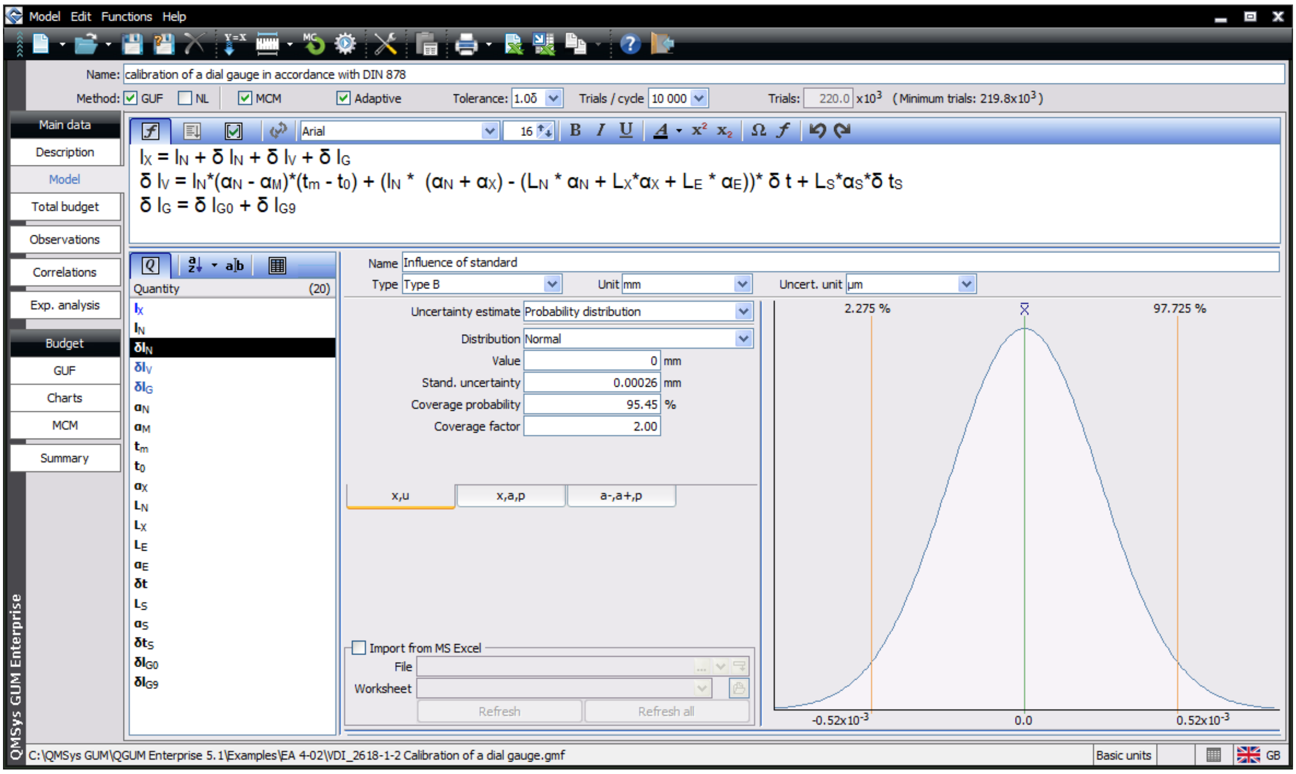
QMSys GUM Software is a standard application that offers the possibility for the user to freely enter or modify the model equation. With this feature, the application can be used to evaluate almost any measurement process. Therefore, a general validation by *Qualisyst Ltd.* for all possible purposes is not possible. The correct calculation can be verified with the help of the examples from the official regulations and guidelines (GUM, Supplement 1 to the GUM, EA-4/02, DAkkS-DKD-3 and EURACHEM/CITAC Guide CG 4), which are part of the installation. These documents are available for free downloading. The results of the validation of the *GUM QMSys software* are shown in the tables in Appendix A.

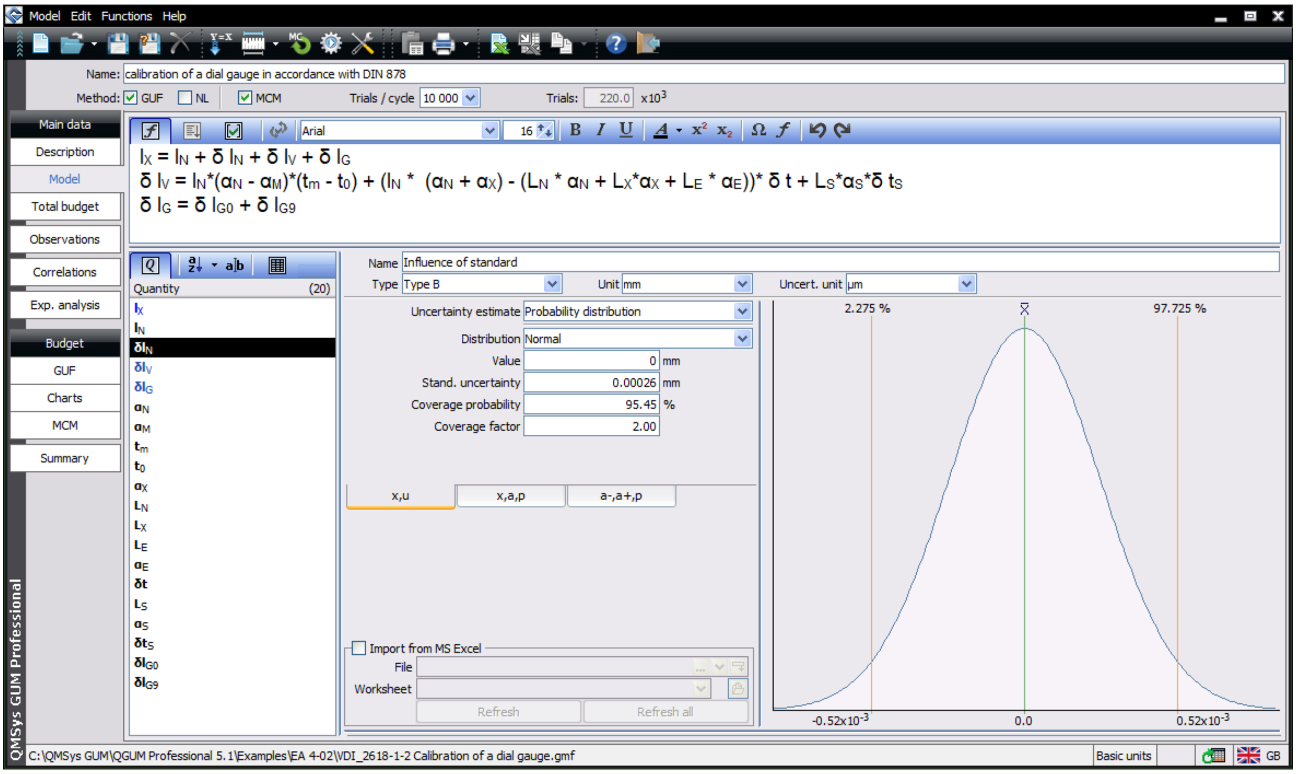
During the development, the emphasis was put on usability and robustness of the ***QMSys GUM Software***. Please contact us by email to qualisyst@qsyst.com in case you face any problems, and we will assist you in solving the problem as soon as possible.

1.3 Editions of the QMSys GUM Software

Main software editions

- **QMSys GUM Enterprise** provides the highest-precision analysis of the measurement uncertainty for all types of measurements by offering a variety of settings for the analysis methods
- **QMSys GUM Professional** offers accurate analysis of measurement uncertainty for linear and nonlinear models with symmetric or asymmetric distribution of the output quantities when using optimized settings for analysis methods.





Additional standalone software editions

The following software editions offer calculating the measurement uncertainties by using the model files, prepared with the software editions **QMSys GUM Enterprise / Professional**.

- **QMSys GUMX Excel Add-In** offers full integration of the measurement uncertainty calculation in MS Excel. The software **QMSys GUMX** also calculates the measurement uncertainty of unlimited number of measurement series (result quantities with identical measurement model) using a model file for only one set of measurements. This functionality is particularly useful when calibrating in several points of the measuring range, in addition, it simplifies the measurement models.
- **QMSys GUM Calculator** is designed primarily for the operators of the measuring instruments with minimum training requirements measurement. The graphical user interface is based on the optimized user interface of the software editions **QMSys GUM Enterprise** with some restrictions of the editing functions for the models of the measurement process, which prevents unwanted change of the developed models. The edition **QMSys GUM Calculator** offers the same methods for calculating the measurement uncertainty, and statistical and graphical evaluation as the software edition **QMSys GUM Enterprise**.
- **QMSys GUML Developer Library** is a dynamic-link-library for integration of the measurement uncertainty calculation in custom specific projects as software for processing the raw measurement data, calibration management systems, LIMS, etc.

The functionality of the different editions is shown in the following tables:

Key features

Function	GUM Enterprise	GUM Professional	GUM Calculator	GUMX Excel Add-In
Modelling the measurement process	✓	✓	×	×
Models for flow measurements in open channels	✓	×	✓	✓
Several output quantities	✓	✓	✓	✓
Indexed input and output quantities	✓	✓	✓	✓
Expert analysis	✓	✓	✓	×
GUM Method for linear models (GUF)	✓	✓	✓	✓
GUM Method for nonlinear models (GUF-NL)	✓	✓	✓	✓
Monte Carlo Method (MCM)	✓	✓	✓	✓
Calculation of nonlinear uncertainty contributions	✓	✓	✓	✓
Suitable for linear and quasi-linear models	✓	✓	✓	✓
Suitable for non-linear models	✓	✓	✓	✓
Proof of capability and compliance assessment	✓	✓	✓	✓
Uncertainty budget	✓	✓	✓	✓
Extended analysis of several output quantities	✓	✓	✓	✓
Reports and export	✓	✓	✓	✓
Single user version on a desktop computer	✓	✓	✓	✓
Portable version on a USB memory stick	✓	✓	✓	×
Server version (concurrent user licenses)	✓	✓	✓	×

Modelling of the measurement process

Function	GUM Enterprise	GUM Professional	GUM Calculator	GUMX Excel Add-In
Free definable model equations	✓	✓	×	×
Models for flow measurements in open channels	✓	×	×	×
Number of the input and output quantities	unlimited			
Catalog of measurement units	✓	✓	×	×
Correlation matrix of the input variables	✓	✓	×	×
Validation of the correlation matrix	✓	✓	×	×
Optimizing of the correlation matrix	✓	✓	×	×

Note: The software editions *QMSys GUM Calculator* and *QMSys GUMX Excel Add-In* use the model files, prepared with the software editions *QMSys GUM Enterprise / Professional*.

Function	GUM Enterprise	GUM Professional	GUM Calculator	GUMX Excel Add-In
Type A Input Quantities	✓	✓	✓	✓
Method of Observation	• Direct - individual values or group values • Indirect - free definable measurement cycles			
Number of observations	unlimited			
Data import via clipboard	✓	✓	✓	✓
Import from Microsoft Excel file	✓	✓	✓	✓
Determining of the standard uncertainty	✓	✓	✓	✓
Correlation analysis of the observations	✓	✓	✓	✓
Statistical analysis, histogram	✓	✓	✓	✓
Type B Input Quantities	✓	✓	✓	✓
Estimate of the uncertainty	• Expanded uncertainty with normal or t-distribution • Standard uncertainty with normal or t-distribution • Limit of error with rectangular distribution • Relative limit of error with different distributions • Probability distribution • Automatic calculation of the standard uncertainty of the molar mass of chemical compounds			
Probability distributions	• Normal distribution • t-distribution • Triangle distribution • U-shaped distribution • Exponential distribution • Cosine distribution	• Log-normal distribution • Rectangular distribution • Trapezoidal distribution • Curvilinear trapezoidal distribution • Square distribution • Half-Cosine distribution		
Entering the distribution parameters	• Value and standard uncertainty • Value and half-width of the distribution area • Lower and upper limits			
Relative uncertainty error	Entered in %, calculating the degrees of freedom acc. to GUM, G.3			
Import from Microsoft Excel file	✓	✓	✓	✓
Import from *.gmf files	✓	✓	✓	✗

Expert Analysis

Function	GUM Enterprise	GUM Professional	GUM Calculator	GUMX Excel Add-In
Linearity test	Yes, calculated in six points for each input variable			✗
Validation of the results	Yes, value and combined standard uncertainty			✗
Analysis of the probability distribution of the output quantities	Yes, symmetry and type of the probability distribution			✗
Checking for correlated input quantities with a finite degree of freedom	✓	✓	✓	✗
Checking for non-linear correlated input quantities	✓	✓	✓	✗
Checking for non-linear non-normally distributed input quantities	✓	✓	✓	✗

Methods for calculation of the measurement uncertainty

Function	GUM Enterprise	GUM Professional	GUM Calculator	GUMX Excel Add-In
GUM Method for linear models (GUF)	✓	✓	✓	✓
Calculation of sensitivity coefficients	✓	✓	✓	✓
Calculation of the effective degrees of freedom	✓	✓	✓	✓
Calculation of the expansion factor for any coverage probability	• Normal distribution • Rectangular distribution • Trapezoidal distribution		• t-distribution • Triangle distribution • Other symmetric distributions	
Expanded uncertainty	✓	✓	✓	✓
Validation of the GUF Method	✓	✓	✓	✗

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Function	GUM Enterprise	GUM Professional	GUM Calculator	GUMX Excel Add-In
GUM Method for nonlinear models (GUF-NL)	✓	✓	✓	✓
Non-linear sensitivity analysis	✓	✓	✓	✓
Sensitivity indices of higher order	up to 3rd order	up to 2nd order		
Calculation for correlated input variables (all distribution types)	✓	✓	✓	✓
Calculation of the effective degrees of freedom (also for correlated input variables)	✓	✓	✓	✓
Calculation of the expansion factor for any coverage probability	• Normal distribution • Rectangular distribution • Trapezoidal distribution		• t-distribution • Triangle distribution • Other symmetric distributions	
Expanded uncertainty, coverage interval	✓	✓	✓	✓
Validation of the GUF-NL Method	✓	✓	✓	✗
Monte Carlo Method (MCM)	✓	✓	✓	✓
Number of simulations	10^4 to 10^8	10^4 to 10^6		
Random number generators - Period	• CMWC4096 by Dr. Marsaglia - 6,58*10^{39460} • Mersenne Twister - 4,32*10^{6001} • Wichmann/Hill - 2,63*10^{36}			
Adaptive Monte Carlo Procedure	✓	✗	✗	✗
Calculation for correlated input variables (all distribution types)	✓	✓	✓	✓
Validation of the Monte Carlo method	✓	✗	✗	✗
Expanded uncertainty, coverage interval (also for asymmetric distributions)	✓	✓	✓	✓

Evaluation of the results

Function	GUM Enterprise	GUM Professional	GUM Calculator	GUMX Excel Add-In
Uncertainty budget	✓	✓	✓	✓
Sensitivity, relative contribution, and Pareto diagram	✓	✓	✓	✗
Statistical analysis of the resulting quantities	✓	✓	✓	✓
Proof of capability	✓	✓	✓	✓
Compliance assessment	✓	✓	✓	✓
Documentation fields	✓	✓	✓	✓
Extended Analysis of Several Output Quantities	✓	✓	✓	✓
Summary table of the resulting quantities	✓	✓	✓	✓
Regression analysis - calculation of the equation of measurement uncertainty for a particular range	✓	✓	✓	✓
Correlation analysis of the resulting quantities	✓	✓	✓	✓
Diagrams	✓	✓	✓	✓
Print and Export	✓	✓	✓	✓
Custom RTF templates (MS Word)	✓	✓	✓	✗
Custom Excel templates	✓	✓	✓	✓
Export to clipboard	✓	✓	✓	✓
Export to Microsoft Excel	✓	✓	✓	✓

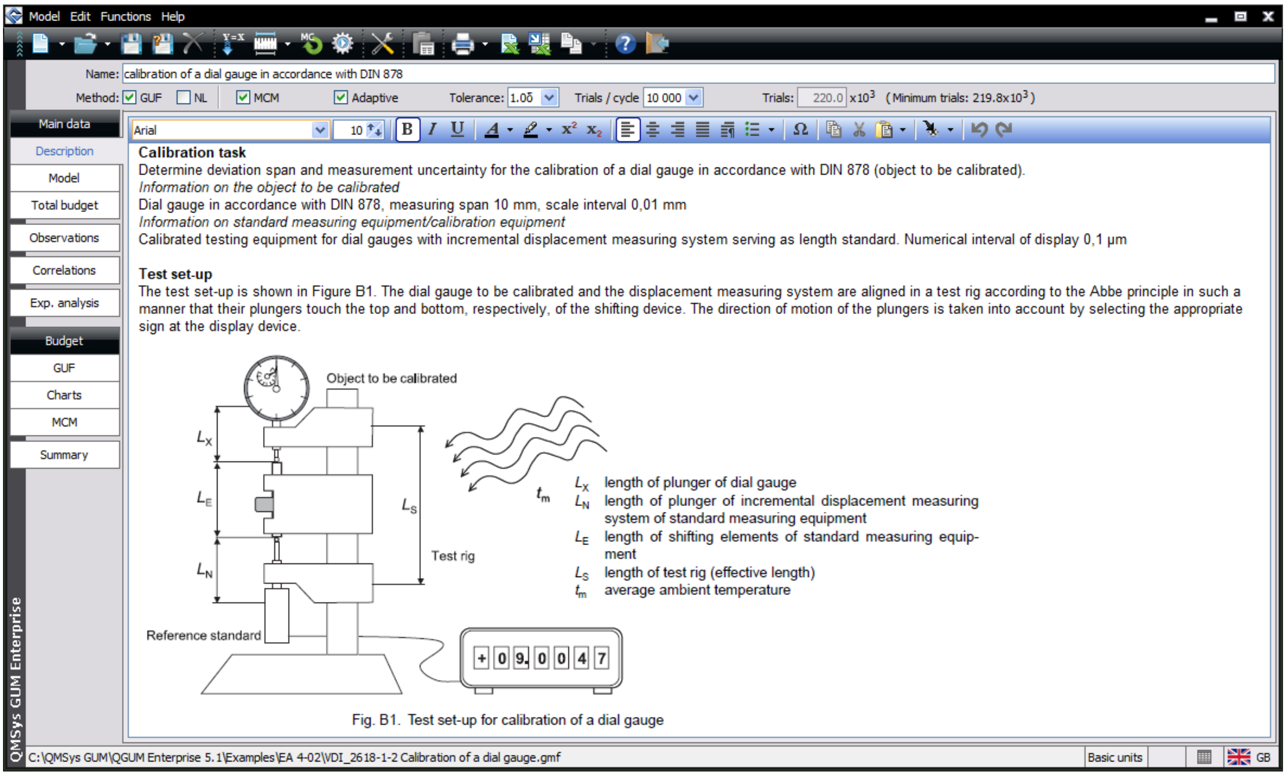
2. Software Description

The graphical user interface of the ***QMSys GUM software*** is based on several views, which are further structured by dialog pages. The title of the project and the settings for the methods for calculating the measurement uncertainty are entered in the upper range of the program window.

The following data are positioned on separate views:

- ***Main data*** - the registers *Description*, *Model* and *Total budget* are available. By selecting the different registers, the corresponding data can be viewed or edited.
- ***Observation*** - this view processes the values of type A quantities.
- ***Correlations*** - known correlations between the input quantities are entered in a matrix of correlation coefficients.
- ***Expert analysis*** - the software performs an advanced analysis of the model and determines the appropriate methods for the following calculation of the uncertainties.
- ***Budget*** - this view presents the results of the analysis in the registers *GUF*, *Charts* and *MCM*
- ***Summary*** - documentation field for summarizing the results.

The program menu and the toolbar provide specific functions for the selected view.



2.1. View Main data

2.1.1. Register Description

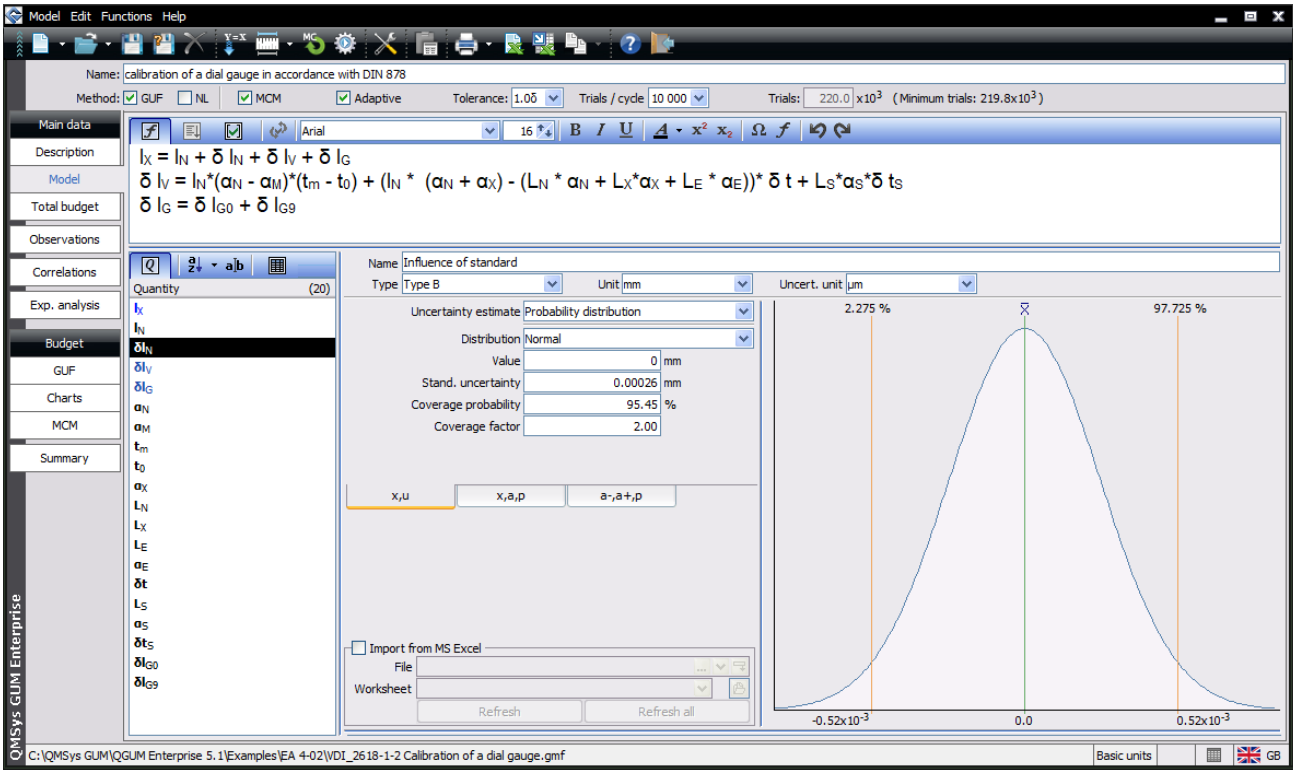
In the *Description* register, a general description of the measurement task can be entered. This data are used for informational purposes and is part of the printout.

Images can be pasted from the clipboard or with the *Insert object* button in the toolbar. Other objects (files) can also be imported and saved in the file of the uncertainty analysis. Double click on the inserted object will start the appropriate program to view or edit the object.

2.1.2. Register Model

The dialog page *Model* in the view *Main data* holds the mathematical model of the uncertainty analysis and the parameters of all quantities.

In the upper field on this page, the equations of the mathematical model can be entered. The model equations are the starting point for all subsequent calculations by the software. It is always possible to insert new quantities into the equation, and to rename or to delete existing quantities. Additional functions are available in a toolbar above the equation field.



When a model equation is more complex and contains a large number of input quantities, it is advisable to split it into smaller parts by introducing interim results, and thereby to make it easier to understand.

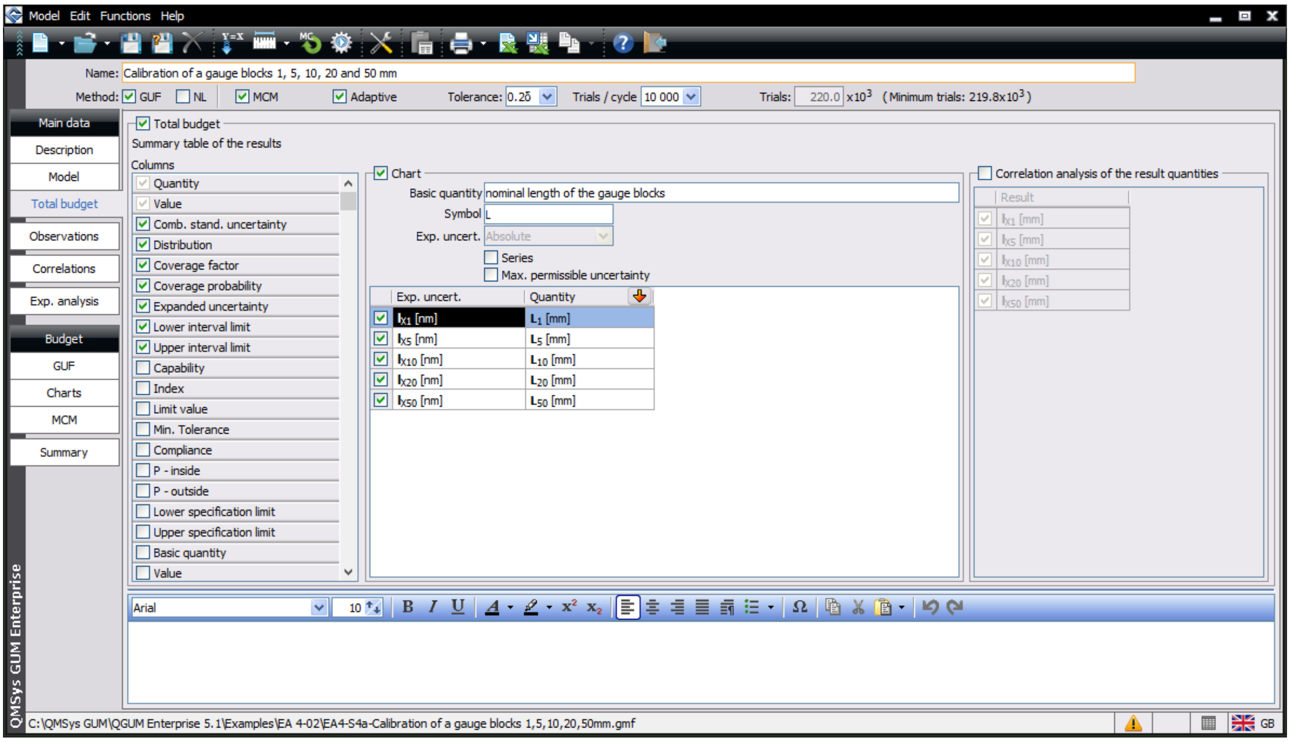
After a new entry or a change in the mathematical model, the syntax of the equations is checked, and the list of quantities is rebuilt or updated. The current data of the selected quantity are displayed to the right and may be edited. The following table provides an overview of the different quantity types.

Type	Description	Comment
Result	Measurand	This type identifies the output quantity and is set automatically by the program.
Interim result	Internal measurand	The program sets this type automatically. Switching to the type "Result" is possible.
Type A	Repeatedly observed quantity	The value and the standard uncertainty of the quantity are evaluated by using statistical analysis of measurement series. Optionally an estimate of the standard uncertainty can be specified.
Type B	Not-repeatedly observed quantity	The value and the standard uncertainty of the quantity are evaluated using means other than statistical analysis of measurement series. For these quantities, the appropriate distribution is selected and parameterized.
Constant	Mathematical constant	For mathematical constants without uncertainty, only the value can be entered.

A basic unit of the quantity value and additional unit for the measurement uncertainty can be assigned to every quantity in the model. The program provides an adequate database with SI units and some other commonly used units outside the SI. New custom measurement dimensions and units can be added to the database.

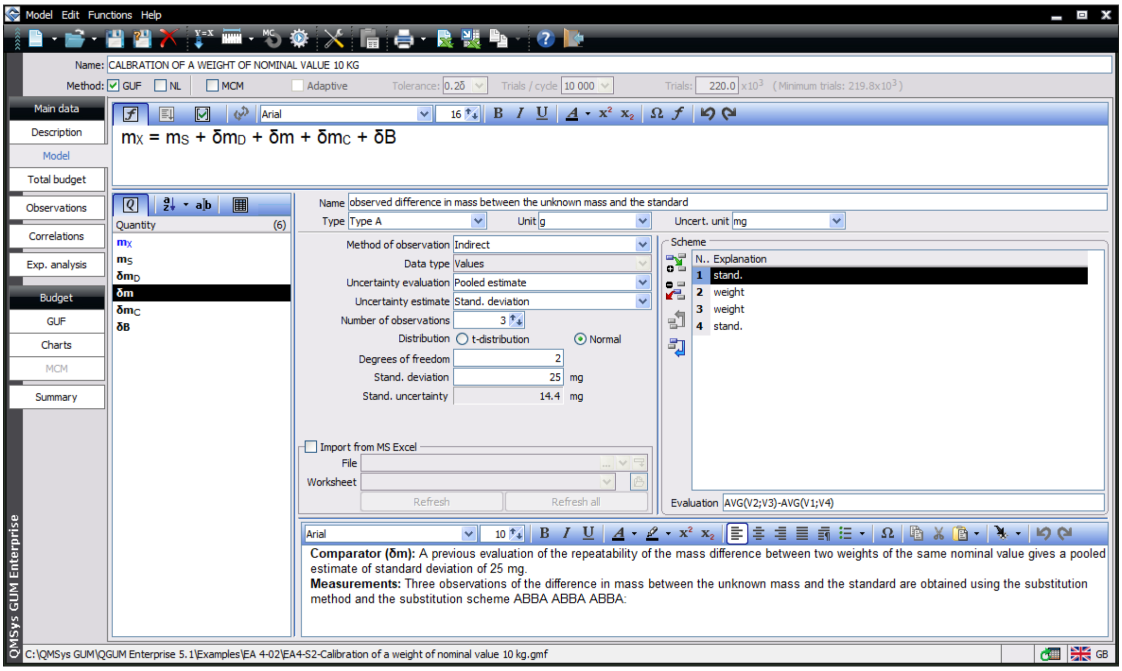
2.1.3. Register Total budget

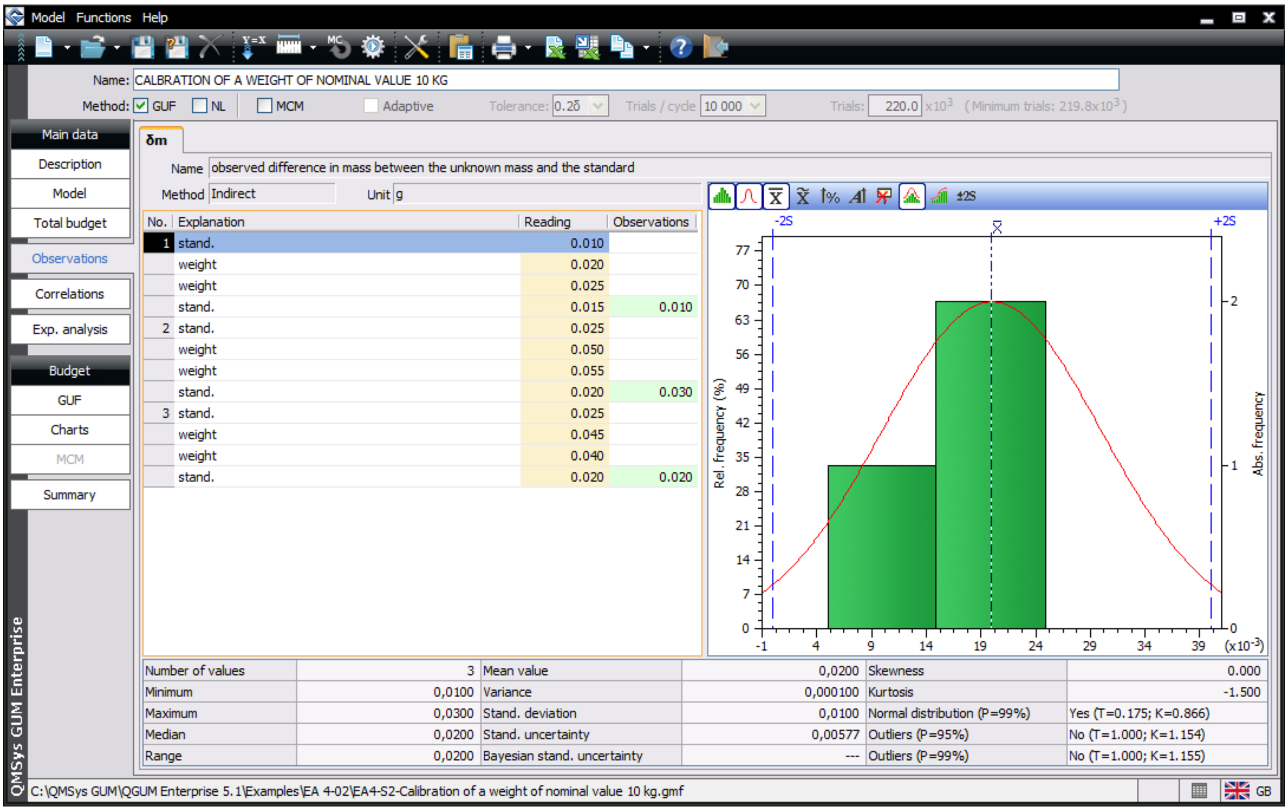
In the page *Total budget,* you can activate the summary of the results in a table and set the parameters for the regression analysis or the correlation analysis of the resulting values. Automatic entering is possible with the built-in search function.



2.2. View Observation

The view *Observation* processes the values of repeatedly observed quantities. The data are typed into a table, the structure of which depends on the method of observation.





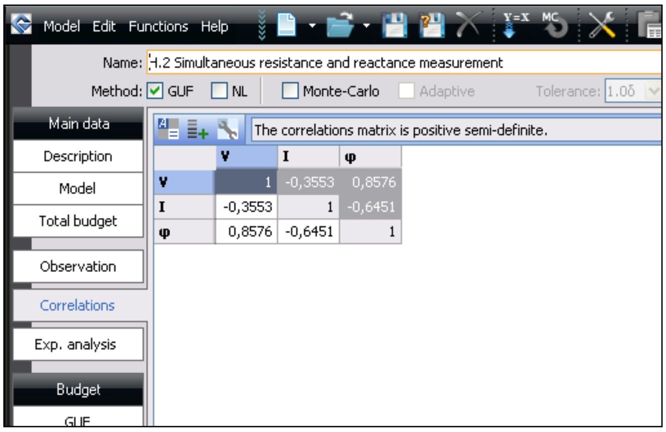
The data for an observed quantity of type A can be imported from the clipboard or from a MS Excel file. The data are read in, checked, and inserted in the observation table. Any existing data will be replaced by the imported data.

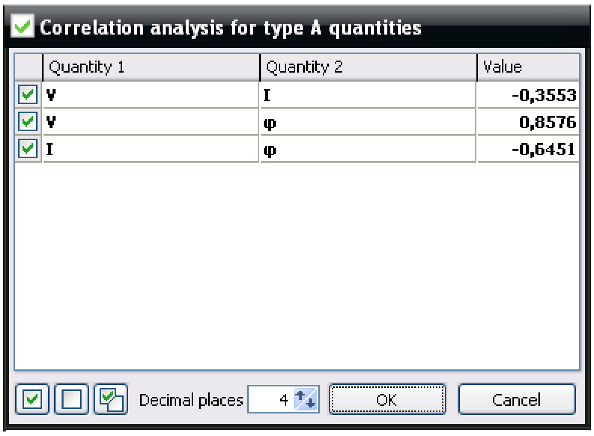
When valid data for all observations (or readings) have been entered, the statistical information including the mean value, the standard deviation, the standard uncertainty, and the Histogram of the data will be displayed.

2.3. View Correlations

In the *Correlations* page, known correlations between the input quantities are entered in a matrix of correlation coefficients. The software analyzes the correlation matrix (Eigen value decomposition) and checks if the matrix is positive semi-definite.

The button *Correlation analysis for type A quantities* will start an analysis for possible correlation between measurands. A prerequisite for a correlation analysis is that the number of observations of the two quantities must be equal and that all the observations are filled in and are valid.





If correlations between input values are taken into consideration, the description field should contain the reason why, and where the correlation coefficients came from.

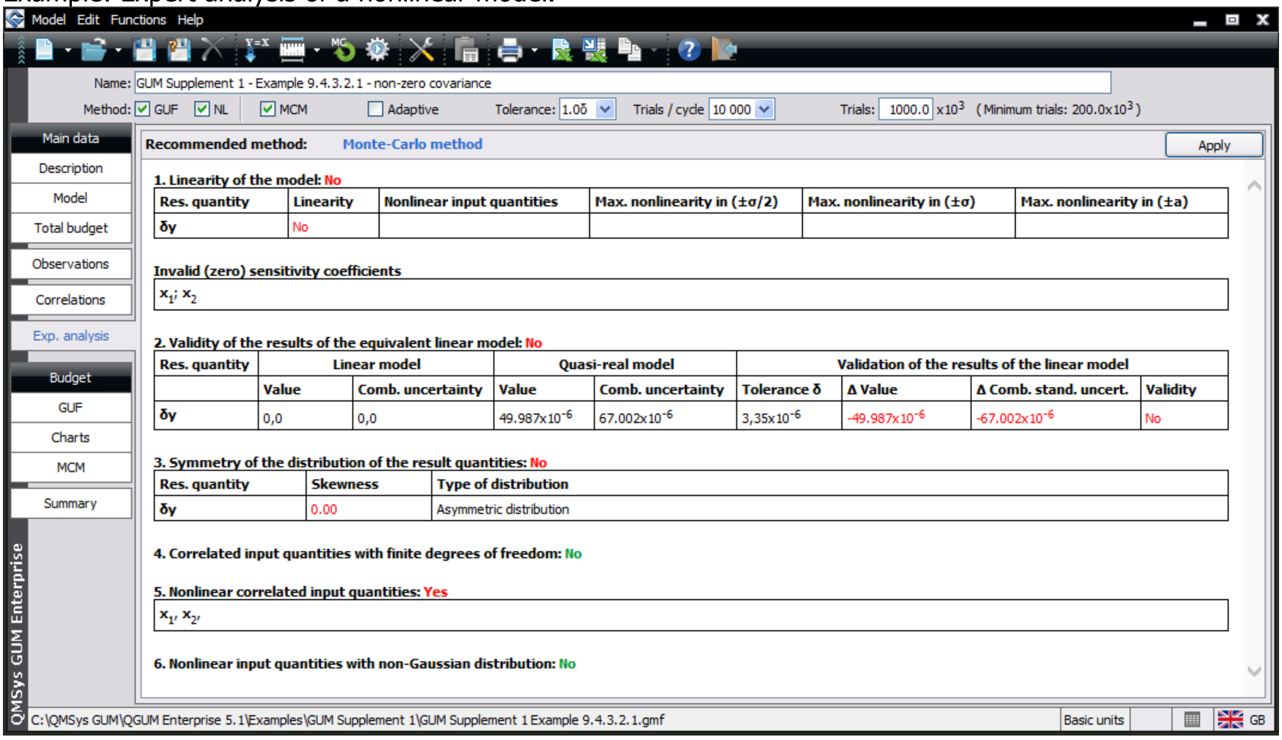
2.4. View Expert Analysis

The view *Expert analysis* presents the results of the expanded analysis of the model. The software checks the conditions for the application of the different methods and determines the appropriate methods for the following calculation of the measurement uncertainty.

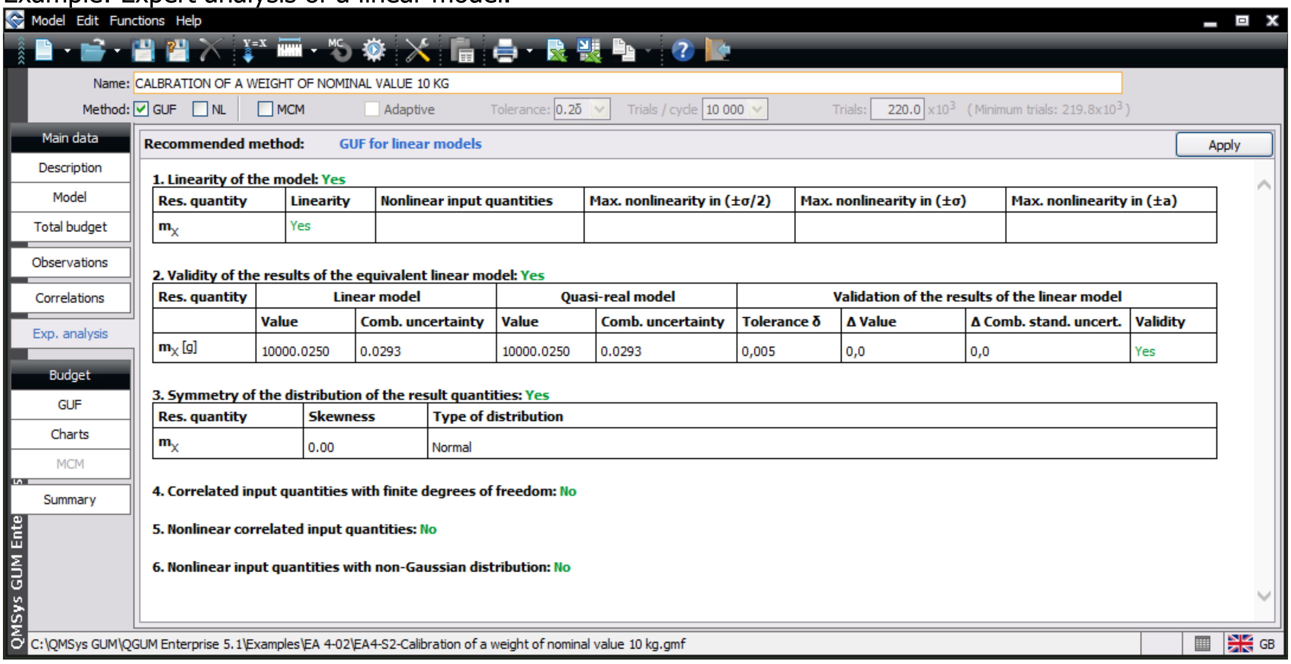
The following tests and calculations are performed:

- Linearity test for each input quantity in sixth areas of the distribution interval
- Calculation of the results of the equivalent linear model and the quasi-real model
- Validating the results of the equivalent linear model (value and combined standard uncertainty)
- Analysis of the distribution of the result quantities, determination of the symmetry and the distribution type
- Check for correlated input quantities with a finite degree of freedom
- Check for non-linear correlated input quantities
- Check for non-linear non-normally distributed input quantities.

Example: Expert analysis of a nonlinear model.



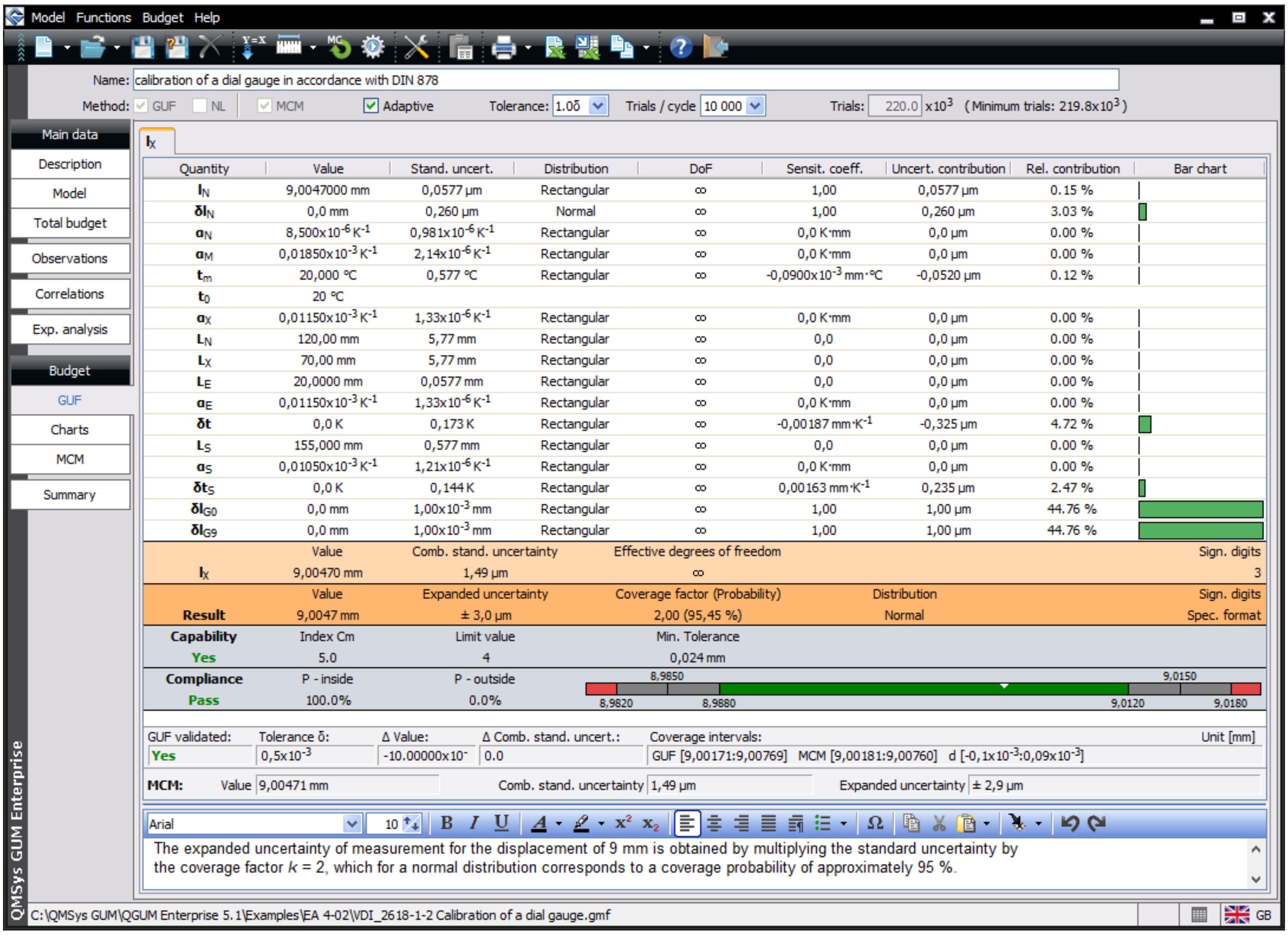
Example: Expert analysis of a linear model.



2.5. View Budget

The result of the analysis is presented in pages *GUF* and *Monte Carlo* of the *Budget* view.

The page *GUF* shows a clearly structured measurement uncertainty budget in a table form. This table holds all used quantities with their quantity names and values, the associated standard uncertainty and effective degrees of freedom, the sensitivity coefficient automatically derived from the model equation and the contribution to the standard uncertainty of the result of the measurement. The *Interim results* are only shown with the value and the standard uncertainty. Additional columns can be activated in the *Budget* menu.



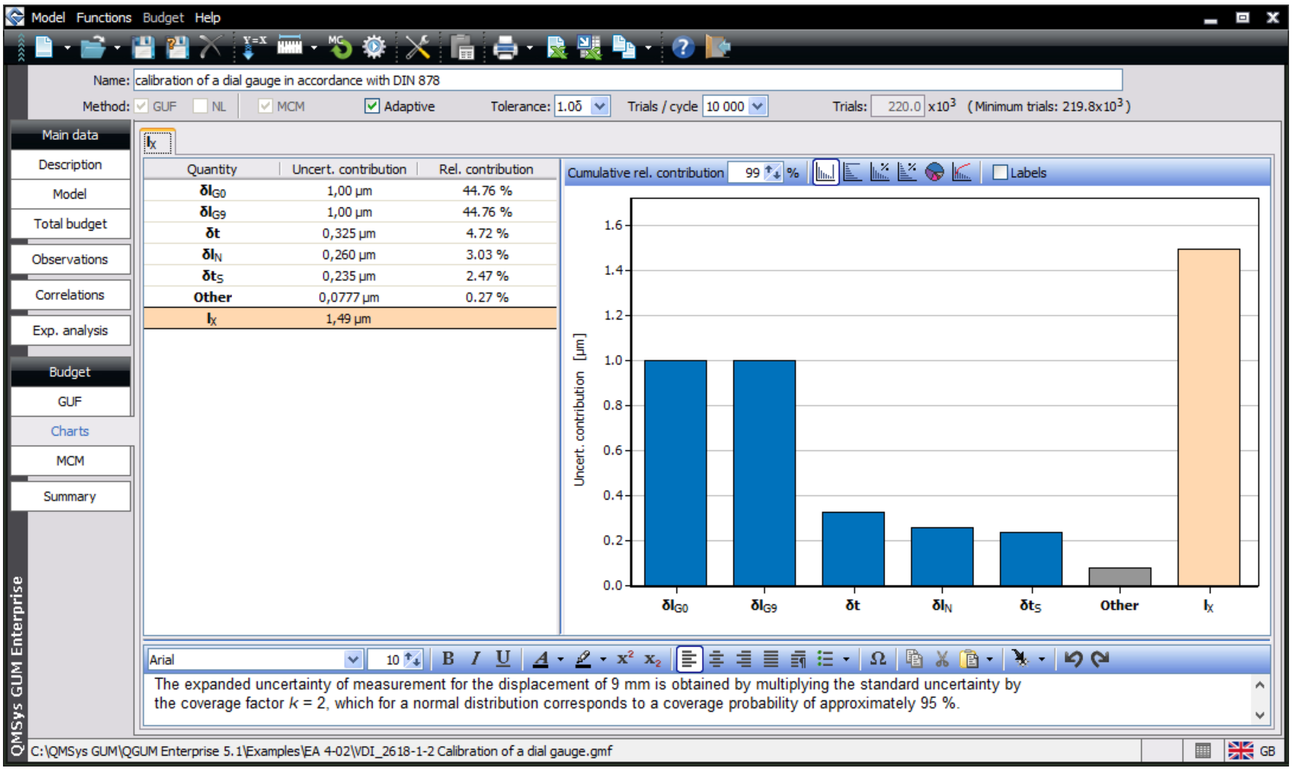
The result quantity is displayed in the bottom line with its value, the corresponding combined standard uncertainty, and the degrees of freedom. Finally, the complete result of the examination is presented as a value with associated expanded uncertainty and automatically or manually selected coverage factor. The results are automatically rounded and displayed in E-Format if necessary.

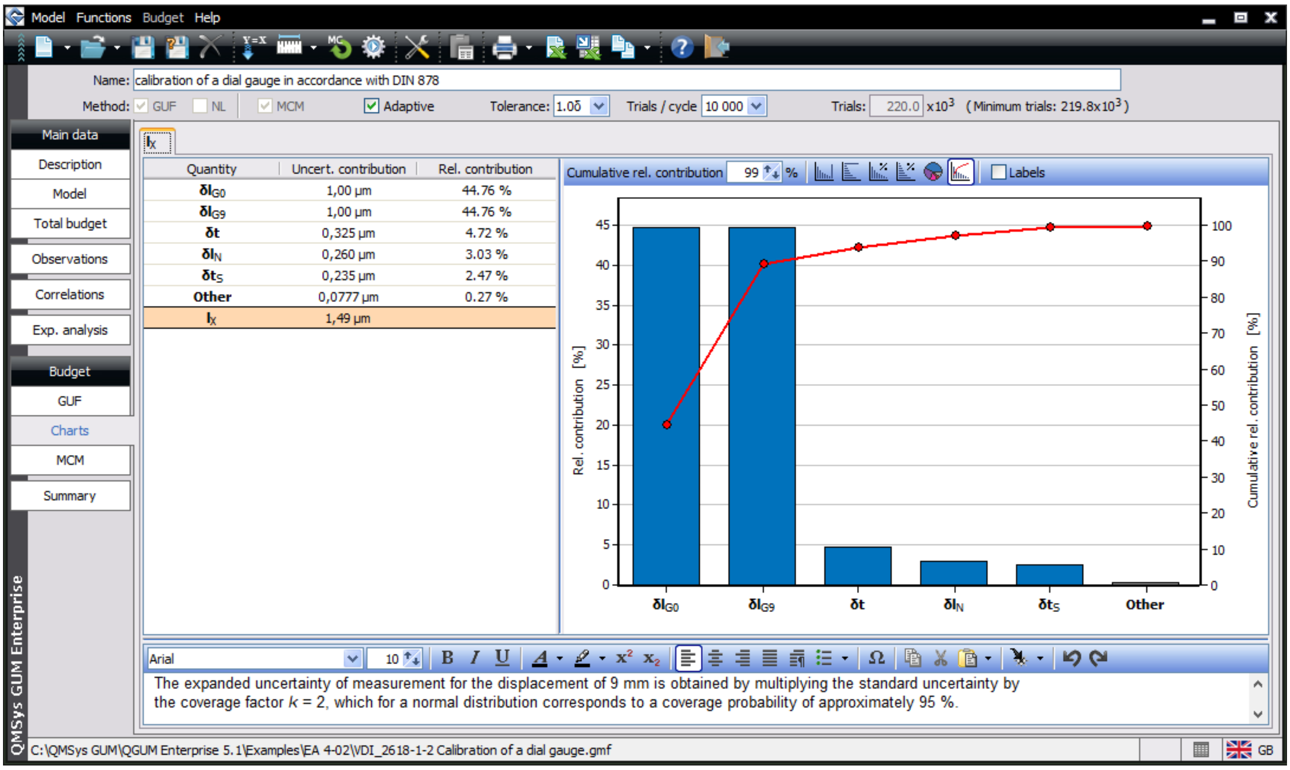
The page *Charts* helps the user quickly to identify the most significant sources of uncertainty. The software offers several types of charts and adjustable limit of the cumulative relative uncertainty contribution.

The *Monte Carlo method* displays a histogram, statistical parameters of the estimated distribution of the result quantities and validation of the results. For result quantities with asymmetric distribution, the program estimates the shortest coverage interval, the asymmetric expanded measurement uncertainty, and the asymmetric coverage factor.

The *Total budget* offers the following additional analysis:

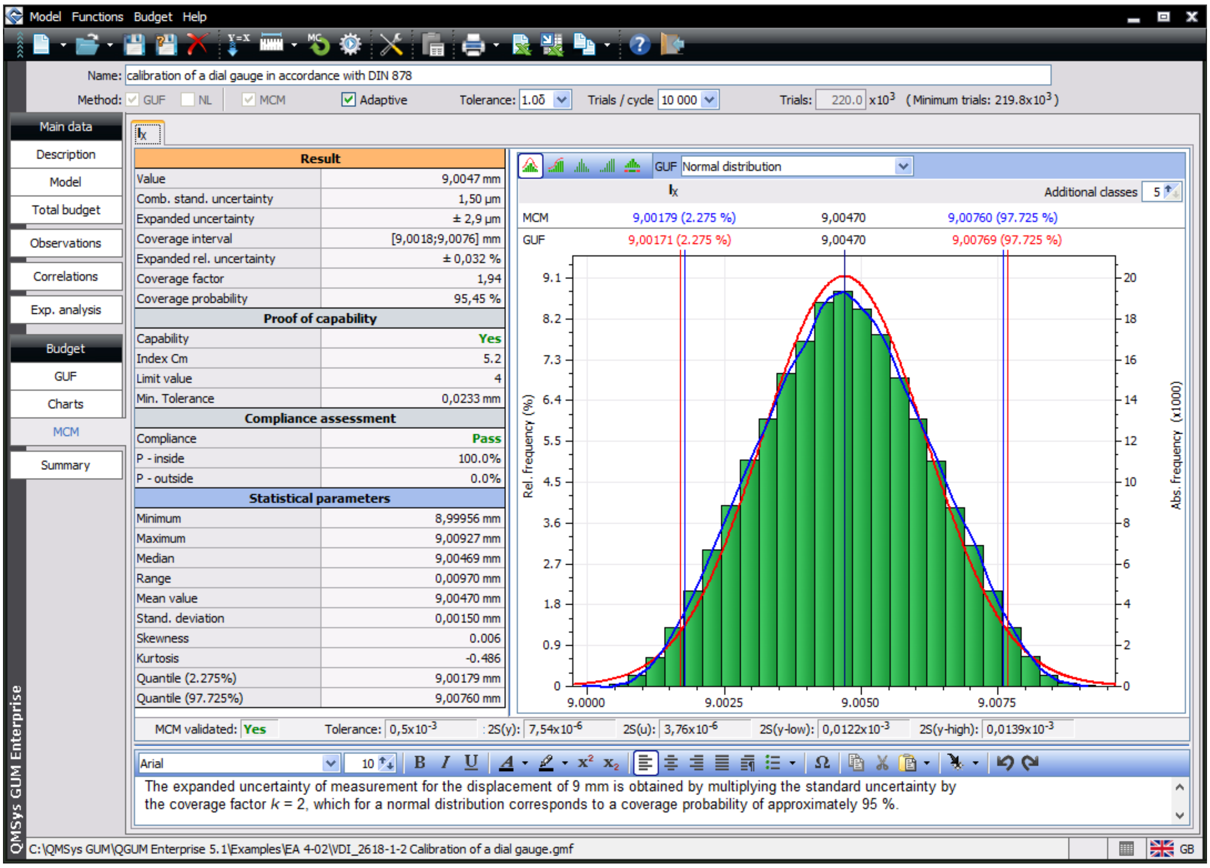
- Regression analysis and calculation of the equation of the expanded measurement uncertainty for a certain measurement range
- Diagrams of the expanded measurement uncertainty for a certain measurement range
- Correlation analysis of the result quantities.

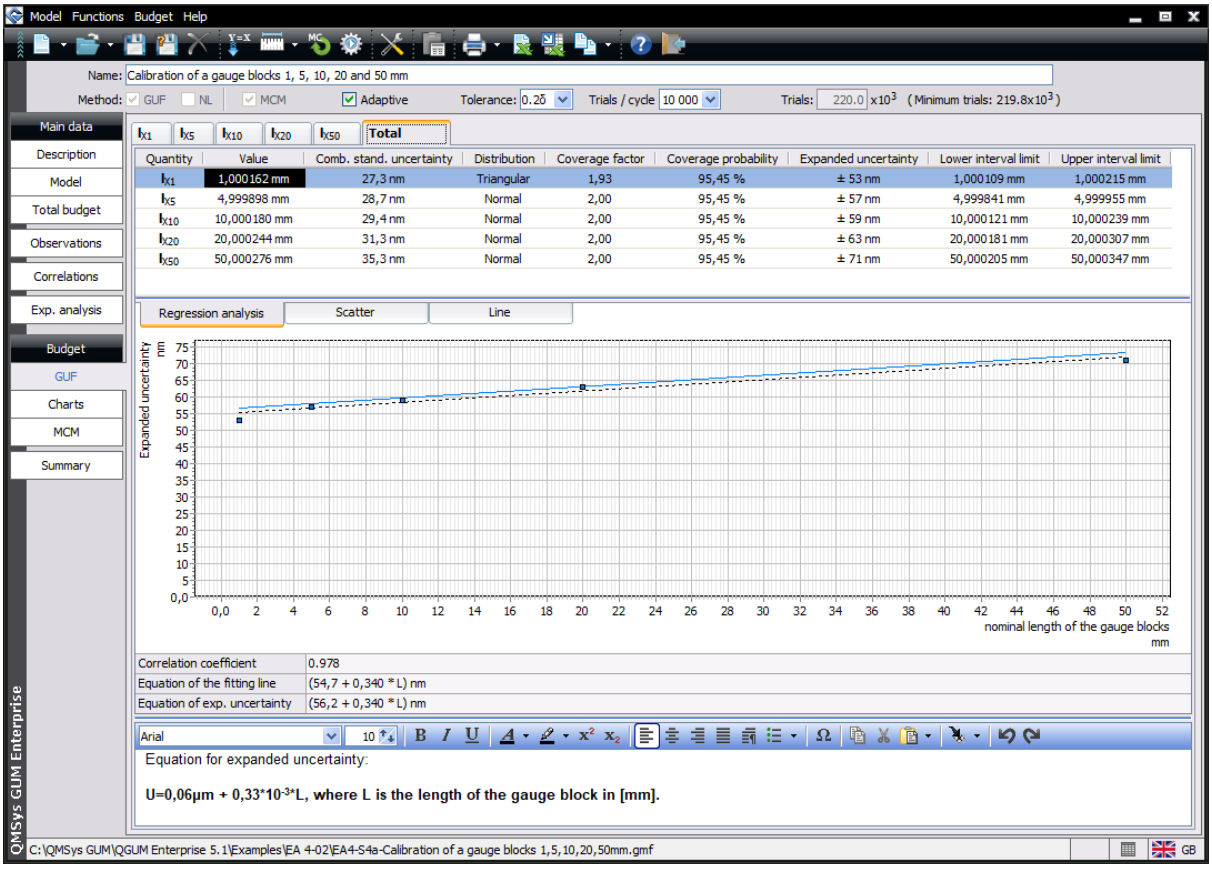




The software automatically validates the results of the GUF Method by comparing the values, the combined standard uncertainties, and the limits of the coverage intervals. The numerical tolerance δ in this comparison is calculated based on the combined standard uncertainty and the number of significant digits (2 to 5). The software offers an alternative calculation of the tolerance δ as a percentage of the combined standard uncertainty. Should the comparison be positive, then the GUM uncertainty framework can be used on this occasion and for sufficiently similar models in the future. Otherwise, consideration should be given for using MCM or another appropriate method instead.

The result of the uncertainty analysis together with all input data can be printed with the help of configurable templates as a report. All input texts are part of the printout and are used for documentation purposes.





Each analysis can be completely saved in a file with a selectable name. In this way, the examination is available at any time for a later review or editing. Each saved analysis can be used as a starting point for new uncertainty analyses using the same model, but with new and changed data.

3. Program Menu, Toolbars, and Input Fields

3.1. Program Menu and Main Toolbar

GB	**Language**	Selects another language. Language files are normal text files in the program folder *Languages* and can be open with programs like MS Notepad.
	Move Up **Move Down**	Toolbar moves up or down.
	New	Creates a new uncertainty analysis. • Model: All types of measurements - default option for developing new models of the measurement process • Model: Measurement of flow in open channels - models with extended functions for the assessment of uncertainty in flow measurements made using the velocity-area method and computed by the mid-section and mean-section method described in the standards ISO 748:2007 and ISO 1088:2007 • Import: Import of files from the *GUM Workbench* program.
	Open	The *Open* command reads in an existing uncertainty analysis from a file. The *Recent files* folder in the *Model* menu lists the previous files that you have opened.
	Save	Saves all changes made in the loaded file.
	Save as ...	The "Save as..." window will pop up. Here you can give the new name and path of the current uncertainty analysis.
	Cancel	After a safety message appears, all changes are rejected.
	Model analysis	The program will analyze the model equation and produce a list of symbols.
	System of measurement units	Defining a supplementary system of measurement units and switching between the basic and the supplementary system.
	MC Simulation	Starts a new simulation for the Monte Carlo method.
	Automatic uncertainty analysis	Starts the expert analysis, then selects the appropriate method (GUF or MCM) and automatically calculates the measurement uncertainty.
	Properties	Calls a window, containing settings for the current uncertainty analysis.
	Import measured values from the clipboard	This function imports measured values from the clipboard for an observed quantity.
	Print	The *Print* command creates a report. It contains all given data and descriptions as well as the uncertainty budget.
	Export to MS Excel	Exports data from ***QMSys GUM Software*** to MS Excel.
	Result export in MS Excel file	Exports the data of the results in MS Excel template.
	Copy	Copies the selected data or graphic into the clipboard.
	Help	Opens this file, containing help-information about the program.
	Exit	An exit from the program.

Menu Model -> Preferences
The *Preferences* command from the *Model* menu lets you set the general settings for the numerical representation, the Monte Carlo method, and other options.

Menu Model -> Catalog of units
This command opens the catalog of measurement units for editing. New units can also be inserted or edited; standard units cannot be changed.

3.2. Input Fields

Methods to calculate the measurement uncertainty

GUM-Method
The following settings are available:

- *GUF* - checkbox for enabling / disabling the GUF-Method for linear models
- *NL* - checkbox for enabling / disabling the GUF-NL method for nonlinear models

Monte Carlo Method
The following settings are available:

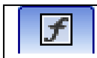
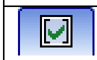
- ***MCM*** - checkbox for enabling / disabling the Monte Carlo method
- ***Adaptive*** - checkbox for adaptive Monte Carlo procedure
- ***Tolerance*** – value from 0.1*δ up to 1.0*δ for the stabilization of the parameters of the simulation
- ***Trials/cycle*** - number of simulations (trials) per cycle
- ***Trials*** - total number of simulations - from 10^3 up to 10^8.

The minimum number of trials according to ***GUM Supplement 1*** is computed automatically.
A basic implementation of an adaptive Monte Carlo procedure involves carrying out an increasing number of Monte Carlo trials until the results have stabilized in a statistical sense. The result, the combined uncertainty and the limits of the coverage interval are deemed to have stabilized if twice the standard deviation associated with them is less than the numerical tolerance δ, associated with the combined standard uncertainty (***GUM Supplement 1, NPL Report DEM-ES-010 and NPL Report DEM-ES-011***). A diagram is showing the stabilization for each output quantity during the simulation.

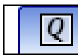
Fields for quantity name, units and other texts
Additional Greek characters are available over the combination of keys "*Ctrl + s*" or over the context menu of the right mouse button.
In the fields for measurement units superscript and subscript formats are available over the combinations of keys "*Ctrl + Up Arrow*", "*Ctrl + Down Arrow*" or over the context menu of the right mouse button.

3.3. Additional Toolbars

Field for model equations

	***Model equations*** - in this view the equations of the mathematical model can be entered or edited.
	***Preview of the equations*** - this view displays the full form of the model equations, when indexed quantities are defined.
	***Validation of the measurement units in the equations*** – in this view is checked the consistent use of the measurement units according to the rules of the SI system.
	***Updating the validation of the measurement units*** – with this button the unit check can be started again after correcting inconsistent units in the model equations.

List of quantities

	***Edit quantities*** - in this view the parameters of the selected quantity can be edited.
	***Sort quantities*** - the order of the quantities can be changed with the button.
	***Rename quantity*** - this button opens a window for entering the new designation of the selected quantity.
	***Review quantities*** – this view shows a table preview with the names and the units of the quantities.

Fields for model equations and descriptions
To these fields stands a button bar for formatting the texts. The selected font type and size in the equations field applies to the entire field, not to individual elements. The other formatting functions can be set individually.

Import of data from MS Excel file

	Selection of a new or already associated MS Excel file.
	Selecting a worksheet from the MS Excel file.
	The linked file and worksheet will be replaced with the newly selected file and worksheet for all input quantities.
	Opens the MS Excel file.

Method of observation for type A quantities, indirect measurements

	Insert	Measuring point insert.
	Delete	Measuring point delete.
	Move Up	Measuring point upward shift.
	Move Down	Measuring point downward shift.

Window Observation - Graphical display

	Classes	Show / Hide classes.
	Function	Show / Hide density function.
	Mean value	Show / Hide mean value.
	Median	Show / Hide median.
	Left axis	Selecting the scale type for the left axes: None, Absolute or Relative.
	Right axis	Selecting the scale type for the right axes: None, Absolute or Relative.
	Labels	Selecting the type of bar labels: None, Absolute or Relative.
	Histogram	Show histogram.
	Cumulative Histogram	Show cumulative histogram.
±2S	**Quantile Limits**	Setting the histogram limits: None; ± 1S; ± 2S; ± 3S; ± 4S; ± 5S; ± 6S.

View Budget – Monte Carlo Method

	Histogram	Show histogram.
	Cumulative Histogram	Show cumulative histogram.
	Probability Plot	Show probability plot.
	Cumulative Probability Plot	Show cumulative probability plot.
	Histogram and compliance assessment	Show histogram and zones of the compliance assessment.
GUF ...	**Distribution**	Selection of distribution for the GUF-Method. For t-distribution, the degrees of freedom (3-100) are automatically calculated. If the degrees of freedom are over 100, the normal distribution is used. For trapezoidal distribution, the shape factor is calculated automatically.

4. Preferences, Model Properties

In ***QMSys GUM software*** the user can configure several aspects of the programs in the dialog windows *Preferences* and *Properties*. The options in the *Preferences* dialog window apply to the program and in the *Properties* dialog window for the current uncertainty analysis. The *Preferences* dialog is called via the menu command *Model* -> *Preferences* and *Properties* window is opened with the toolbar button *Properties* or via the menu *Functions* -> *Properties*.

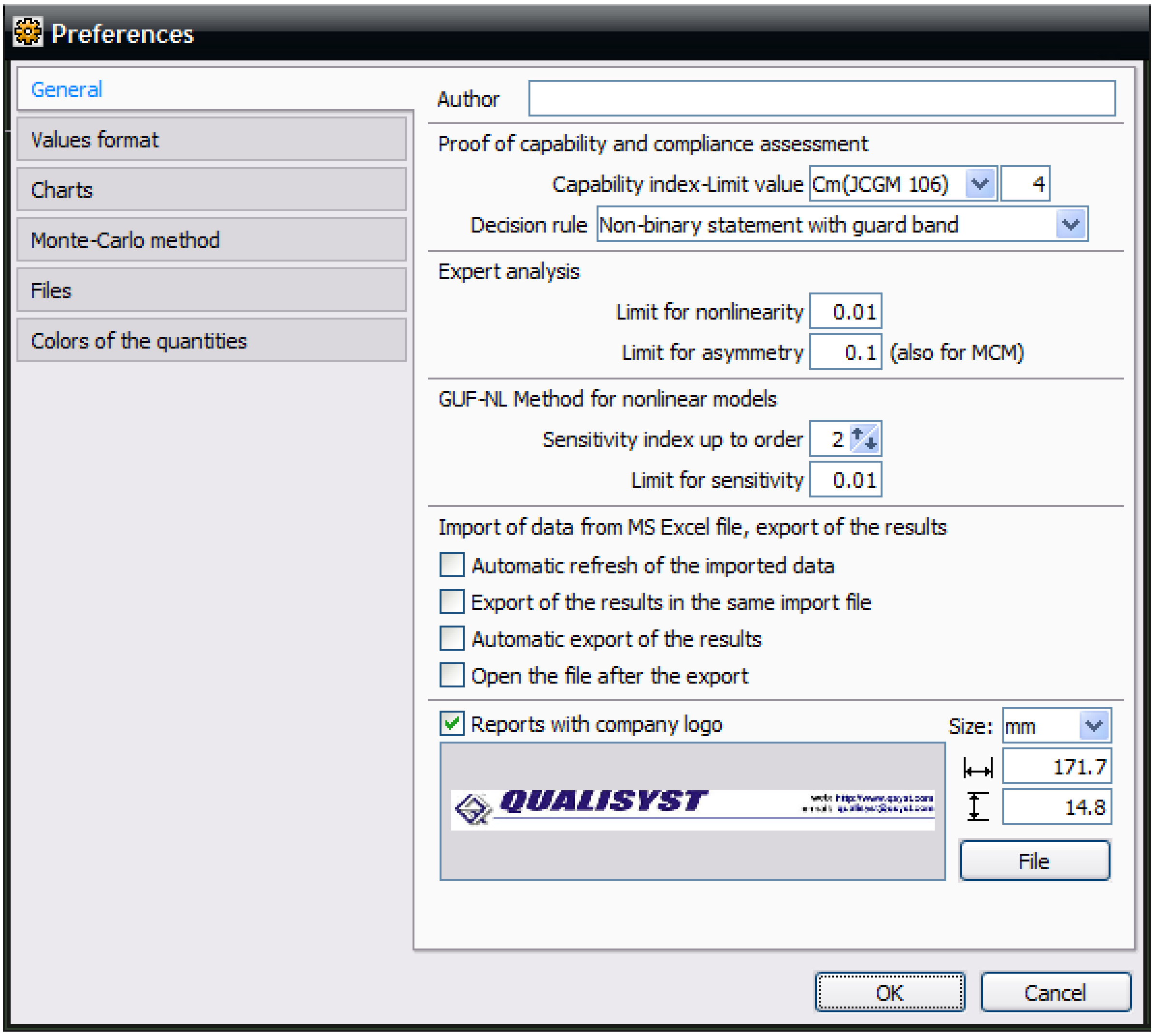
The *Preferences* dialog window is structured with the help of four register tabs grouping together different configuration settings. All changes in the settings are automatically saved. The options in the *Preferences* dialog window are used as default settings for any new analysis.

File-related options are set in the *Properties* window or in the text boxes on the appropriate program views and pages. In the *Properties* window, the first author and date of creation of the uncertainty file, the last editor and the date of modification, and the version of the file are also shown. The change in these settings has an effect only on the current analysis and is saved only in the file.

Some setting options in the editions ***GUM Professional*** and ***GUM Calculator*** are fixed and cannot be changed.

4.1. General

On the *General* tab the following settings are edited:



- Author - the author field contains the default value for the author of an uncertainty analysis. The author and the current date are saved in the field *Created* when a new measurement uncertainty file is created. When editing existing uncertainty analysis, the current editor and date are saved in the field *Modified*.

- Proof of capability and compliance assessment - in this area, the default settings for the proof of capability and compliance assessment are selected.

- Expert analysis - in these fields are entered the limits for nonlinearity of the model, for asymmetry of the distribution of the result quantities. The following table lists the default settings of the limits and the expected errors in the combined measurement uncertainty under typical and worst-case conditions:

Parameter	Limit	Average Error	Max. Error
Nonlinearity	0,05	± 0,5 %	1 %
Asymmetry	0,5	± 2 %	4 %
Sensitivity (GUF-NL)	0,01	± 0,25 %	0,5 %

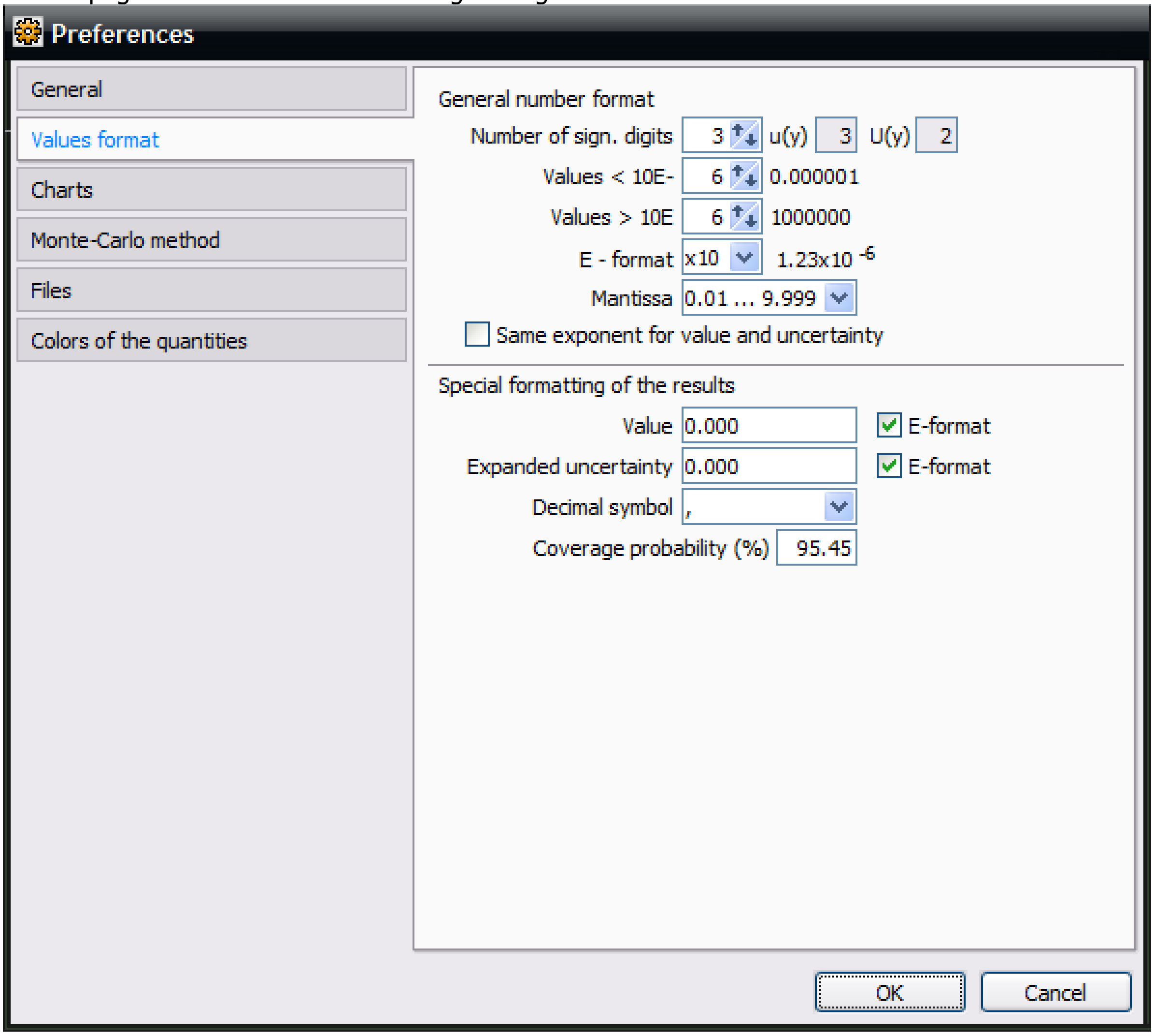
- GUF-NL Method for nonlinear models - in these fields the order of the sensitivity index and the limit for sensitivity are defined. The edition ***GUM Enterprise*** provides calculation of sensitivity indices to third order (simultaneous interaction of two and / or three input quantities). The editions ***GUM Professional*** and ***GUM Calculator*** calculate sensitivity indexes to second order (simultaneous interaction of two input quantities). In the table of the measurement uncertainty budget only uncertainty contributions of higher order with sensitivity greater than or equal to the entered limit are shown.

- Import of data from MS Excel – this option holds the checkbox for enabling / disabling the automatic updating of data imported from MS Excel files. Additionally, can be adjusted the export of results in the same or another file. If the automatic export is selected, the results will be written in the selected file automatically after each calculation of measurement uncertainty and preparation of the budget. The file containing the exported results can also be automatically opened for further processing.

- Reports with company logo - in this area, the settings for the company logo on the uncertainty of measurement reports will be carried out. A graphic file with your company logo is selected with the *File* button. After that, the logo is automatically converted and stored in the file "Logo.dat" in the program folder. There is also the possibility of adjusting the size of the company logo on the report. The including of the company logo in the report can also be turned on or off in the corresponding check box.

4.2. Values Format

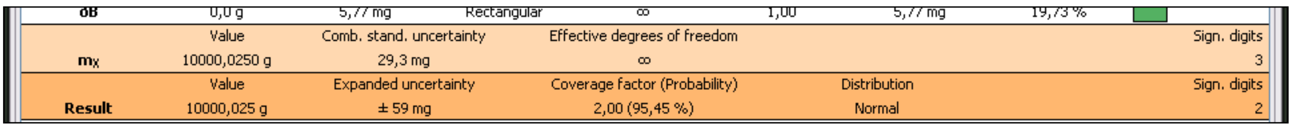
On the page *Values format* the following setting are edited:



- General number format - in this area are defined the number of significant digits and the range for numbers, which will be displayed as an exponential expression (E-format). All numbers smaller than the number, defined in the *Values < 10E* field or bigger than the number, defined in the *Values > 10E* field will be given in the E-format. Additionally, you can set the e-format (e.g., $x10^{-6}$, E-6, $.10^{-6}$) and the mantissa. The exponent is always a multiple of three and is chosen so that the mantissa is in the range given by the *Mantissa* field. The option *Same exponent for value and uncertainty* forces the same exponent on the value and the associated uncertainty of a quantity.

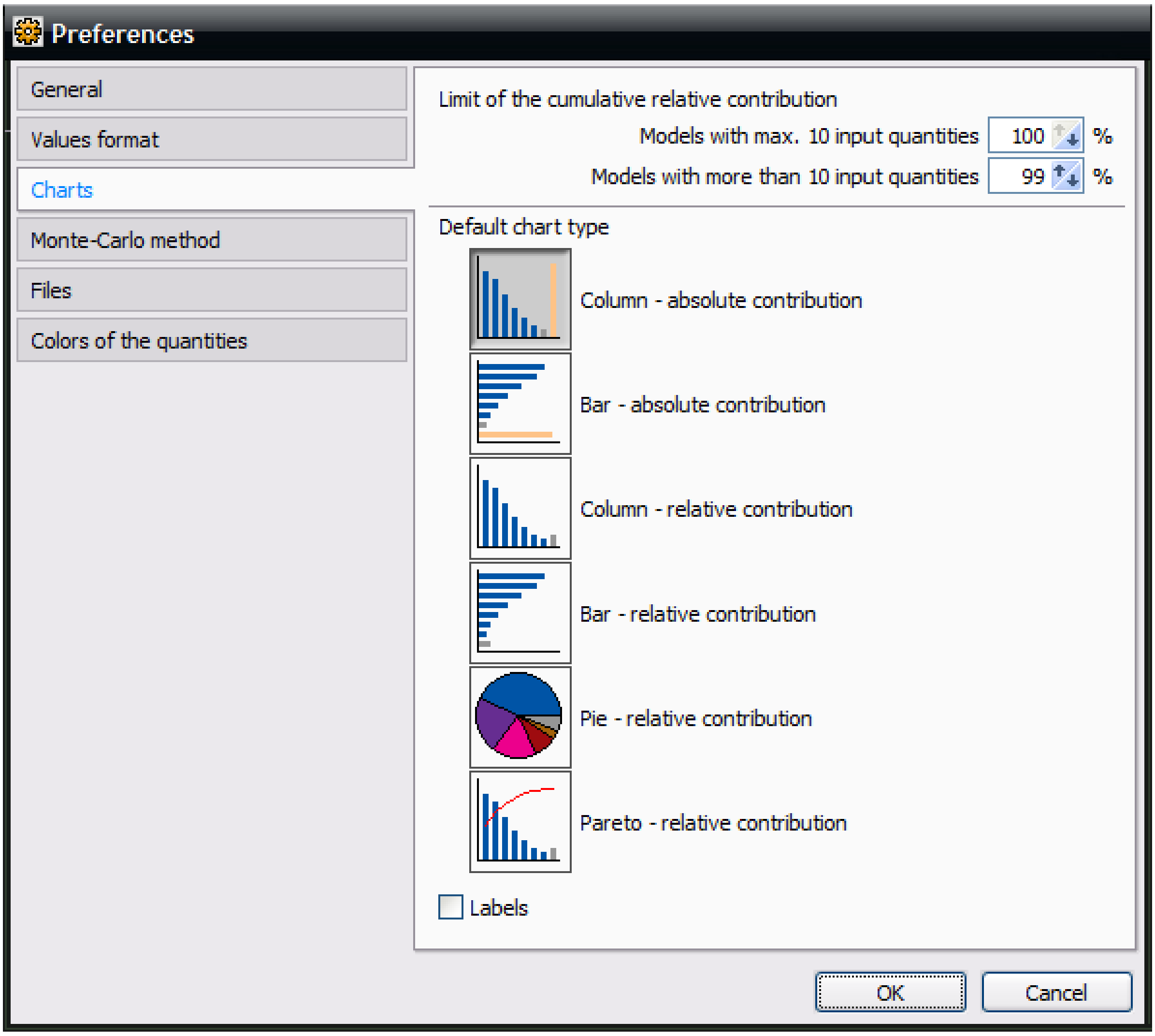
- Special formatting of the results - in these fields the default settings for the special format of the value and the expanded uncertainty are entered. The default *coverage probability*, the options for *E-format* and *Decimal symbol* of the result quantity can also be predefined. To format fractions, or numbers with decimal points, include digit placeholders: # (number sign) displays only significant digits and does not display insignificant zeros; 0 (zero) displays insignificant zeros, if a number has fewer digits than there are zeros in the format.

If the special formatting of the result quantities in the view *Main data* on the register page *Model* is not selected, the expanded measurement uncertainty will be formatted with the given number of significant digits minus one.



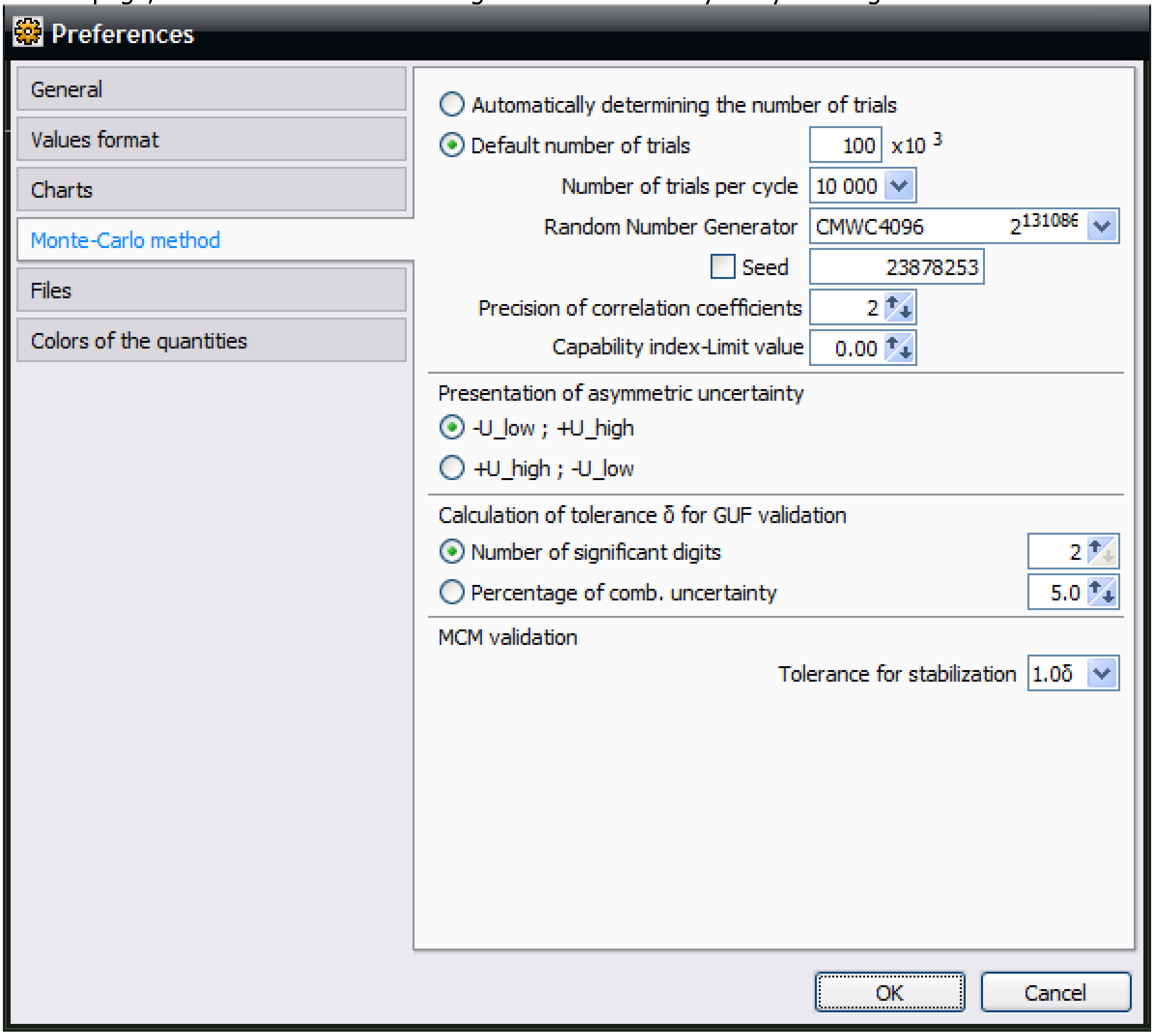
4.3. Charts

On this page can be selected the default chart type for the graphical illustration of the most significant sources of uncertainty. The software offers several types of charts and adjustable limit of the cumulative relative uncertainty contribution for simple models (max. 10 input quantities) and complex models.



4.4. Monte Carlo Method

On this page, the user can edit the settings for the uncertainty analysis using the Monte Carlo method:



- General settings - at this area the total number of trials (simulations), the number of simulations in a cycle, the random number generator, the fixed seed, and the precision of the correlation coefficient between the simulated values (number of decimal places) are defined. A higher precision of the correlation coefficients can also increase the computing time.

- Presentation of asymmetric uncertainty - here the order of presentation of the asymmetric areas of the expanded uncertainty is selected, for example, [-0.14, +0.08] or [+0.08, -0.14].

- Calculation of tolerance δ for GUF-validation - the numerical tolerance δ is determined based on the combined standard uncertainty and the number of significant digits (2 or 3). The software offers an alternative calculation of the tolerance δ as a percentage of the combined standard uncertainty, which allows a more accurate validation with a given probability.

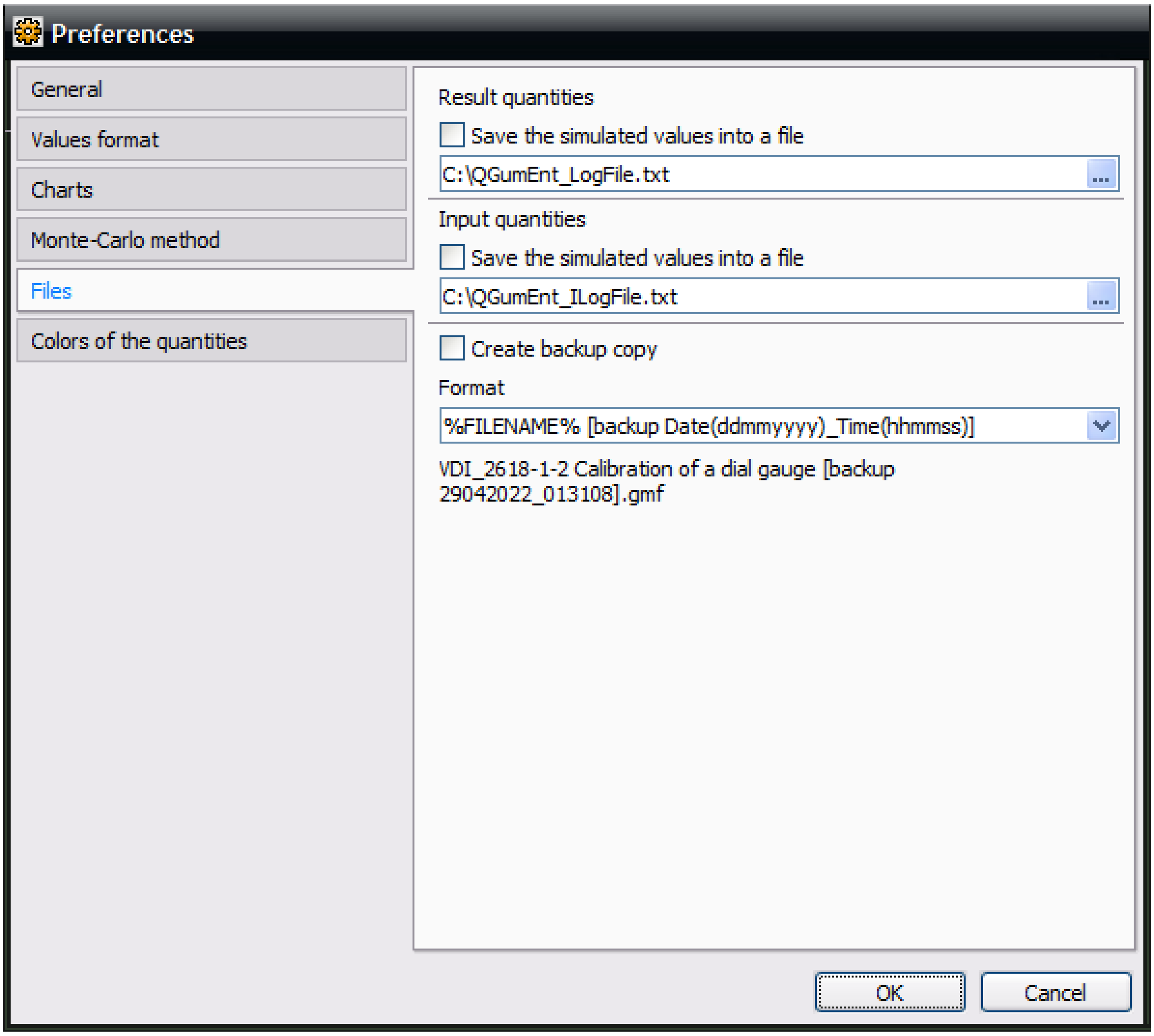
  Examples:

Comb. uncertainty (Significant digits)	Significant digit for calculating δ	Tolerance δ	Tolerance δ as a percentage of the combined uncertainty
0,1 (1)	2	0,05	50,00%
0,9 (1)	2	0,05	5,56%
0,10 (2)	3	0,005	5,00%
0,90 (2)	3	0,005	0,56%

- Validation of the Monte Carlo method - at this point the numerical tolerance for the stabilization of the parameters of the resulting distribution is given. The possible values are from 0.1*δ to 1.0*δ; *GUM Supplement 1* recommends the value 0.2*δ.

4.5. Files

On the *Files* page the following settings are edited:



- Result quantities – with this option the user can enable the storing of the simulated values of the result quantities in a specific file. The activation of this option will increase the computing time.

- Input quantities - this option enables the storing of the simulated values of the input quantities in a specific file. The activation of this option will increase the computing time.

- Create backup - this setting enables or disables the automatic creation of backup copy file when saving an existing uncertainty analysis. Pre-defined formats of the file name are available in the select list.

4.6. Colors of the quantities

For each quantity can be selected different font color to display the short names of the quantities in general list. By default, the undefined quantities are with grey font, results with blue font and all other quantities with black font.

5. Performing an Uncertainty Analysis

The steps below outline the basic procedure for creating an uncertainty analysis with the *GUM QMSys Software*:

1. A new blank uncertainty analysis starts with the command ***New*** / button. Alternatively, the user can open and edit an existing analysis, and save it under a new name.

2. On the ***Description*** page in the view ***Main data*** the user can enter descriptive title and general description of the procedure.

3. The mathematical equations of the measurement process are entered in the page ***Model*** (see also the structure of the model equation). With the ***Model analysis*** command / button the program will analyze the model equations and produce a list of quantities.

4. Processing of the quantities - the quantity type and other necessary details are entered for the quantity, selected left in the list.

5. Observation data for quantities of type A are inserted in the view ***Observation***.

6. The correlation coefficient between the input variables are analyzed or edited on the ***Correlations*** page in the view ***Main data***.

7. In the ***Total budget*** page the user can activate the summarizing of the results in a table and set the parameters for the regression analysis or the correlation analysis of the result quantities.

8. With the selection of ***Expert analysis*** view an expanded analysis of the model is carried out. The software checks the conditions for the application of the different methods and determines the appropriate methods for the following calculation of the measurement uncertainty. With the *Apply* button the settings for the following calculation of the measurement uncertainty are set automatically. Manual selection and adjustment of the method for the uncertainty analysis is also possible.

9. With the selection of the ***Budget*** view the software applies the selected methods for the calculation of the measurement uncertainty. The result is displayed in the pages ***GUF*** and / or ***Monte Carlo*** in the view ***Budget***.

10. Generated reports can be stored, printed, or sent via e-mail.

11. The uncertainty analysis is saved using the command / button ***Save*** or ***Save as***. For large models with many quantities, it is recommended to save the analysis in different stages of the analysis process.

The button *Automatic uncertainty analysis* in the main toolbar performs the steps 8 and 9 – conducting the expert analysis, selecting the appropriate method (GUF, MCM) and calculating the measurement uncertainty.

Note: For non-linear models do not insert any additional contributions that take into account the higher order terms and this way correct the nonlinearity of the model - as in DKD-3 Example 4, uncertainty contribution u(δα, At). With the ***GUF-NL*** method for nonlinear models the program automatically determines the uncertainty contributions of higher order terms and takes them into account when calculating the combined measurement uncertainty.

5.1 Validation of the GUM Uncertainty Framework Using the Monte Carlo Method

The GUM uncertainty framework can be expected to work well only for linear and nonlinear models with symmetric distribution of the result quantities. Moreover, further requirements regarding covariance, degrees of freedom and probability distribution of the input quantities should be considered. It is not always straightforward to determine whether all the conditions for its appropriate application hold. Indeed, the degree of difficulty of doing so would typically be considerably greater than that required to apply the Monte Carlo method. Therefore, since these circumstances cannot readily be tested, any cases of doubt should be validated. Since the range of validity for the Monte Carlo method is wider, it is recommended to use both methods and compare the results.

The software automatically validates the results of the GUF Method by comparing the values, the combined standard uncertainties, and the limits of the coverage intervals. The numerical tolerance δ in this comparison is calculated based on the combined standard uncertainty and the number of significant digits (2 or 3). The program offers an alternative calculation of the tolerance δ as a percentage of the combined standard uncertainty.

Should the comparison be positive, then the GUM uncertainty framework can be used on this occasion and for sufficiently similar models in the future. Otherwise, consideration should be given to using MCM or another appropriate method instead.

5.2. Rounding of Numbers

The numerical values in the uncertainty budget are rounded automatically according to the rules in EA - 4/02 point 6.3:

- The standard uncertainty, the combined uncertainty, the sensitivity coefficient, and the uncertainty contribution are rounded to the specified number of significant digits, default value is 3.
- The expanded uncertainty is rounded to the specified number of significant digits minus one, default value is 2. The software offers the possibility to enter separate special formatting for the value and expanded uncertainty for each result quantity.
- The value of each quantity is shown with the precision of the corresponding standard or expanded uncertainty.
- The coverage factor is rounded to three significant figures.
- For the roundness procedure, the rules for rounding from numbers are used (ISO 31-0: 1992, appendix B).
- If the numerical value of the uncertainty decreases due to the roundness by more than 5%, the rounded-up value is displayed.

Decimal symbol for values when specifying the quantity, in measurements and evaluations is determined by the setting of MS Windows.

5.3. Pseudo-Random Number Generators

In the editions ***QMSys GUM Enterprise*** and ***QMSys GUM Professional*** you can choose between many different generators.

Generator	**Period 2^x**	**Period 10^x**
CMWC4096 by Dr. Marsaglia	2^{131086}	$6,58*10^{39460}$
Mersenne Twister	$2^{19937} - 1$	$4,32*10^{6001}$
ISAAC	2^{8295}	$1,10*10^{2497}$
xor4096	2^{4096}	$1,05*10^{1233}$
TT800	2^{800}	$6,61*10^{240}$
AES	2^{128}	$3,39*10^{38}$
Kiss123	2^{123}	$1,07*10^{37}$
Enhanced Wichmann-Hill	2^{121}	$2,63*10^{36}$
Taus113	2^{113}	$1,05*10^{34}$
Taus88	2^{88}	$3,09*10^{26}$
Salsar	2^{70}	$1,17*10^{21}$
Standard	2^{32}	$4,97*10^{9}$

By default, the *CMWC4096* generator is used to produce the random numbers. In the menu *Model-Preferences* you can choose another generator.

5.4. Versions of QMSys GUM Files

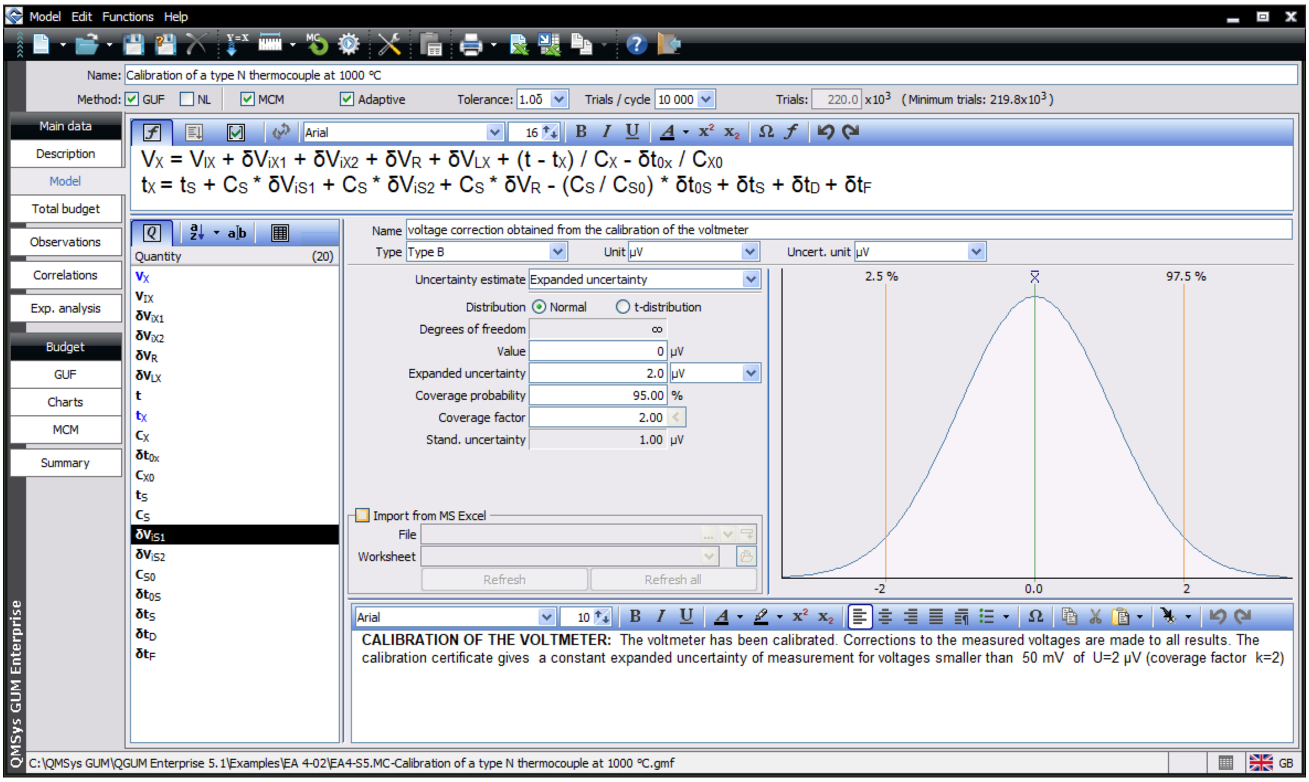
QMSys GUM software saves the uncertainty analysis into binary files with extension "*.gmf". The version of the file corresponds to the version number of the program, with which the file is last saved. There are different versions of the file structure (4.8 to 4.12, 5.1), all of which are backward compatible. A new version of the ***QMSys GUM software*** can open and process files created with the older program version, while the files saved with a newer software version are no longer readable with an older version.

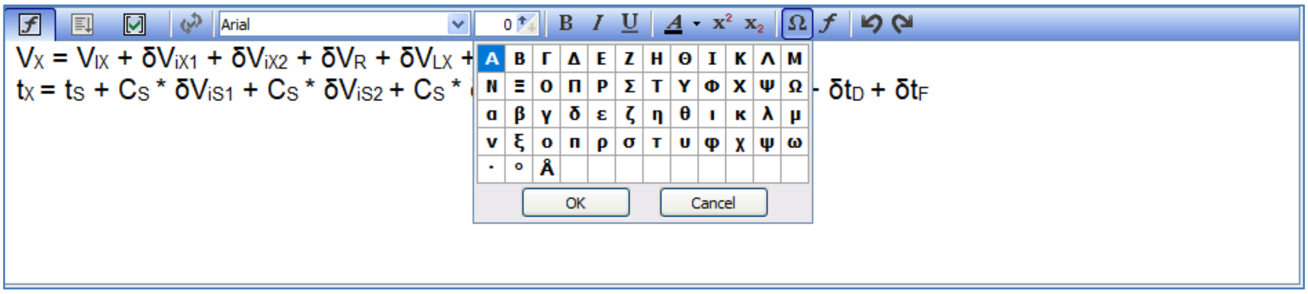
The different editions of the ***QMSys GUM software*** (Enterprise, Professional, Calculator) use the same file structure. Therefore, files created e.g., with the ***QMSys GUM Enterprise*** can be opened with any other edition.

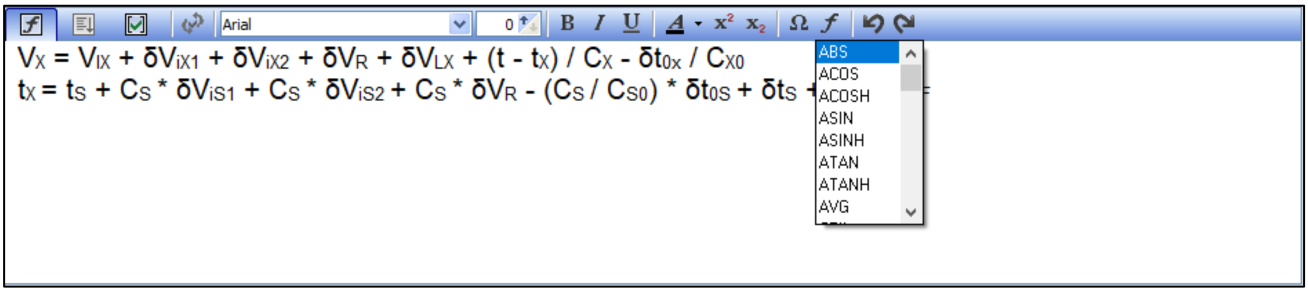
The model files for the Excel Add-In ***QMSys GUMX*** are prepared with the software editions ***QMSys GUM Enterprise / Professional*** and saved in the special format with extension "*.gxl".

6. Creating the Mathematical Model Equations

In the upper field on the dialog page *Model* in the view *Main data* the equations of the mathematical model can be entered. The model equations are the starting point for all subsequent calculations by the software. It is always possible to insert new quantities into the equation, and to rename or to delete existing quantities. Additional functions are available in a toolbar above the equation field.







The individual equations are separated by a line break. With the *Model analysis* command / button or by leaving the analysis equation field the program will analyze the model equations and produce a list of quantities. Note that the equations in the equation field will not wrap around when their length exceeds the visible area. By using the scroll bar, which appears below the equation field, the right-hand side of long entries may be inspected.

The equation field can be resized. When placing the cursor right below the equation field, it will change into a divider symbol. By pressing and dragging, the proportion of the screen used for the equation field and the list of quantities can be adjusted. The field equation can change in size by clicking on the lower separation line and by moving it up or down.

6.1. Structure of the Model Equation

The mathematical model represents the procedure of the measurement and the method of evaluation. It describes how the values of the output quantity are obtained from values of the input quantities. An ideal measurement model relating the input quantities to the output quantity is initially developed. The model is then augmented by terms constituting further input quantities, describing the effects that influence the measurement.

In the GUM approach the input quantities are characterized by probability distributions and treated mathematically as random variables. The probability distributions are chosen such that the estimates of the input quantities correspond the expectations of the probability distributions.

The estimated values of the input quantities, the influence factors and the measurement results are related as follows:

Result Quantity = f(Input quantities; Influence factors)

The right side represents an algebraic expression consisting of an appropriate combination of mathematical operators and functions, as well as numbers and input quantities. The algebraic expression is evaluated according to common mathematical rules of operation when the uncertainty analysis is performed.

The individual equations are separated by a line break.

Especially when the model equation is more complex and has a bigger number of input quantities it should be split into smaller parts (and thereby made easier to understand) by introducing interim result.

The equations for interim results have the same structure as the model equation itself. The order of the equations in the Equation field is irrelevant. For interim results the following rules must be followed:
1. Every interim result must be used on the right side of an equation.
2. Every interim result must be calculable without any direct or indirect use of its own value. Mathematical loops are not allowed!
3. It is allowed to use interim result multiple times on the right side of other equations.

For interim result no separated uncertainty calculation is done. The value is listed in the budget for transparency and traceability reasons.

Explanation texts inside of the model equation field must be enclosed with {}. Text in curly brackets will be ignored in the evaluation of mathematical equations.

The model equation can be changed at any time. With the *Model analysis* command / button or by leaving the analysis equation field the program will analyze the modified model equations and actualize the list of quantities. The Equation field can only be left if the equation is mathematically correct. If this is not the case, an error message will be displayed, and the cursor is positioned at the point where the error was detected.

If any changes to a quantity name have been made, the new name will replace the old name in the quantity list. All data connected with the old quantity will also be deleted.

A change of a quantity name without losing the assigned data can be made in the list of quantities with second click on the selected quantity or with double click when selecting the quantity. Additional Greek characters are available over the combination of keys "*Ctrl + s*" or over the context menu of the right mouse button. Superscript and subscript formats are available over the combinations of keys "*Ctrl + Up Arrow*", "*Ctrl + Down Arrow*" or over the context menu of the right mouse button. By pressing *ENTER* for confirmation after the name was changed (or by leaving the quantity name field) the model equation is updated automatically, and the corresponding entries are retained. The change of the quantity name can be cancelled with the *ESC* button.

6.2. Operators and Functions

Operations with higher priority are executed before those with a lower priority. In case of equal priority, execution proceeds from left to right, except when raising to a given power, in which case execution proceeds from right to left. The sequence of operations can be controlled by putting certain parts in parentheses and thereby giving them a different priority.

Function names are reserved names and should not be used as quantity names. The arguments for the trigonometric functions should be expressed in radians. Likewise, the result of any arc-function will be given in radians.

Following table lists the possible mathematical operators and functions:

Function	Syntax	Description
Quantity	**Cx**	The selection of an appropriate name is up to the user, but it must be done carefully. In many cases, the quantity name could be deviated from a name of a physical quantity. Quantity names can contain any combination of letters or digits and are handled case sensitive. The names of the build in functions and the names SUB, DVN, MULT, POWER, DIV, MOD are not valid as quantity names.
+	**Ca+Cb**	Addition.
-	**Ca-Cb**	Subtraction.
*****	**100*Cb**	Multiplication.
/	**Ca/100**	Division.
^	**Cb^x**	Raise to the power of x.
ABS	**ABS(Ca-Cb)** **ABS(Ca)**	Absolute value.
ACOS	**ACOS(Ca)**	Arc cosine, result in [rad].
ACOSH	**ACOSH(Ca)**	Arc cosine hyperbolic, result in [rad].
ACOT	**ACOT(Ca)**	Arc cotangent, result in [rad].
ACOTH	**ACOTH(Ca)**	Arc cotangent hyperbolic, result in [rad].
ASIN	**ASIN(Ca)**	Arc sine, result in [rad].
ASINH	**ASINH(Ca)**	Arc sine hyperbolic, result in [rad].
ATAN	**ATAN(Ca)**	Arc tangent, result in [rad].
ATANH	**ATANH(Ca)**	Arc tangent hyperbolic, result in [rad].
AVG	**AVG(Ca;Cb)**	Mean value ( ), max. 30 arguments are permitted.
COS	**COS(Ca)**	Cosine, argument in [rad].
COSH	**COSH(Ca)**	Cosine hyperbolic, argument in [rad].
COT	**COT(Ca)**	Cotangent, argument in [rad].
COTH	**COTH(Ca)**	Cotangent hyperbolic, argument in [rad].
DEG	**DEG(Ca)**	Converts radians to degrees.
EXP	**EXP(Ca)**	Returns "e" raised to the power of argument. The constant "e" equals 2.71828182845904, the base of the natural logarithm.
FACT	**FACT(Ca)**	Returns the factorial of an argument.
LN	**LN(Ca)**	Returns the natural logarithm of an argument. Natural logarithms are based on the constant "e".
LOG	**LOG(Ca;2)**	Returns the logarithm of an argument to the base you specify. If base is omitted, it is assumed to be 10.
LOG2	**LOG2(Ca)**	Returns the base-2 logarithm of an argument.
LOG10	**LOG10(Ca)**	Returns the base-10 logarithm of an argument.
MAX	**MAX(Ca;Cb)**	Returns the largest value in a set of arguments, max. 30 arguments are permitted.
MIN	**MIN(Ca;Cb)**	Returns the smallest value in a set of arguments, max. 30 arguments are permitted.
PI	**PI**	Returns the number 3.14159265358979, the mathematical constant pi, accurate to 15 digits.
PRODUCT	**PRODUCT(Ca;Cb)**	Product of the arguments, max. 30 arguments are permitted.
RAD	**RAD(Ca)**	Converts degrees to radians.
SIN	**SIN(Ca)**	Sine, argument in [rad].
SINH	**SINH(Ca)**	Sine hyperbolic, argument in [rad].
SQRT	**SQRT(Ca)**	Returns a positive square root.
STDEV	**STDEV(Ca;Cb)**	Estimates standard deviation based on a set of arguments, max. 30 arguments are permitted.
SUM	**SUM(Ca;Cb)**	Adds all the arguments, max. 30 arguments are permitted.
SUMSQ	**SUMSQ(Ca;Cb)**	Returns the sum of the squares of the arguments, max. 30 arguments are permitted.
TAN	**TAN(Ca)**	Tangent, Argument in [rad].
TANH	**TANH(Ca)**	Tangent hyperbolic, argument in [rad].
VAR	**VAR(Ca;Cb)**	Estimates variance based on a set of arguments, max. 30 arguments are permitted.
INDEX	**INDEX n=(1:9)**	Quantity index for simplifying the equations of measurement models with repetitive identical expressions.

Error function

Function	Syntax	Description
$\mathbf{erf}(\mathbf{x})$	**ERF(X)**	Error function for a real argument X
$\mathbf{erfc}(\mathbf{x})$	**ERFC(X)**	Complementary error function for a real argument X
$\mathbf{exp}(\mathbf{x}).\,\mathbf{erfc}(\sqrt{\mathbf{x}})$	**EXPERFC(X)**	X >= 0
$\mathbf{exp}(\mathbf{x}^2).\,\mathbf{erfc}(\mathbf{x})$	**EXPERFC2(X)**	X is a real number
$\mathbf{exp}(\mathbf{x}).\,\mathbf{erfc}(-\sqrt{\mathbf{x}})$	**EXPERFCN(X)**	X >= 0

6.3. Models with Indexed Quantities

The indexed quantities can be used for simplifying the models in following cases:

- Models for measurement of area, volume, flow, and other quantities by using numerical integration methods (i.e., approximating definite integrals by finite sums)
- Models for calibration of measuring instruments at several points throughout the calibration range
- Models for calibration of sets of identical standards with different sizes – gauge blocks, plug and ring gauges setting rings, measuring pins, scale weights, and others.

Examples for models using the numerical integration method

- Measurement of liquid flow in open channels – mean-section method

$$Q = \sum_{1}^{n} (b_{n+1} - b_n)\left(\frac{d_{n+1} + d_n}{2}\right)\left(\frac{\bar{v}_{n+1} + \bar{v}_n}{2}\right)$$

- Measurement of liquid flow in open channels – mid-section method

$$Q = \sum_{1}^{n} \bar{v}_n d_n \left(\frac{b_{n+1} - b_{n-1}}{2}\right)$$

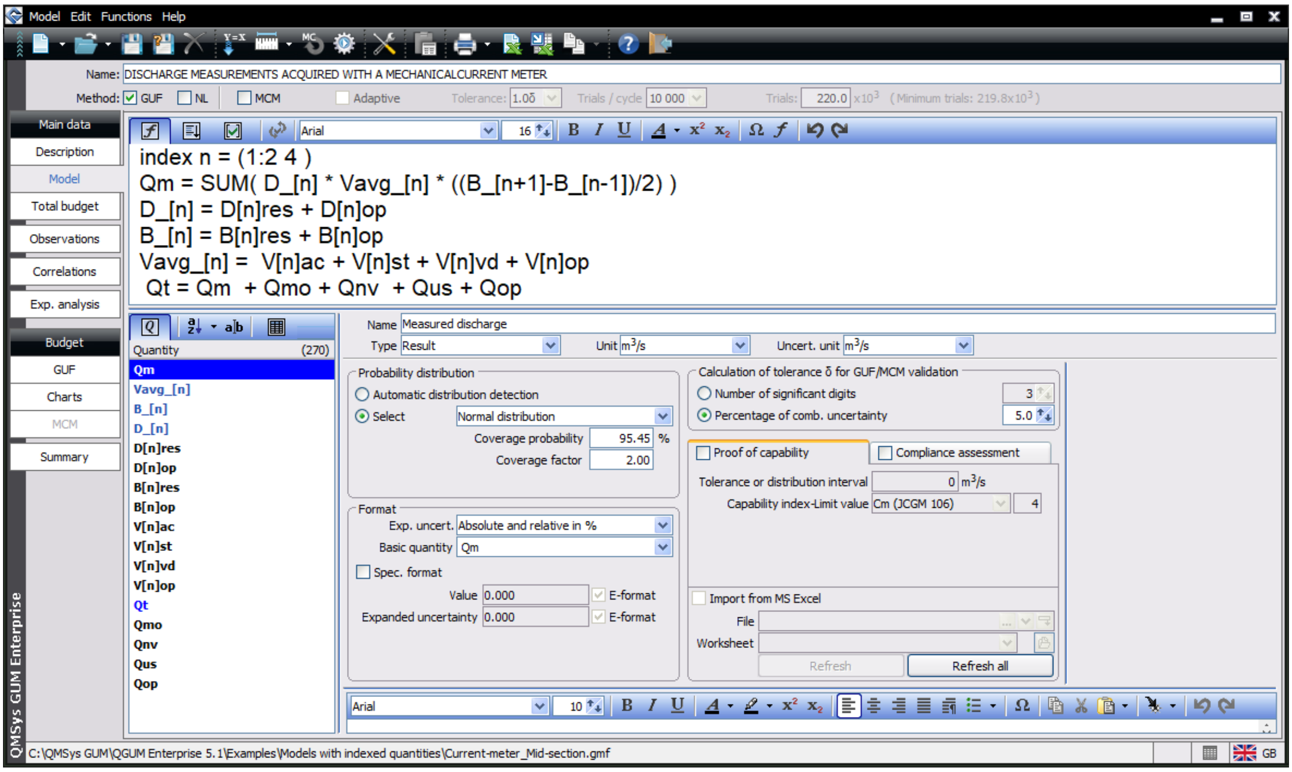
In these cases, the software offers the possibility to define an index and using it in the reduced model equations. Indexes can be entered as interval, as list of index values or mixed:

Index definition	Index values, generated by the software
index n = (1:9)	1;2;3;4;5;6;7;8;9
index n = (LB;1;2;3;4;5;6;7;RB)	LB;1;2;3;4;5;6;7;RB
index n = (LB;1:7;RB)	LB;1;2;3;4;5;6;7;RB

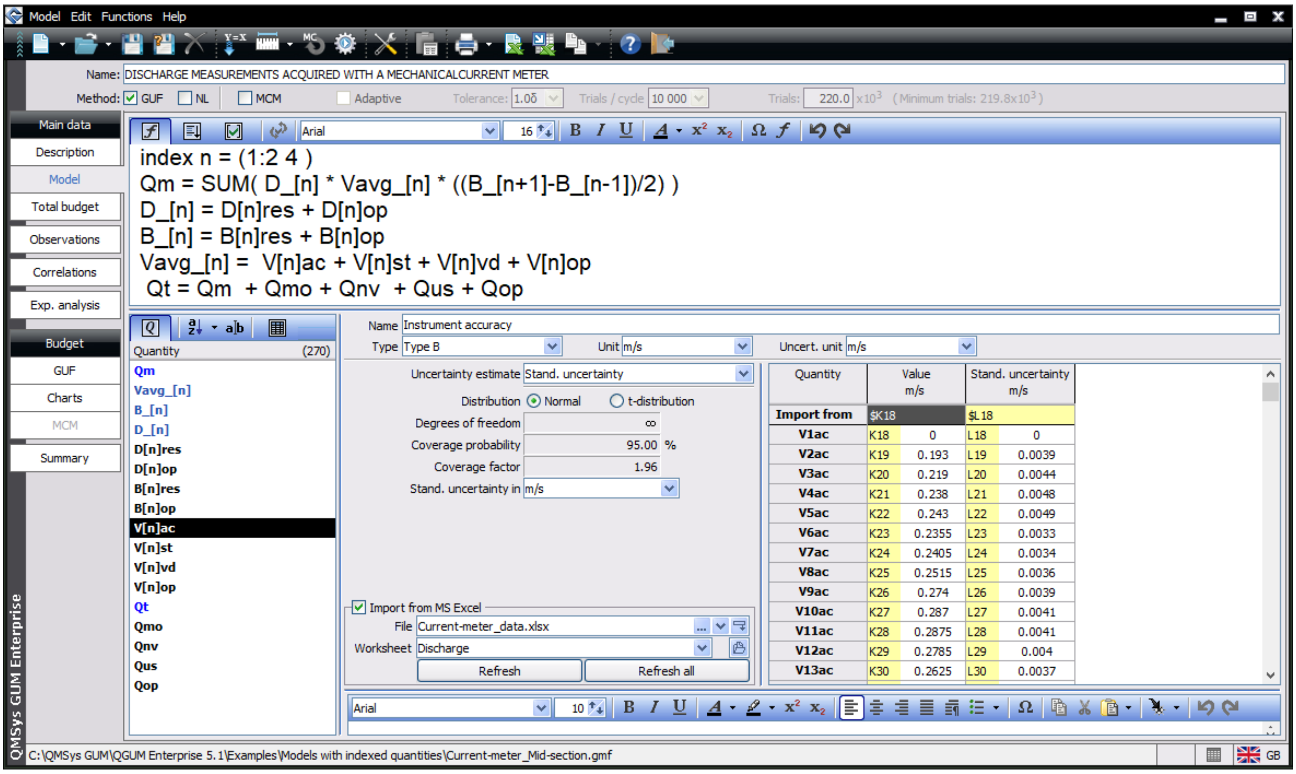
The index name is placed in the model equations after the name of quantity in square brackets []. The first and last index values are absolute limits and when using indexing expressions such as [n-1] or [n+1] for the resulting index numbers outside the defined range will be used respectively the first or last index value.

Several examples of models with indexed quantities can be found in the folder "../Examples/ Models with indexed quantities".

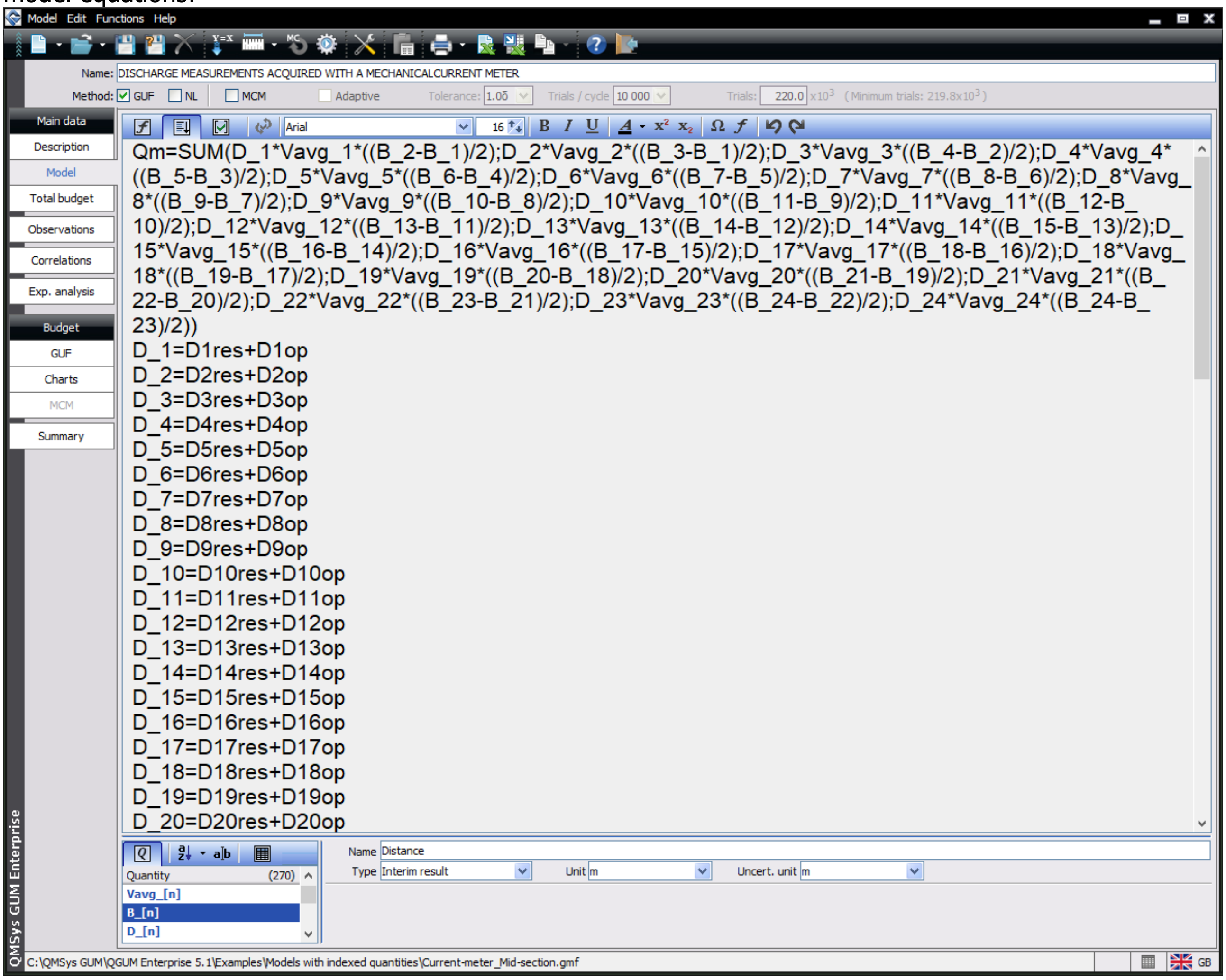
Example of model for numerical integration by using indexed quantities:



In the model equations are used the main quantities with index [n], and the values of each sub-quantity with the respective index are individually edited in the tables. When using import from Excel the program automatically initializes the appropriate cells to the quantities with the corresponding indexes.



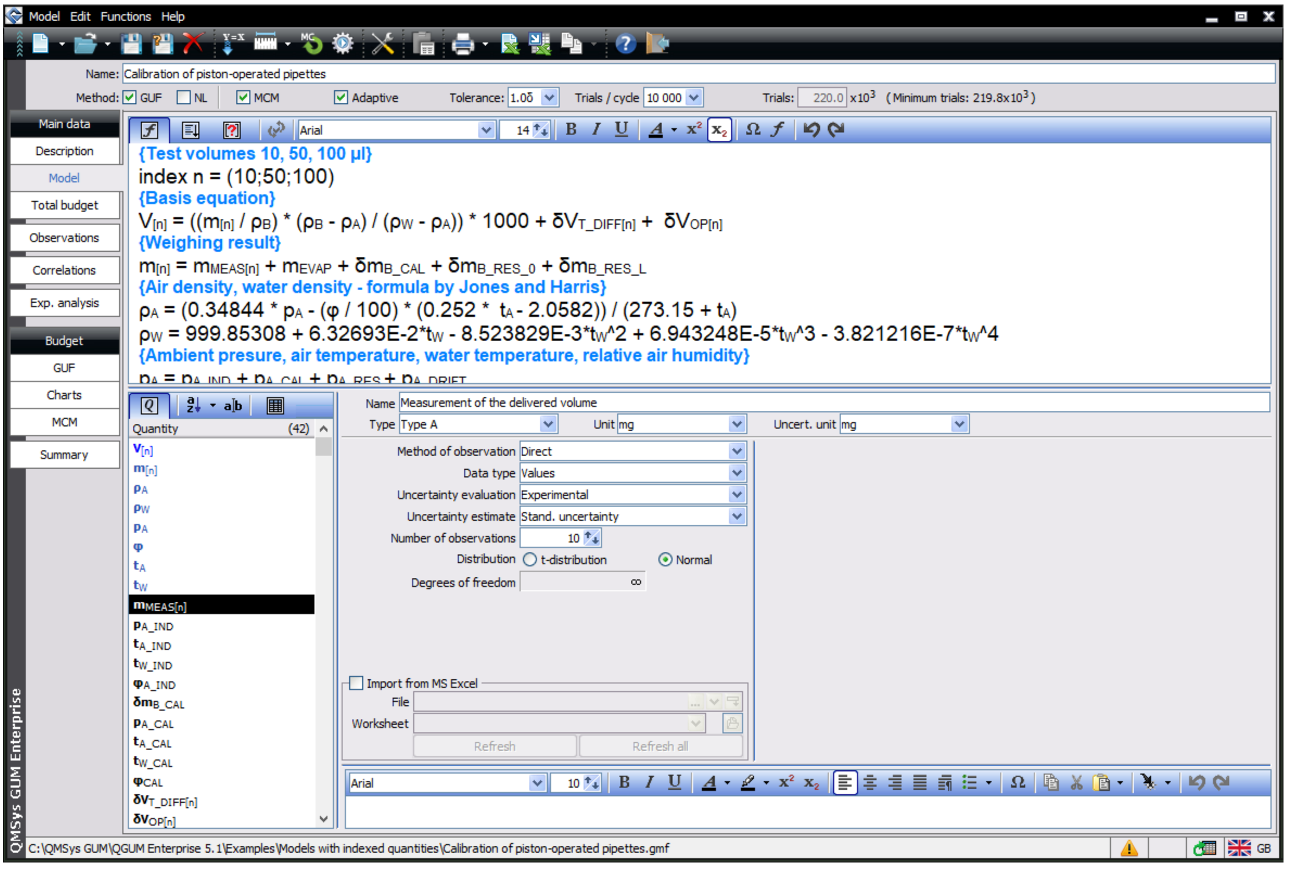
The button "Preview of the equations" in the toolbar of the model field shows a window with the full form of the model equations:

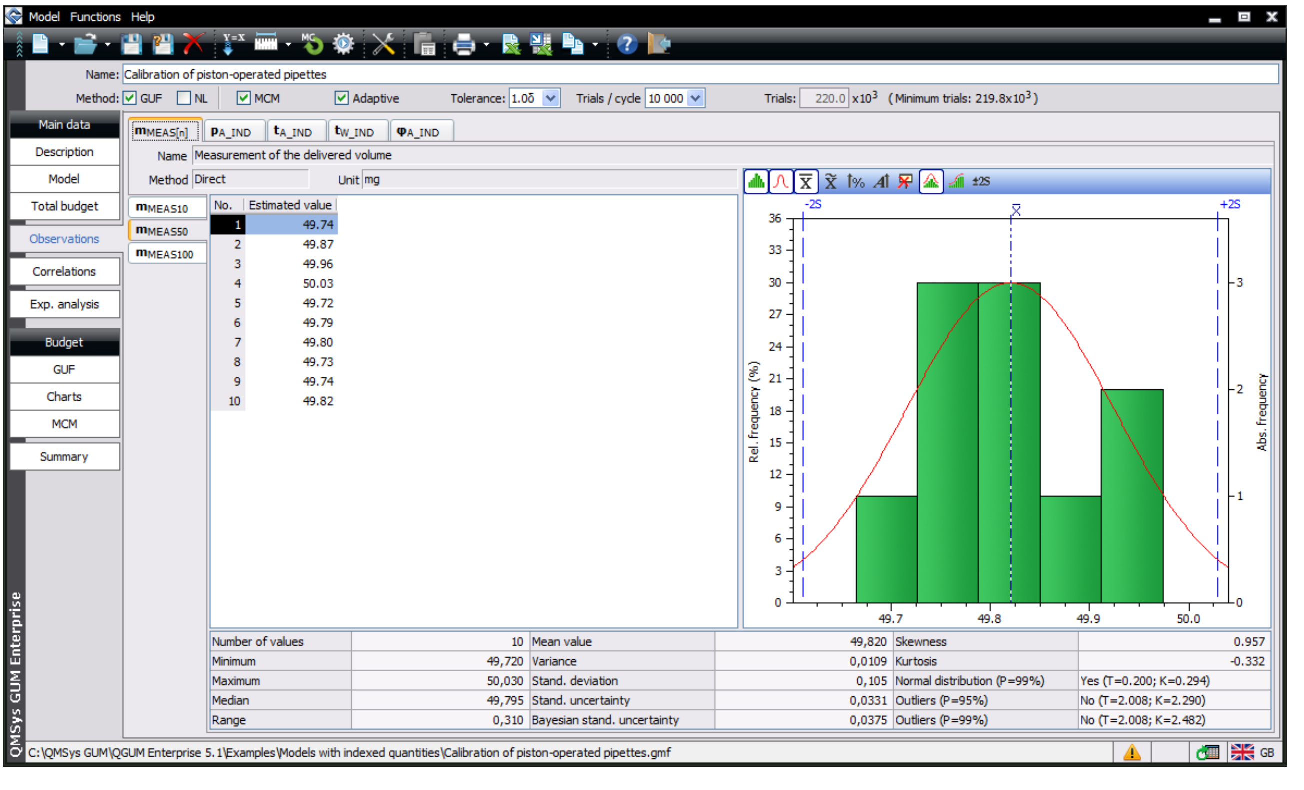


Examples for measurement at several points throughout the calibration range

The indexed quantities can also be used for simplifying the models for calibration at several points throughout the calibration range of the measuring instruments.

Example of model for calibration of piston-operated pipettes at three points:

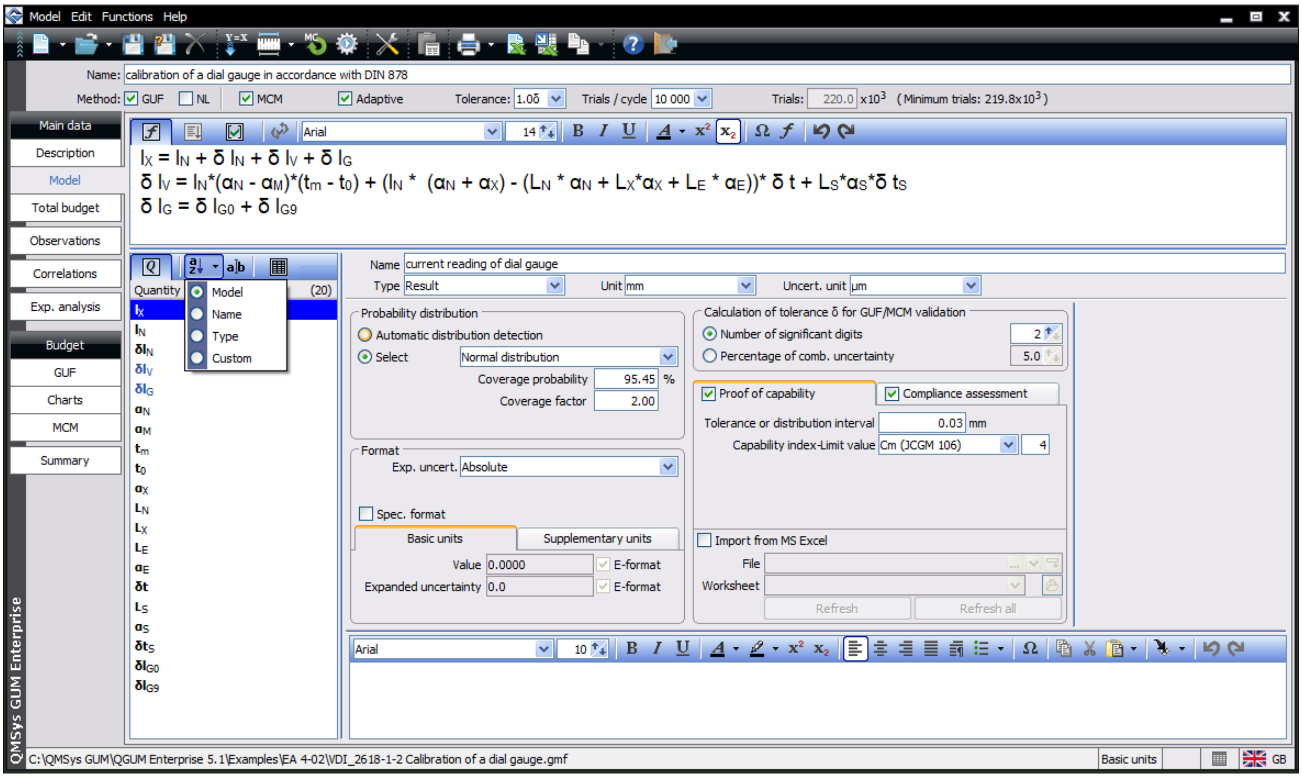




6.4. Changing the Order of the Quantities

By default, the quantities are sorted in the list by the order of appearance in the model equation.

The order of the quantities can be changed with the button  if necessary. This order is saved in the uncertainty analysis file. The next time the file is opened, the quantities will be displayed in the list in the specified order. The order of the quantities is used in all budgets, lists, reports, and exports.



Following sorting options are available:
By model - the order of the quantities results from the order in which they appear in the model equation.
By name - the quantities are sorted alphabetically by the identification in the model equation. The Greek letters are sorted alphabetically after the Latin letters.
By Type - the quantities are sorted according to their type as follows:
- Results
- Interim results
- Type A
- Type B Expanded uncertainty
- Type B Standard uncertainty
- Type B Limit of error
- Type B Relative limit of error
- Type B Molar mass
- Type B probability distribution - these quantities are also sorted by the type of distribution - normal distribution, log normal distribution, t-distribution, rectangular distribution, triangular distribution, trapezoidal distribution, square distribution, U-shaped distribution, cosine distribution, 1/2 cosine distribution, exponential, and curvilinear trapezoidal distribution.

Custom - the user can adjust the order of the quantities by selecting one quantity and dragging to the new position, or by the *Move up* and *Move down* buttons.

When adding quantities to the model equation, the previous selected sort order will be maintained, and the new quantities will appear at the beginning of the list. In this case, the quantity order needs to be maintained manually or a sort option needs to be selected.

7. Quantities

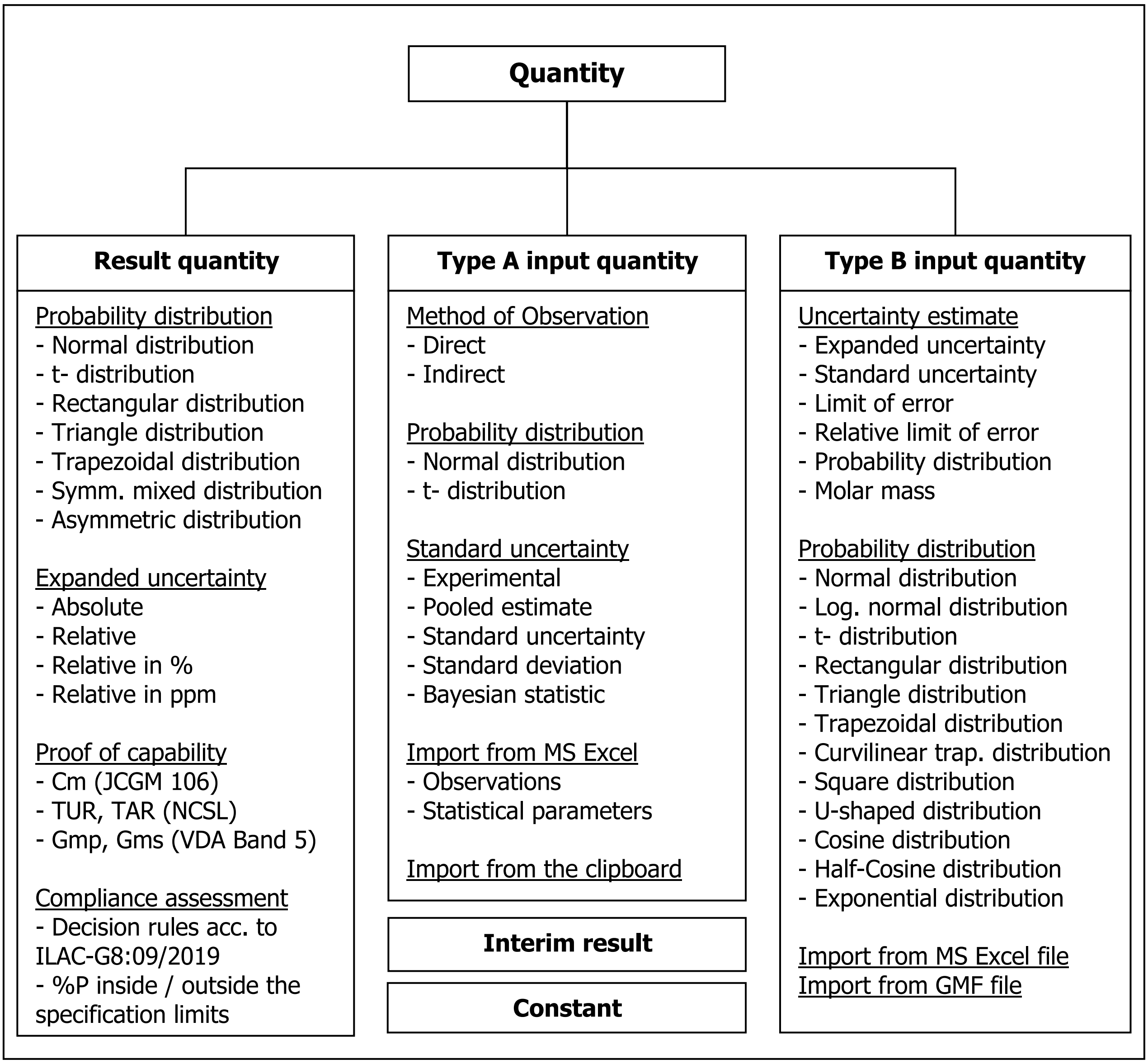
The *Model* page in the view *Mean data* holds the relevant data for every quantity of the uncertainty analysis. To edit the data of a quantity, it should first be selected from the left list. The current data, associated with the selected quantity, is displayed, and may be edited.

Each quantity can be assigned:

- short definition in the *Name* field
- type of the quantity
- units for the quantity value and the standard uncertainty
- documentation text.

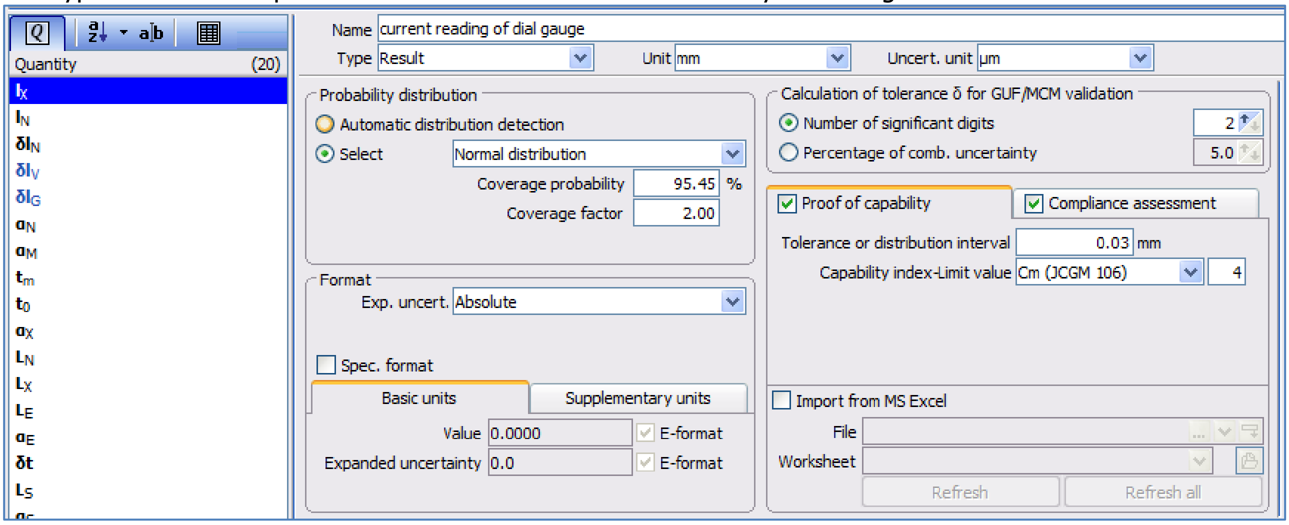
Within the *Type* field every quantity is assigned to an evaluation type, selected from a list. All other fields are dependent on the type of quantity.

The following diagram shows the different quantity types with their respective subtypes. They are described in detail on the following pages.



7.1. Result Quantities

The type of the result quantities is detected and set automatically according to the mathematical model.



<u>Automatic distribution fitting</u>
This option activates the fitting of the observed distribution to a theoretical distribution. The value of the coverage factor is determined automatically to the selected distribution and the specified coverage probability.

<u>Distribution and Calculation of the coverage factor</u>:

- Normal distribution – enter the necessary value into one of the coverage probability and coverage factor fields. Using the Normal distribution, the program will find the other value and will insert these values into the budget table.
- t-distribution - it is assumed that the distribution of the result can be described with a t-distribution and the established degrees of freedom which are calculated by the Welch-Satterthwaite formula. Enter the necessary value into the coverage probability field. Using the t-distribution and the estimated degrees of freedom, the program will calculate the coverage factor.
- Rectangular distribution - enter the necessary value into the coverage probability field and the program will calculate the coverage factor automatically.
- Triangular distribution - enter the necessary value into the coverage probability field and the program will calculate the coverage factor automatically.
- Trapezoidal distribution - enter the necessary values into the coverage probability field and shape factor field, the program will calculate the coverage factor. Calculation of the shape factor is offered by activated Monte Carlo method.
- Symmetric mixed distribution - you can enter any validated value of the coverage probability and coverage factor.

For result quantities with asymmetric distribution, the program estimates the shortest coverage interval, the asymmetric expanded measurement uncertainty and asymmetric coverage factor (*GUM Supplement 1)*.

<u>Format of the expanded uncertainty:</u> Absolute; Absolute and relative; Absolute and relative in %; Absolute and Relative in ppm.

<u>Calculation of tolerance δ for GUF / MCM validation</u> - the numerical tolerance δ is calculated on the basis of the combined standard uncertainty and the number of significant digits (2 to 5). The software offers an alternative calculation of the tolerance δ as a percentage of the combined standard uncertainty, which allows a more accurate validation with a given probability.

<u>Proof of capability and compliance assessment</u> - in this area, the settings for the verification of the capability of the measuring system respectively the measurement process and for the conformity assessment of the measurement results can be specified. The limit values can be imported from an Excel file.

7.1.1. Proof of Capability of Measurement Processes and Measuring Systems

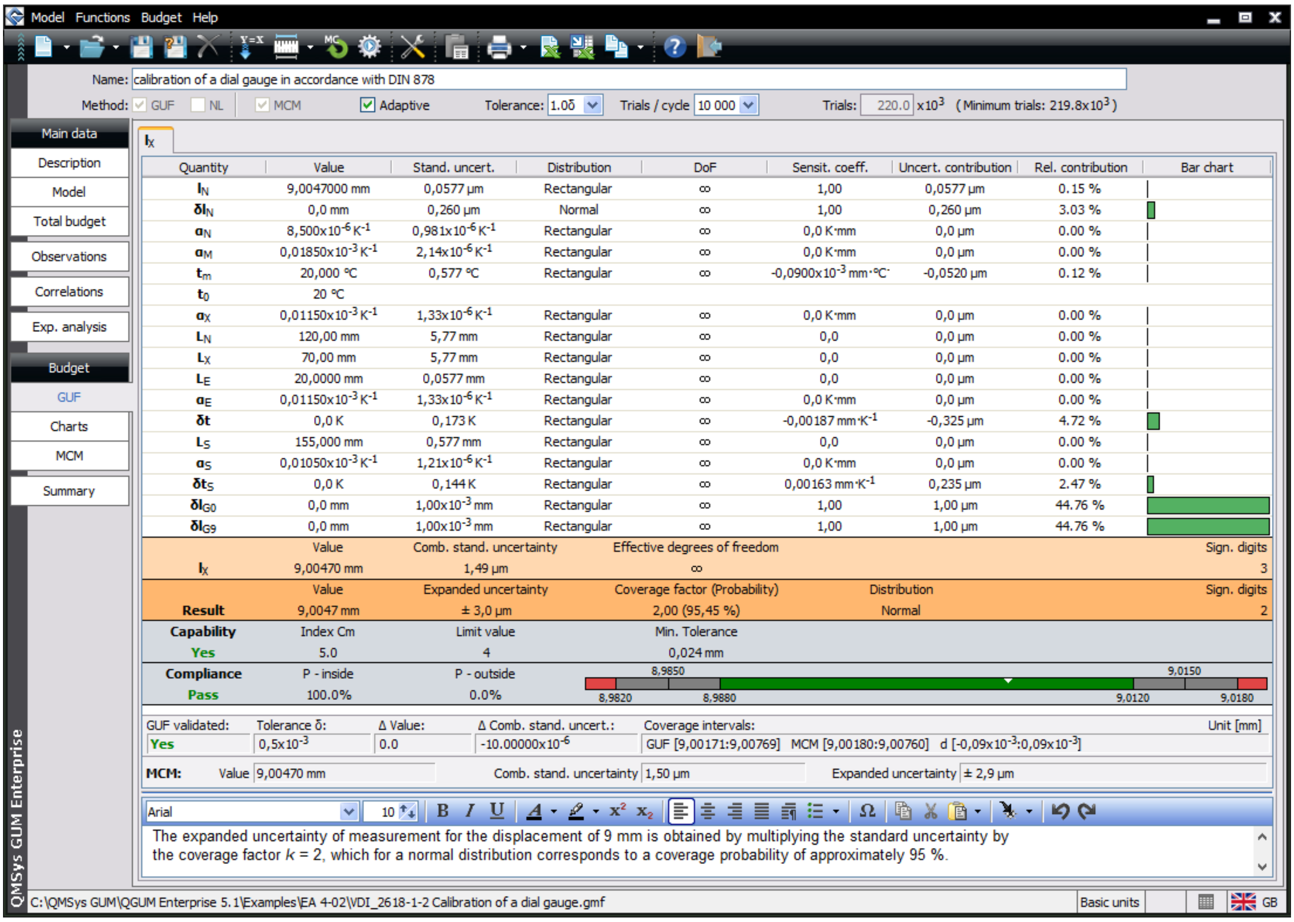
The basic approach is to set the uncertainty in relation to the tested tolerance and to use this relation as an evaluation criterion. The program offers several capability indexes for evaluating the capability of measuring systems and measurement processes:

Reference	Capability Index	Formulae*, Default Limit
JCGM 106	Cm (Measurement capability index)	$C_m = \frac{T}{2*U}$ ≥ 4
NCSL Glossary of Metrology related Terms	TUR (Test Uncertainty Ratio)	$TUR = \frac{T}{2*U_{MP}}$ ≥ 4:1
	TAR (Test Accuracy Ratio)	$TAR = \frac{T}{2*U_{MS}}$ ≥ 10:1
VDA Volume 5	Gmp (Measurement process capability)	$g_{MP} = \frac{2*U_{MP}}{T}*100\%$ ≤ 30%
	Gms (Measuring system capability)	$g_{MS} = \frac{2*U_{MS}}{T}*100\%$ ≤ 15%

*T – Tolerance or distribution interval, U_{MP} – expanded uncertainty of the measurement process, U_{MS} – expanded uncertainty of the measuring system.

For result quantities with asymmetric distribution in the above formulas is used the coverage interval instead of 2*U. In the calibration or verification of a measuring instrument, a specified requirement is often expressed in terms of a maximum permissible error (T=2*MPE). For one-sided tolerances is entered the distribution interval of the values, given from the manufacturing process.

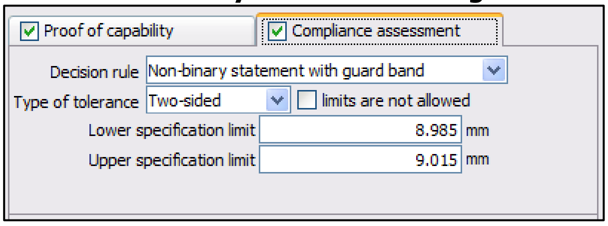
To classify measuring systems and processes, the software calculates the minimum tolerance, at which the measuring system or the measurement process is currently still capable.



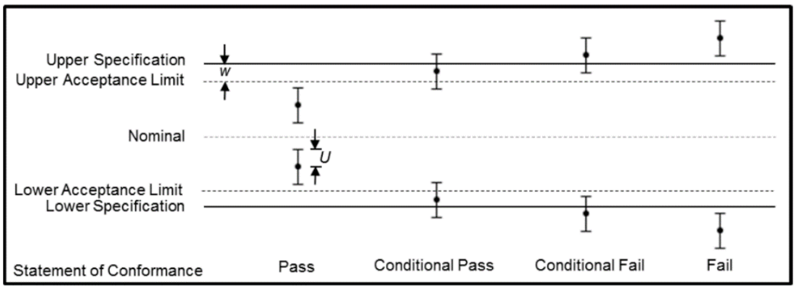
7.1.2. Compliance Assessment of Measurement Results

The software offers several decision rules in accordance with the guide ILAC-G8:09/2019:

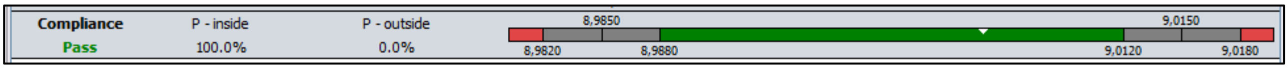
- **Non-binary statement with guard band (w = U)**



The evaluation follows ILAC-G8:09/2019, par. 4.2.3:

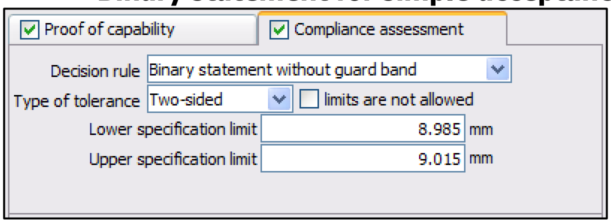


U = 95% expanded measurement uncertainty

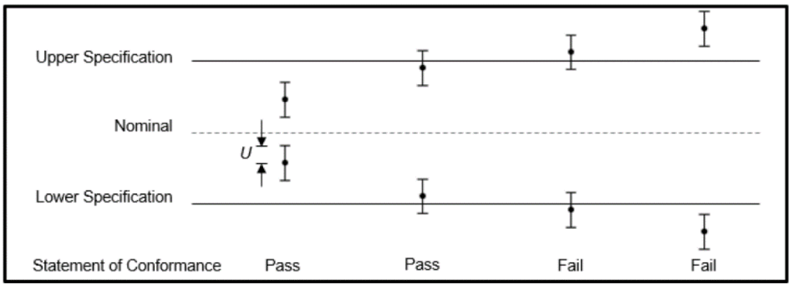


The measurement uncertainty is considered. Evaluation results are reported as:
- Pass - the measured result is inside the acceptance limits
- Conditional Pass - the measured result is outside the acceptance limits but inside the specification limits
- Conditional Fail - the measured result is outside the specification limit by a margin less than the measurement uncertainty
- Fail - the measured result is outside the specification limit by a margin more than the measurement uncertainty
- N/A - uncertainty is greater than the stated tolerance, therefore it is not possible to determine compliance or otherwise.

- **Binary statement for simple acceptance rule**



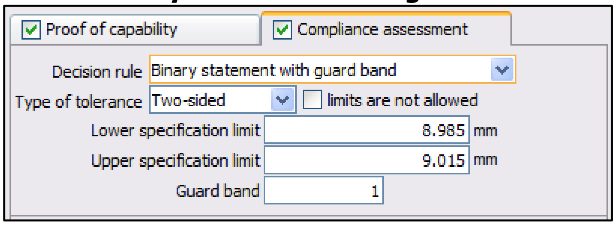
The evaluation follows ILAC-G8:09/2019, par. 4.2.1:



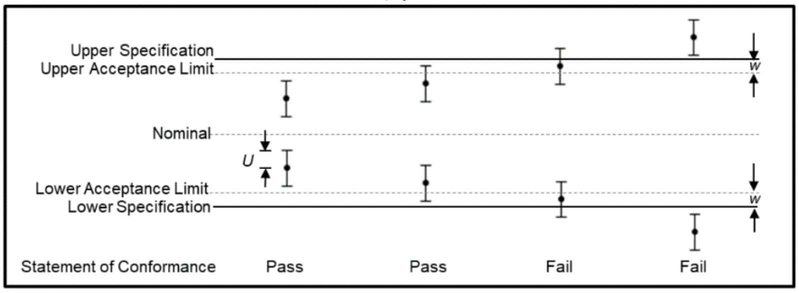
U = 95% expanded measurement uncertainty

The measurement uncertainty is not considered. Evaluation results are reported as:
- Pass - the measured result is inside the specification limits
- Fail - the measured result is outside the specification limits.

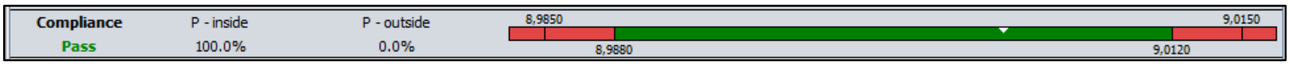
- **Binary statement with guard band**



The evaluation ILAC-G8:09/2019, par. 4.2.2:



U = 95% expanded measurement uncertainty



The measurement uncertainty is considered. Evaluation results are reported as:
- Pass - the measured result is inside the acceptance limits.
- Fail - the measured result is outside the acceptance limits.
- N/A –uncertainty is greater than the stated tolerance, therefore it is not possible to determine compliance or otherwise.

While it is common to use a guard band w = U, there may be cases where a multiplier other than 1 is more appropriate. The user can define arbitrary multiple of U between -3 and 3 to have applied as guard band.

The table below provides examples of different guard bands to achieve certain levels of specific risk, based on the customer application:

Decision rule	Guard band w	Specific Risk
6 sigma	3 U	PFA < 1 ppm
3 sigma	1,5 U	PFA < 0.16%
ILAC G8:2009 rule	1 U	PFA < 2.5%
ISO 14253-1:2017	0,83 U	PFA < 5%
Uncritical	-1 U	Item rejected for measured result outside [LSL-U; USL+U], PFR < 2.5%

PFA - Probability of False Accept, PFR - Probability of False Reject

7.2. Interim Results

The type of *Interim result* is detected and set automatically according to the mathematical model. Only following data can be edited: short definition in the *Name* field, type of the quantity, units for the quantity value and the standard uncertainty, and a documentation text.

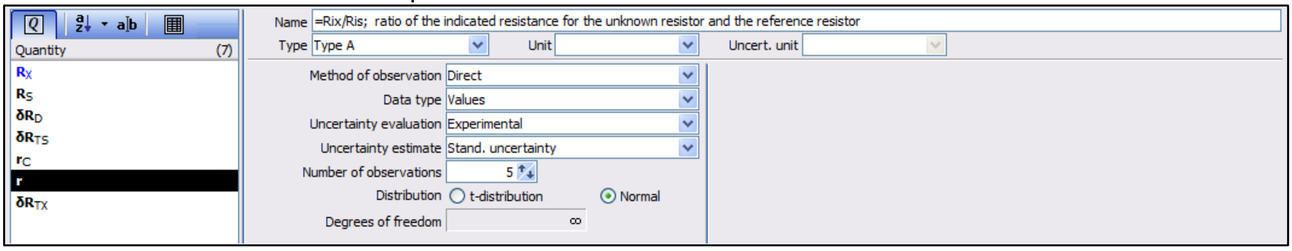
Switching to the type "Result" is possible; in this case the uncertainty budget is also calculated.

7.3. Type A Input Quantities

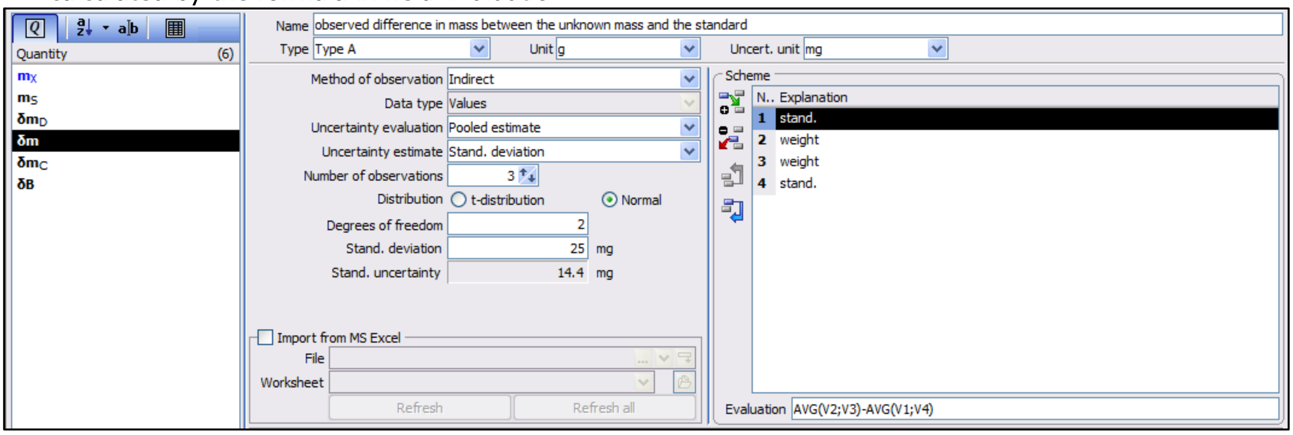
For repeated observations evaluated by statistical methods, the following data are necessary to define:

<u>Method of observation:</u>

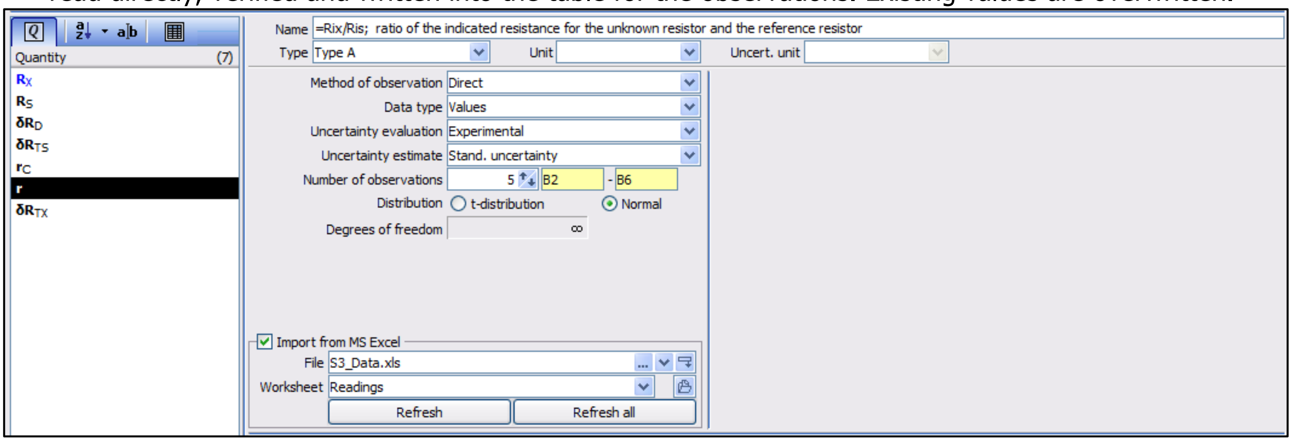
- *Direct* - Observations (individual values or group values) are entered directly in the observation table. The number of measurements is equal to the number of observations.



- *Indirect* – the examination is performed in cycles. The reference value and the unknown value are measured alternately. Each reading is identified through "V + No. of the reading". The value of the observation is calculated by the formula in field *Evaluation*.



- *Direct / Indirect* with import from a MS Excel file - the observations (individual values) or the readings are read directly, verified and written into the table for the observations. Existing values are overwritten.



Uncertainty evaluation - in this field, the user can switch between experimental determination of the standard deviation or giving a pooled estimate from a prior evaluation:

- Experimental – this type offers two uncertainty estimates:
  - Standard uncertainty with normal or t-distribution
  - Bayesian standard uncertainty with t-distribution (GUM Supplement 1)

  The Bayesian evaluation is a useful extension for small numbers of observations (usually smaller than 10). It is possible with the Bayesian evaluation completely to avoid the using of the effective degrees of freedom calculus in the uncertainty evaluation.

- Pooled estimate – this type offers three uncertainty estimates:
  - Standard uncertainty with normal or t-distribution
  - Standard deviation of the sample with normal or t-distribution
  - Bayesian standard uncertainty with t-distribution (GUM Supplement 1)

The following table provides an overview of the calculation formulae, the minimum number of observations and the minimum degrees of freedom for the different types of *Uncertainty evaluation*.

Uncertainty Evaluation	Uncertainty Estimate	Formulae	Normal distribution		t-distribution	
			n	DoF	n	DoF
Experimental	Standard uncertainty	$u(x)=\frac{s_{Exp}}{\sqrt{n}}$	≥2	$f_t=n-1$	≥4	$f_t=n-1$
	Bayesian standard uncertainty	$u(x)=\sqrt{\frac{n-1}{n-3}}\cdot\frac{s_{Exp}}{\sqrt{n}}$	-	-	≥4	$f_t=n-1$
Pooled estimate	Standard uncertainty	u(x)	≥1	$f_t=n-1$	≥1	≥3
	Standard deviation of the sample	$u(x)=\frac{s_P}{\sqrt{n}}$	≥1	$f_t=n-1$	≥4	$f_t=n-1$
	Bayesian standard uncertainty	$u(x)=\sqrt{\frac{f_P}{f_P-2}}\cdot\frac{s_P}{\sqrt{n}}$	-	-	≥1	≥3

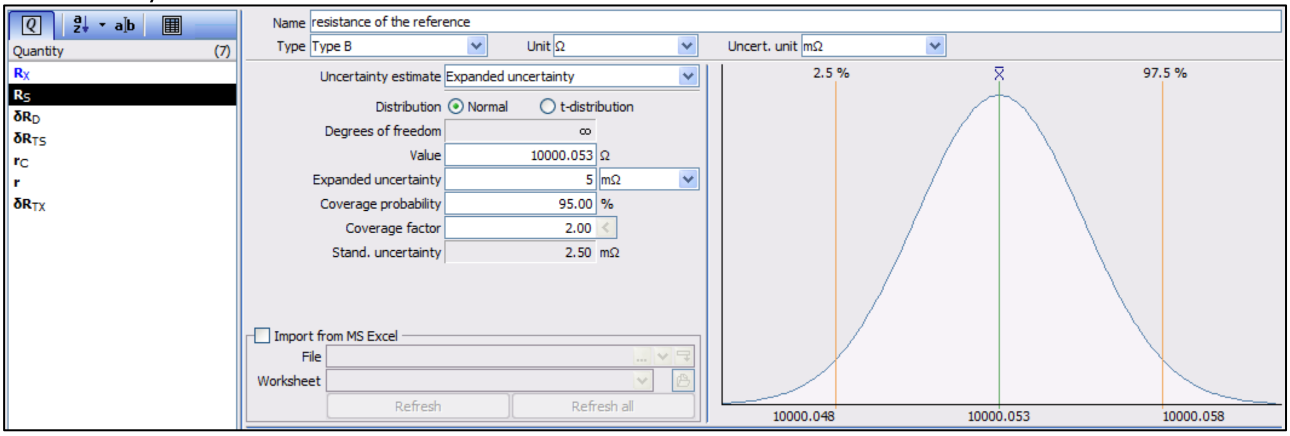
Number of observations (n) – this field contains the number of individual values or group values, used for the evaluation. The number of readings is determined according to the selected method of observation.

The field *Number of observations* can take value 1, if the field *Uncertainty Evaluation* has been set to *Pooled estimate*.

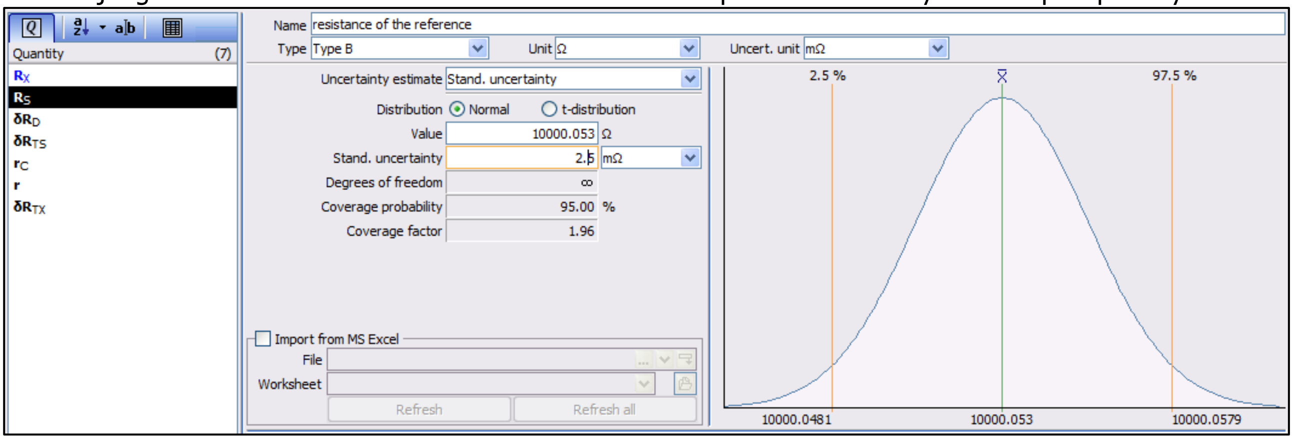
7.4. Type B Input Quantities

The Type B evaluation of standard uncertainty is the evaluation of the uncertainty associated with the estimate of an input quantity by means other than the statistical analysis of a series of observations. The following options for entering the input parameters of type B input quantities are available:

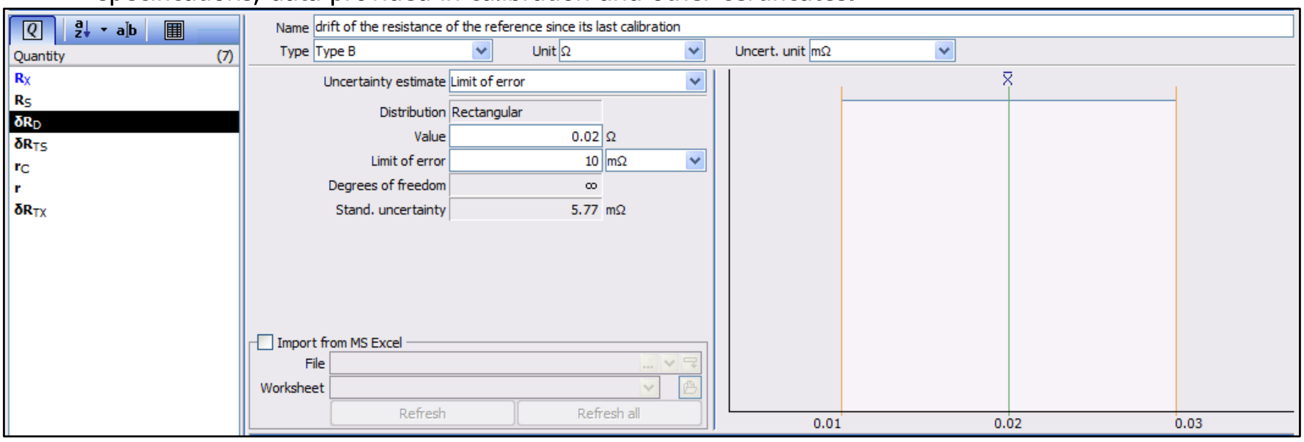
- **Expanded uncertainty** – *Distribution, Degrees of freedom, Value, Expanded uncertainty, Coverage factor* and *Coverage probability* are requested. The data for the input quantity come from an uncertainty analysis or a certificate.



- **Standard uncertainty** - *Distribution, Degrees of freedom, Value* and *Standard uncertainty* are requested. The data for the input quantity come from an uncertainty analysis, a certificate or a scientific judgement based on all available information on the possible variability of the input quantity.



- **Limit of error** - *Value* and *Limit of error* are requested. The information comes from manufacturer's specifications, data provided in calibration and other certificates.



The expanded uncertainty, the standard uncertainty and the limit of error can be calculated relative to the estimate of the same or a different input quantity. In this case after the field for the uncertainty is selected one of the following units: "(relative)", "% (rel.)", "‰ (rel.)" or "ppm (rel.).

The measurement uncertainty, respectively the limit of error, is calculated by using the linear equation:

$U = C + W*X$

where:

U - expanded or standard uncertainty, respectively limit of error

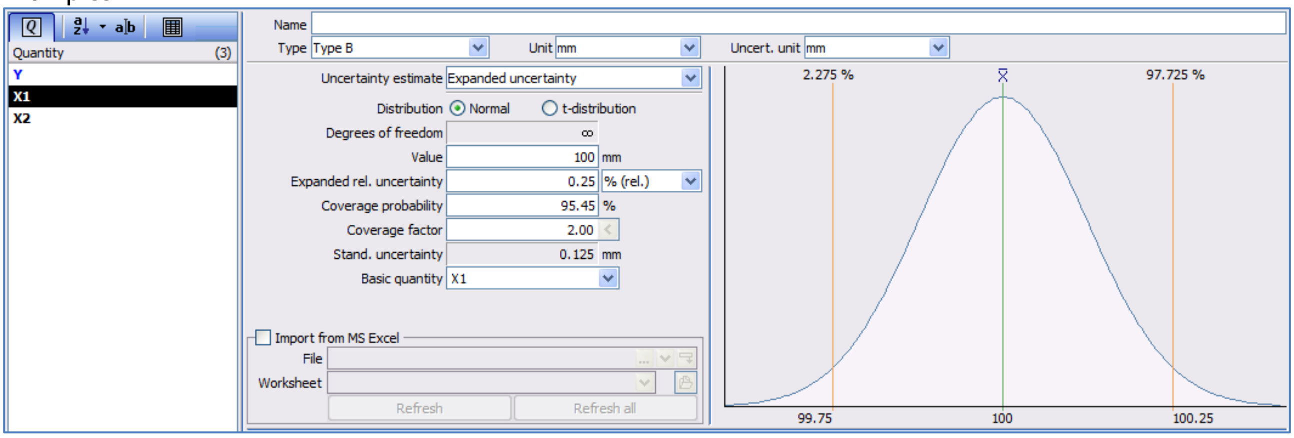
X - estimate of the same or a different input quantity

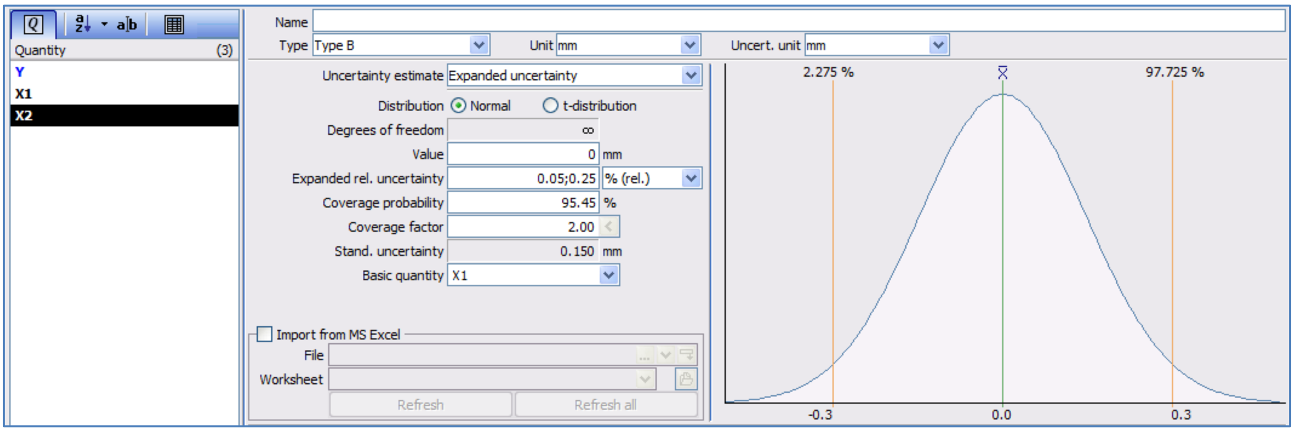
C - constant term of the measurement uncertainty in the quantity unit

W - relative term (slope) of the measurement uncertainty.

This formula is often used to indicate in the calibration certificate the expanded uncertainty of the measuring instrument. The parameters are entered in the same order C;W in the field of the measurement uncertainty, separated by semicolons. If the constant is zero, only the relative term (slope) of measurement uncertainty should be entered.

Examples:





QMSys Metrology and measurement software

- **Relative limit of error** - *Distribution, Value, Relative limit of error* and *Minimum absolute limit of error* are requested. The information comes from manufacturer's specifications, data provided in calibration and other certificates.

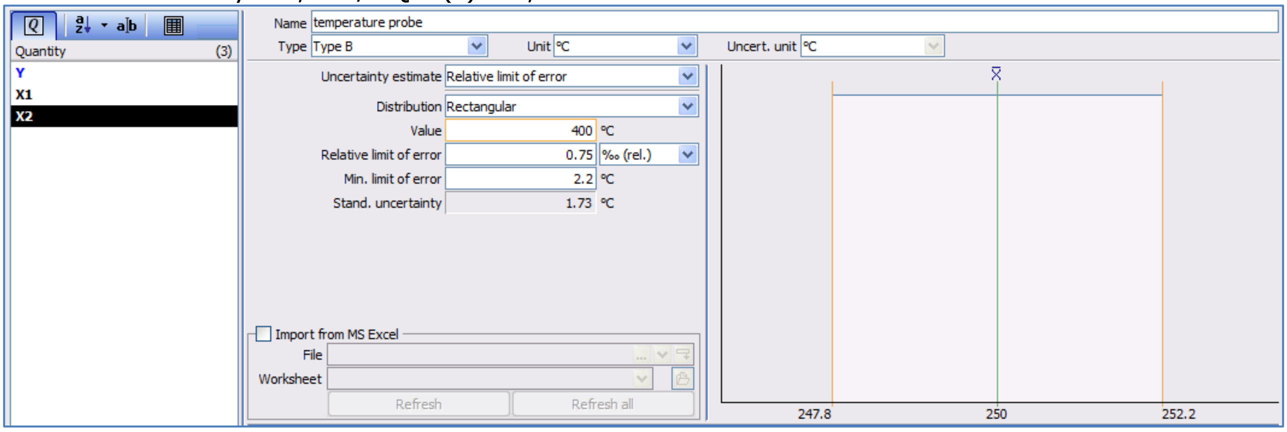
The software offers the following distributions:
- Normal distribution
- Rectangular distribution
- Triangular distribution
- U-shaped distribution.

The relative limit of error can be entered in %, ‰ or ppm, relative to the estimate of the same input quantity. When the normal distribution is selected, instead of error limits the relative standard uncertainty and the minimum absolute standard uncertainty are entered.

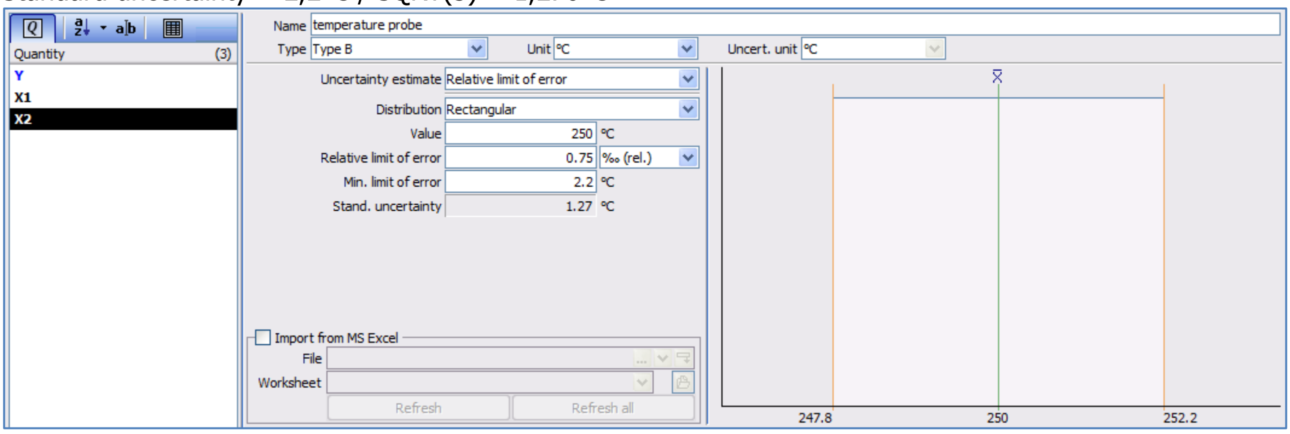
If the absolute limit of error, calculated based on the relative error and the quantity estimation, is greater than the entered minimum limit of error, then it is used in the calculation of the standard uncertainty; otherwise, the entered minimum limit is used for the further calculation of the standard uncertainty.

Example: The accuracy of a temperature probe is ± 2,2 °C or ± 0,75 % of reading (whichever is greater) in the range 0-800 °C.

Reading * Relative limit of error = 400°C * 0,75% = 3,0°C
Limit of error = MAX (3,0°C ; 2,2°C) = 3,0°C
Standard uncertainty = 3,0°C / SQRT(3) = 1,73°C

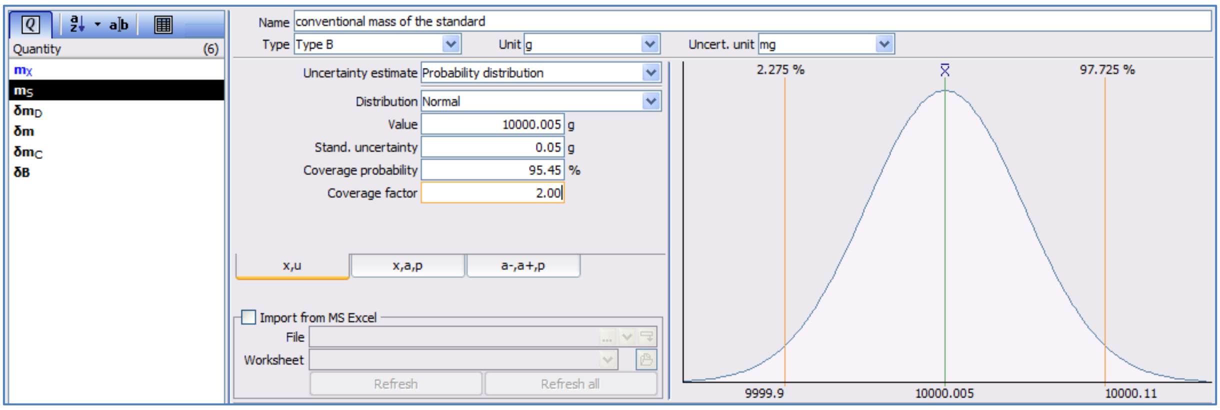


Reading * Relative limit of error = 250°C * 0,75% = 1,875°C
Limit of error = MAX (1,875°C ; 2,2°C) = 2,2°C
Standard uncertainty = 2,2°C / SQRT(3) = 1,270°C

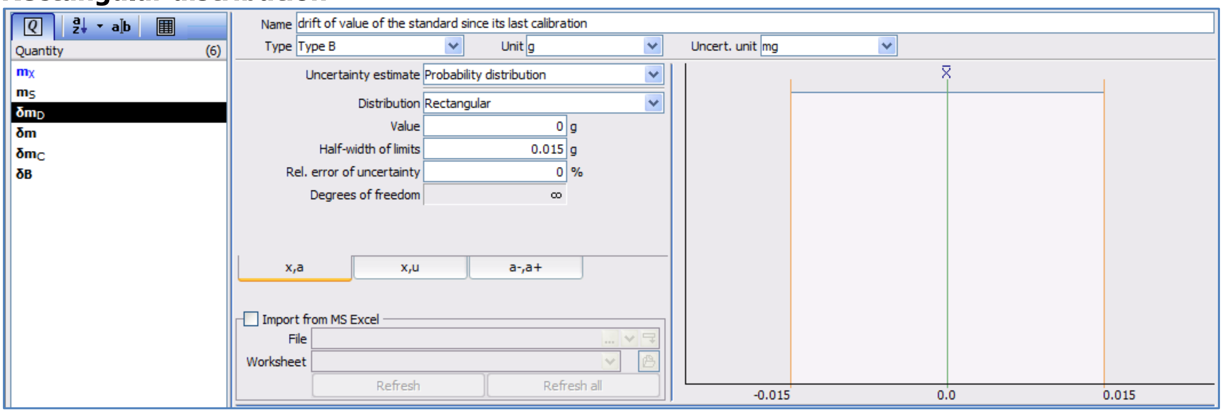


- **Probability distribution** – a known or assumed distribution is given. Additional parameters of the selected distribution will be requested.

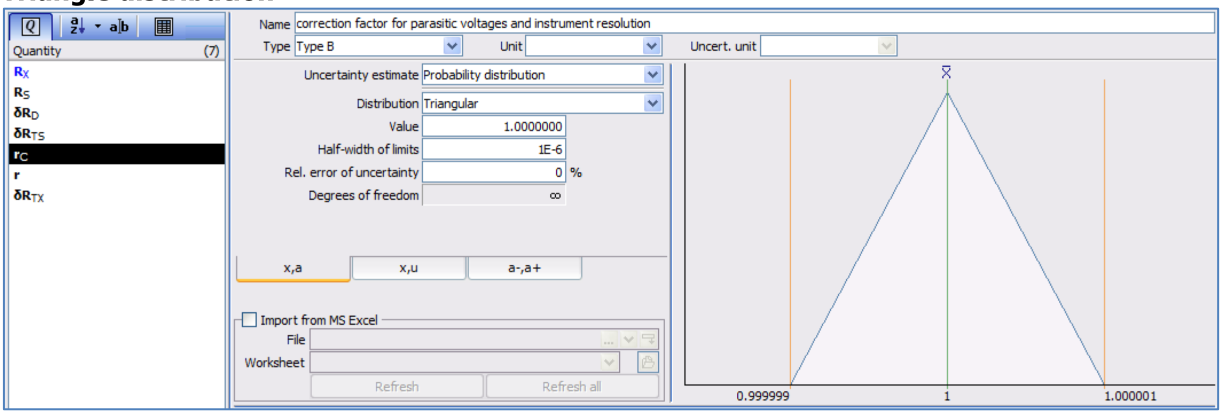
Normal distribution



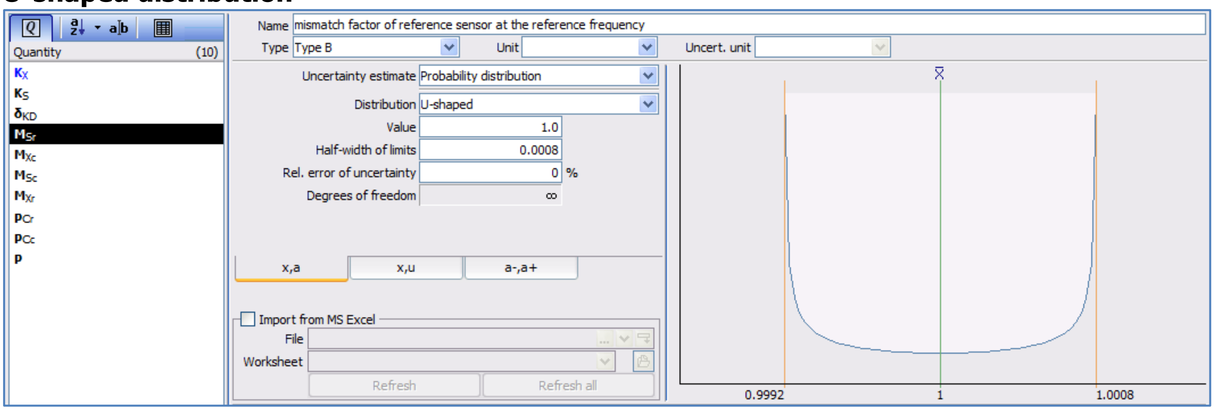
Rectangular distribution



Triangle distribution



U-shaped distribution



Definition options of the probability distributions for type B evaluation of the standard uncertainty

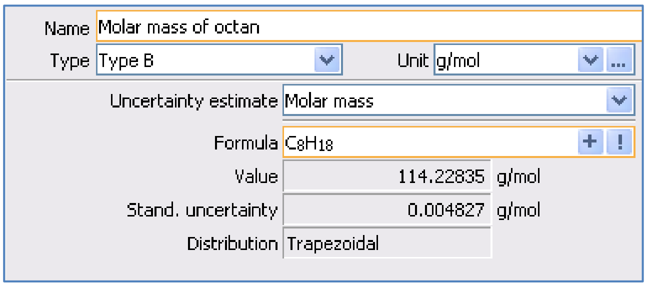
Probability distribution	**Definition options**		
Normal	**μ, σ**	**μ, a, p**	**a- , a+, p**
	- Value - Standard deviation - Coverage probability* - Coverage factor*	- Value - Half-width of limits - Coverage probability** - Coverage factor**	- Upper limit - Lower limit - Coverage probability** - Coverage factor**
Lognormal	**a, μ, σ**	**a, μ(ln(x)), σ(ln(x))**	
	- Limit value - Mean - Standard deviation - Coverage probability*	- Limit value - Mean of the normally distributed ln(x) - Standard deviation of the normally distributed ln(x) - Coverage probability*	
Student's t	**μ, σ, f**	**μ, a, f, p**	**a- , a+, f, p**
	- Value - Standard deviation - Coverage probability* - Degrees of freedom (≥3)	- Value - Half-width of limits - Coverage probability - Degrees of freedom (≥3)	- Upper limit - Lower limit - Coverage probability - Degrees of freedom (≥3)
Rectangular	**μ, a**	**μ, σ**	**a- , a+**
	- Value - Half-width of limits - Relative uncertainty error	- Value - Standard uncertainty - Relative uncertainty error	- Upper limit - Lower limit - Relative uncertainty error
Triangular	**μ, a**	**μ, σ**	**a- , a+**
	- Value - Half-width of limits - Relative uncertainty error	- Value - Standard uncertainty - Relative uncertainty error	- Upper limit - Lower limit - Relative uncertainty error
Trapezoidal	**μ, a, b**	**μ, σ, ß**	**a- , a+ , ß**
	- Value - Half-width of limits (a)** - Half-width of limits (b)** - Shape factor** - Relative uncertainty error	- Value - Standard uncertainty - Shape factor - Relative uncertainty error	- Upper limit - Lower limit - Shape factor - Relative uncertainty error
Curvilinear Trapezoidal (Rectangular with inexactly prescribed limits)	**μ, a, d**	**μ, σ, d**	**a- , a+ , d**
	- Value - Half-width of limits - Interval d - Relative uncertainty error	- Value - Standard uncertainty - Interval d - Relative uncertainty error	- Upper limit - Lower limit - Interval d - Relative uncertainty error
Quadratic	**μ, a**	**μ, σ**	**a- , a+**
	- Value - Half-width of limits - Relative uncertainty error	- Value - Standard uncertainty - Relative uncertainty error	- Upper limit - Lower limit - Relative uncertainty error
U-shaped	**μ, a**	**μ, σ**	**a- , a+**
	- Value - Half-width of limits - Relative uncertainty error	- Value - Standard uncertainty - Relative uncertainty error	- Upper limit - Lower limit - Relative uncertainty error
Cosine	**μ, a**	**μ, σ**	**a- , a+**
	- Value - Half-width of limits - Relative uncertainty error	- Value - Standard uncertainty - Relative uncertainty error	- Upper limit - Lower limit - Relative uncertainty error
1/2 Cosine	**μ, a**	**μ, σ**	**a- , a+**
	- Value - Half-width of limits - Relative uncertainty error	- Value - Standard uncertainty - Relative uncertainty error	- Upper limit - Lower limit - Relative uncertainty error
Exponential	**μ**		
	- Value	- Relative uncertainty error	

* Only for visualization purposes.
** Parameters are interdependent and are recalculated automatically when one of them is changed.

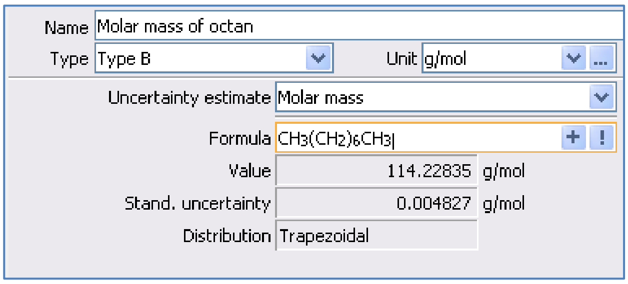
Relative uncertainty errors - the relative error of the summarized estimated value of the uncertainty is entered in %. The program calculates the number of effective degrees of freedom for the inputs type B according to GUM, Equation G.3.

- **Molar mass** – this type offers automatic calculation of the molar mass, the associated standard uncertainty, and the distribution type of chemical compounds. To calculate molar mass of a chemical compound, enter its formula. The software supports both types of formula - molecular and condensed structural, which can contain only letters, numbers, and parenthesis ().

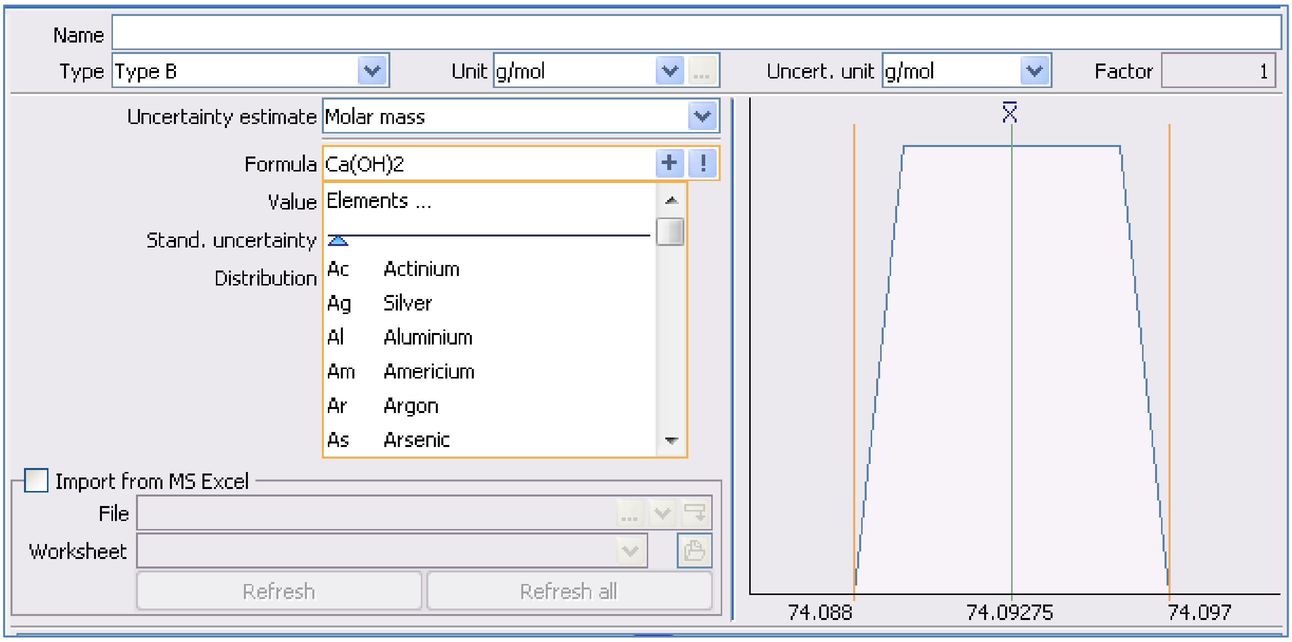
- Molecular formula



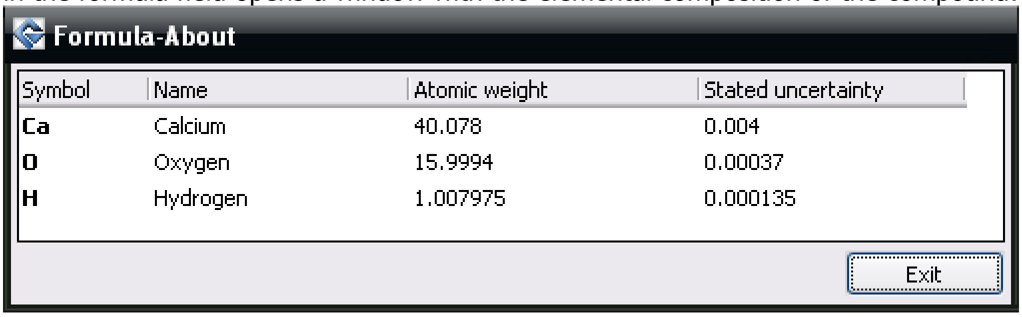
- Condensed structural formula



Chemical elements can be selected from a list with the "+" button:



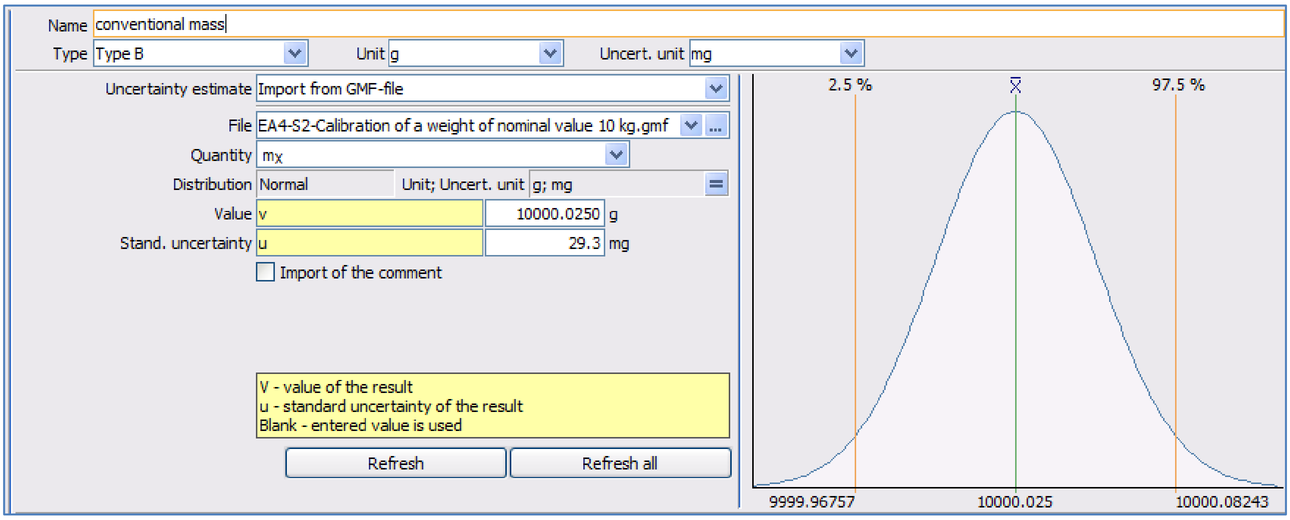
The first row in the list opens a table with the standard atomic weights and the stated uncertainty of all chemical elements. The atomic weights and the uncertainty of the selected element in the list can be edited. The "!" button in the formula field opens a window with the elemental composition of the compound:



The uncertainty in the molar mass of the compound is determined in accordance with the ***EURACHEM/CITAC Guide CG 4 Quantifying Uncertainty in Analytical Measurement***. For each element, the standard uncertainty is found by treating the IUPAC quoted uncertainty as forming the bounds of a rectangular distribution. The corresponding standard uncertainty is therefore obtained by dividing those values by $\sqrt{3}$. The uncertainty contribution of each element is calculated by multiplying the standard uncertainty by the number of atoms. The contributions from different elements are independent, therefore the standard uncertainty of the compound is a square root of the sum of the squares of the element contributions. The reference sources for the standard atomic weights and the stated uncertainty are:

- CIAAW Atomic weights of the elements 2013
- NIST Standard Reference Database 144, Atomic Weights and Isotopic Compositions for All Elements

- **Import from GMF model files** – this type allows the user to reuse the results of other uncertainty analyses previously saved as GMF files for the current analysis.



File - with the button ... a new GMF file is selected and with ˅ an already linked GMF file. The version of the imported GMF file must be version 4.10 or later.

Quantity - selection of the result quantity whose parameters will be imported; in the next fields the distribution and the units of the result quantity are displayed. The button = imports the units of the result quantity in the selected input quantity.

V*alue, Standard uncertainty* - both the value and standard uncertainty of the result, as well as just the value or standard uncertainty, can be imported by entering "V" and "u". The imported numbers can be converted, e.g. if "u*1000" is entered in the first field, the imported standard uncertainty is multiplied by 1000. The operators " +, -, *, / " are allowed. The comment of the result quantity can also be imported to the selected input quantity.

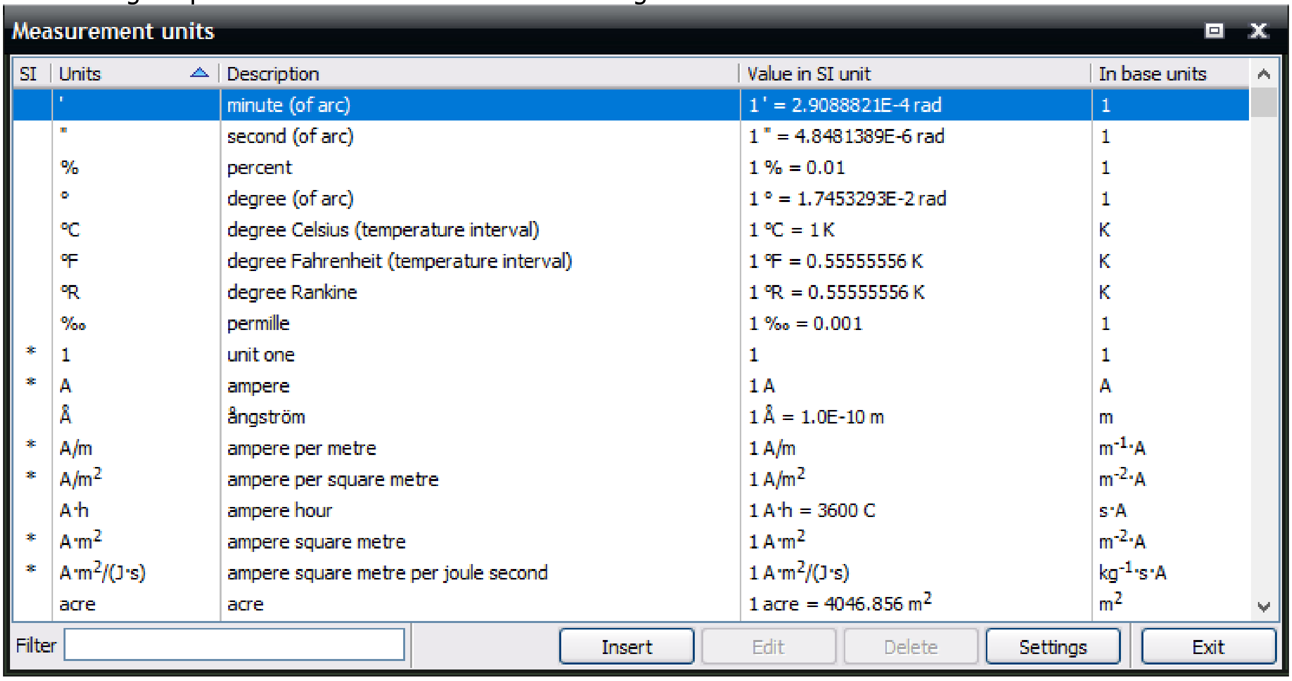
With the *Refresh* button, the data of the selected input quantity will be read again and updated, and with the *Refresh all* button, the data of all input quantities will be updated.

7.5. Use of units

The software offers a sufficient database with almost all SI units and some other common units according to ISO/IEC 80000. In addition, the software checks the use of all units according to the model equations entered.

Catalog of Units

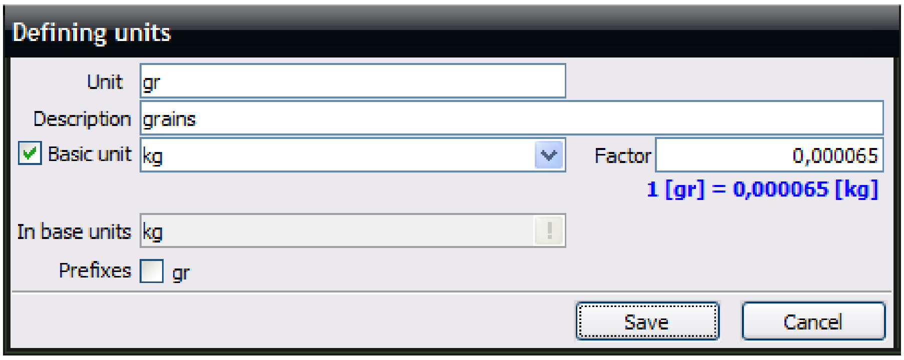
The catalog is opened via the menu "Model -> Catalog of units".



SI	Units	Description	Value in SI unit	In base units
	'	minute (of arc)	1 ' = 2.9088821E-4 rad	1
	"	second (of arc)	1 " = 4.8481389E-6 rad	1
	%	percent	1 % = 0.01	1
	°	degree (of arc)	1 ° = 1.7453293E-2 rad	1
	°C	degree Celsius (temperature interval)	1 °C = 1 K	K
	°F	degree Fahrenheit (temperature interval)	1 °F = 0.55555556 K	K
	°R	degree Rankine	1 °R = 0.55555556 K	K
	‰	permille	1 ‰ = 0.001	1
*	1	unit one	1	1
*	A	ampere	1 A	A
	Å	ångström	1 Å = 1.0E-10 m	m
*	A/m	ampere per metre	1 A/m	m^{-1}·A
*	A/m^2	ampere per square metre	1 A/m^2	m^{-2}·A
	A·h	ampere hour	1 A·h = 3600 C	s·A
*	A·m^2	ampere square metre	1 A·m^2	m^{-2}·A
*	A·m^2/(J·s)	ampere square metre per joule second	1 A·m^2/(J·s)	kg^{-1}·s·A
	acre	acre	1 acre = 4046.856 m^2	m^2

Insertion of new units

New custom measurement units can be added to the database with the *Insert* button. Editing and deleting is possible only for the custom units; standard units cannot be changed.



When defining new units, the basic unit in the SI system is selected in which the corresponding quantity is measured. If no basic unit is selected, the entered unit will be considered as the basic unit for the corresponding quantity.

The factor of the new custom unit to the basic unit must be entered. If the entered unit is also basic unit for the corresponding quantity, the factor is equal to 1.

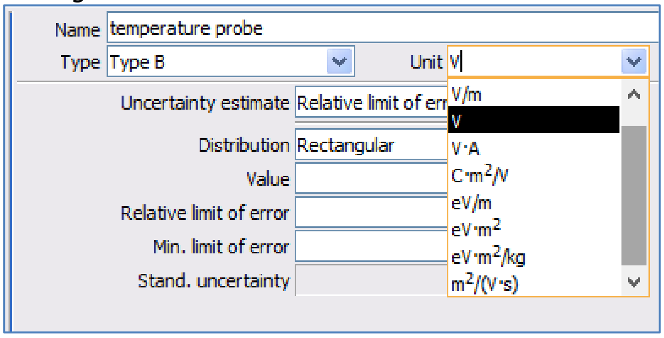
When checking the model equations, the used units are translated into the SI base units, so it is recommended also to enter the translation into the seven SI base units (m, kg, s, A, K, mol, cd). If the basis unit is standard SI unit, the program automatically fills the expression into the SI base units.

Prefix notation for new custom measurement units can also be added.

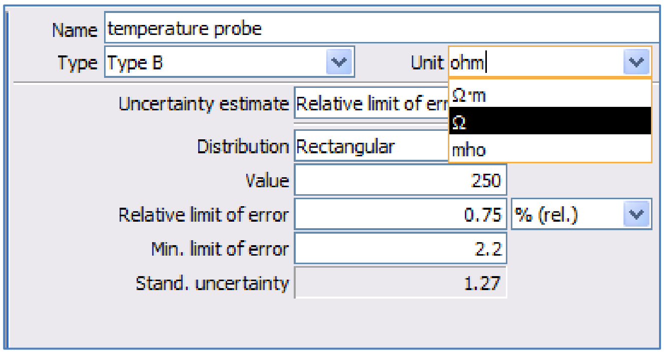
Assigning Units

Each quantity introduced in the model equation can be assigned a unit in the following way:

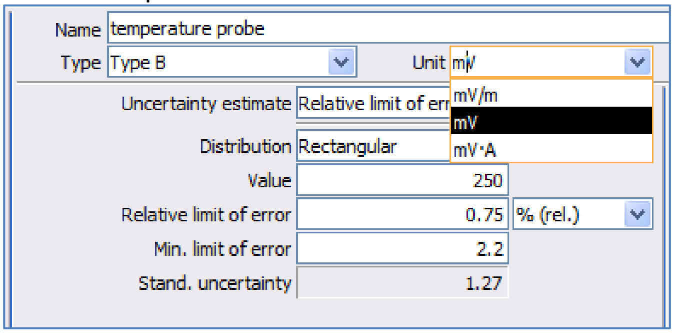
- Direct input of the unit or part of the unit in the field - the software opens a list of matching units from the catalog



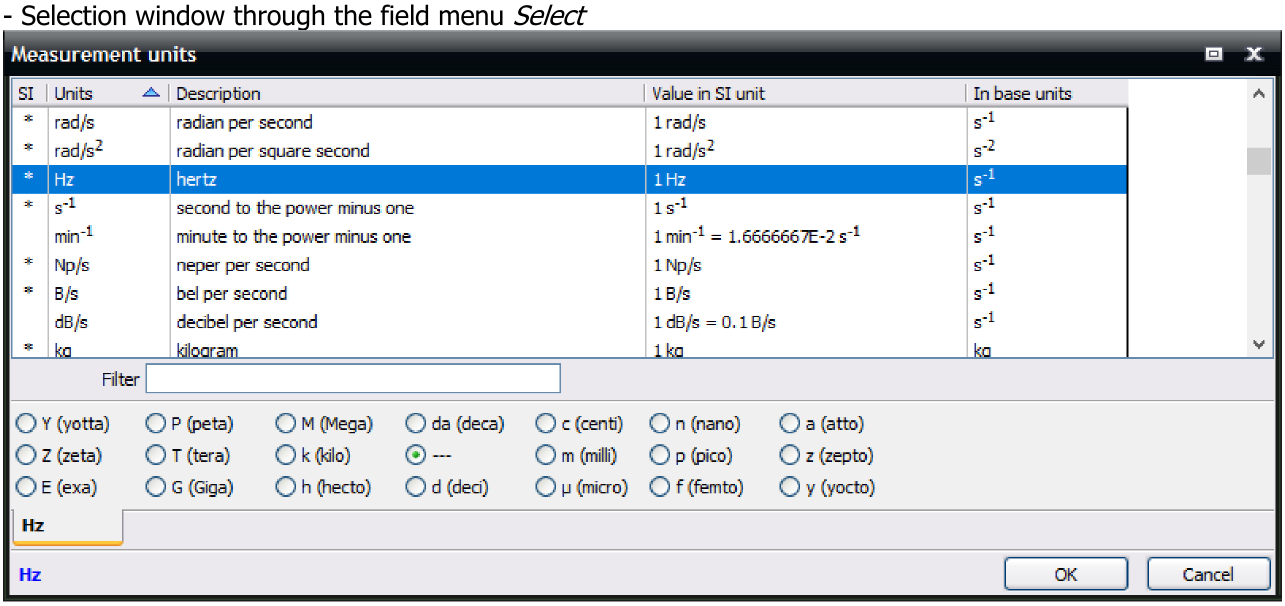
- The name of the unit can also be entered as a search text



- Units with prefix can also be searched



- Selection window through the field menu *Select*

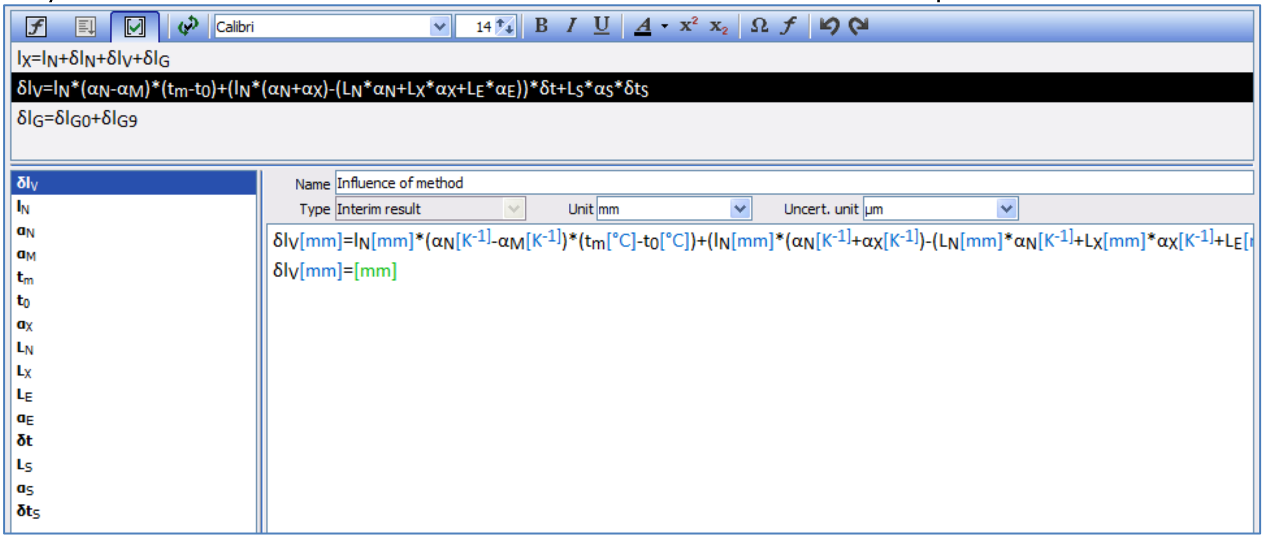


In the fields for units, you can use the context menu of the right mouse button to insert Greek symbols, or to format the characters as superscript or subscript.

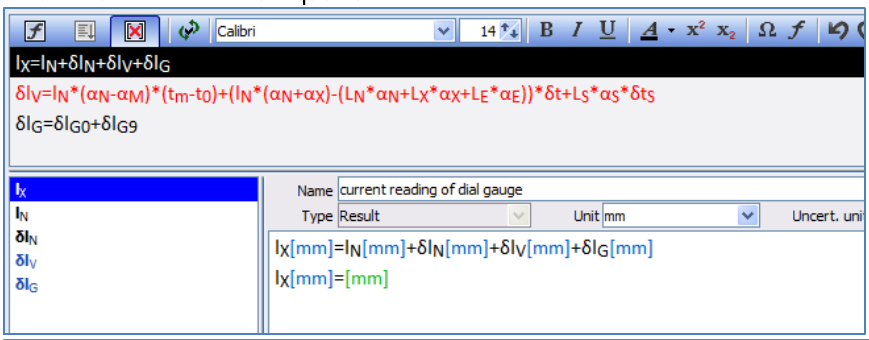
In addition, a unit with a different prefix can be selected for the measurement uncertainty.

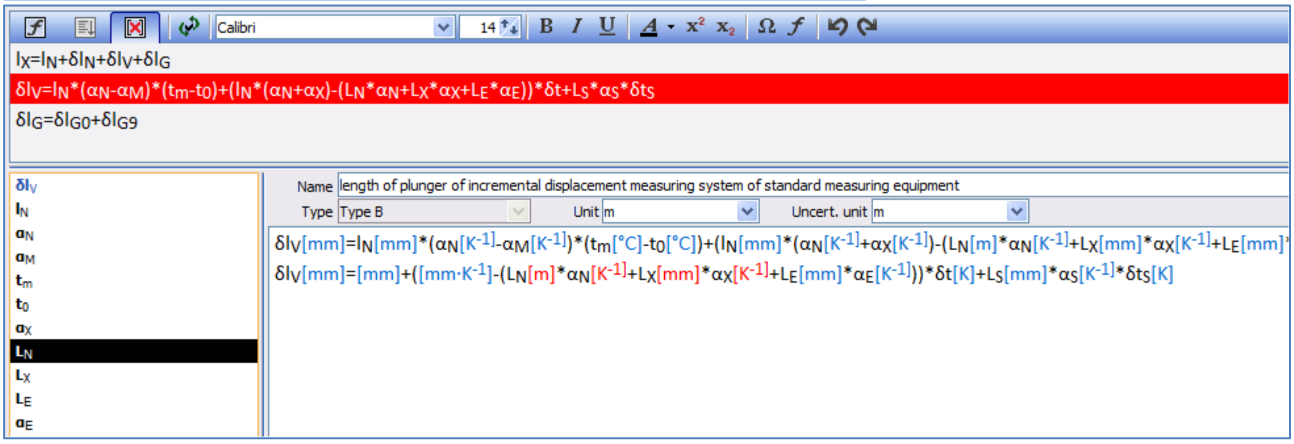
Unit Validation

After the units have been defined for all quantities, the consistent use of the units according to the rules of the SI system can be checked in the "Validation of the measurement units in the equations" view.



Equations with inconsistent use of units are marked with red font. Below are the magnitudes and unit check results for the selected equation.



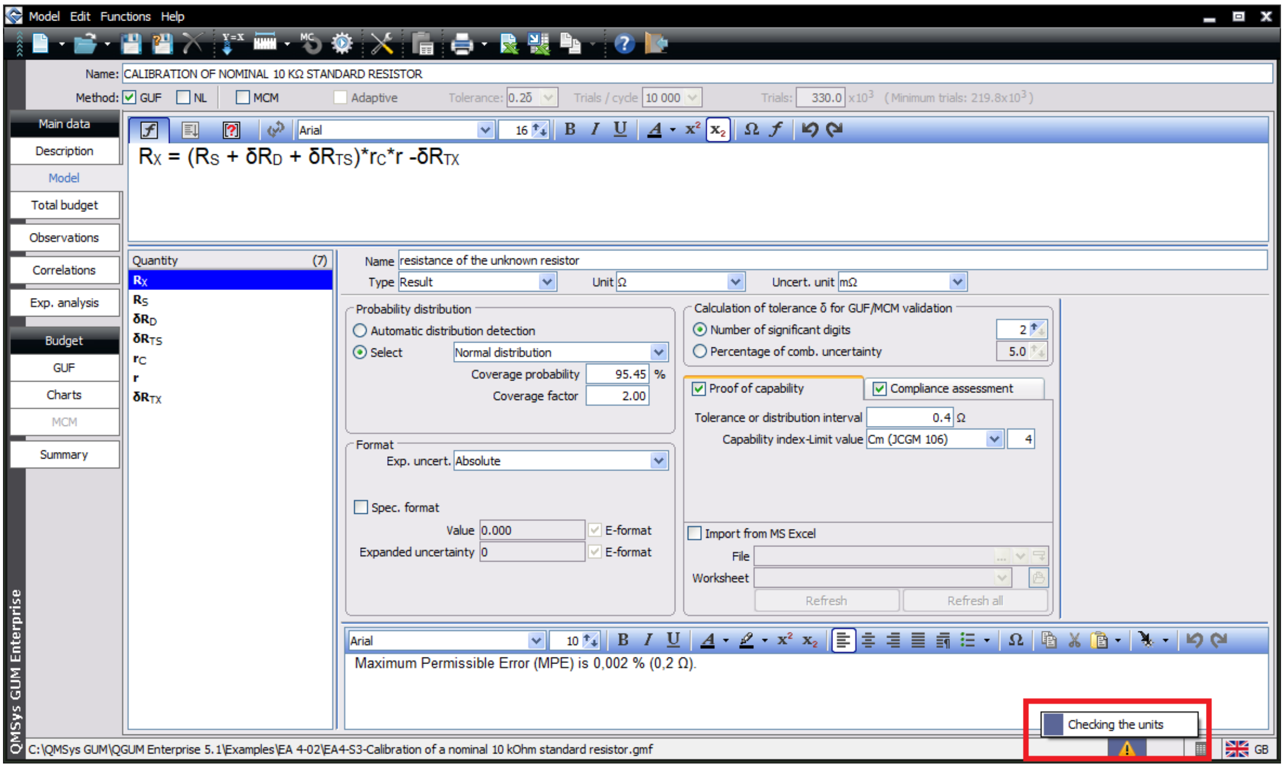


After changing the inconsistent units, you can use the button  to update the unit validation.

Validation of the units does not guarantee that the model equation is also suitable for the intended use and that the calculated values are correct. It only checks whether the use of the units is formally consistent.

If the usage of the units is not consistent, the units or the model should be changed. The warning indicates that it cannot be automatically decided whether the use of the units is consistent. The user decides whether either the warning can be ignored, or whether the units or model should be changed.

When a file is opened, it is checked whether the units used are in the catalog and whether the use of the units has been verified. The result of the check is displayed in the status bar. Selecting the warning with the mouse button hides the pop-up menu for inserting missing units or checking units.



When units that are missing from the catalog are found while loading the model, a window opens in which these units can be replaced or entered in the catalog of measurement units.

The result of the unit check is displayed and saved by the corresponding message. The next time the file is loaded, the result of the unit check is also visualized if necessary.

Kelvin (K) and Degree Celsius (°C)

If quantities are defined in an equation whose units use both °C and K, there is a possibility that absolute and relative temperature specifications are not used correctly. The user should either avoid mixing °C and K, or carefully check that the units have been used correctly.

Conversion factors

In the model equations you have to insert the conversion factors (factors for converting one unit into another unit) as quantities of the constant type and enter the correct value with the correct unit.

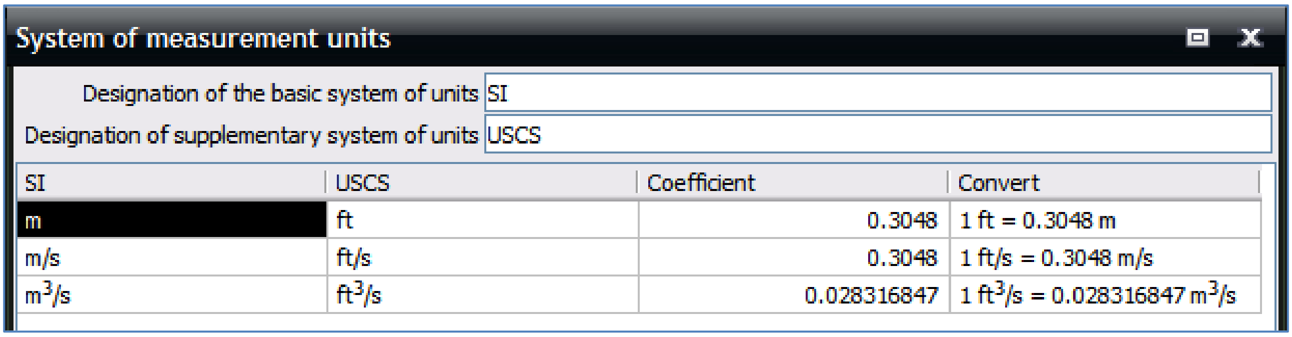
Scaling factors

Unit scaling factors (e.g., mg/mL to mg/L) may be entered into the model equations.

Defining a supplementary system of measurement units

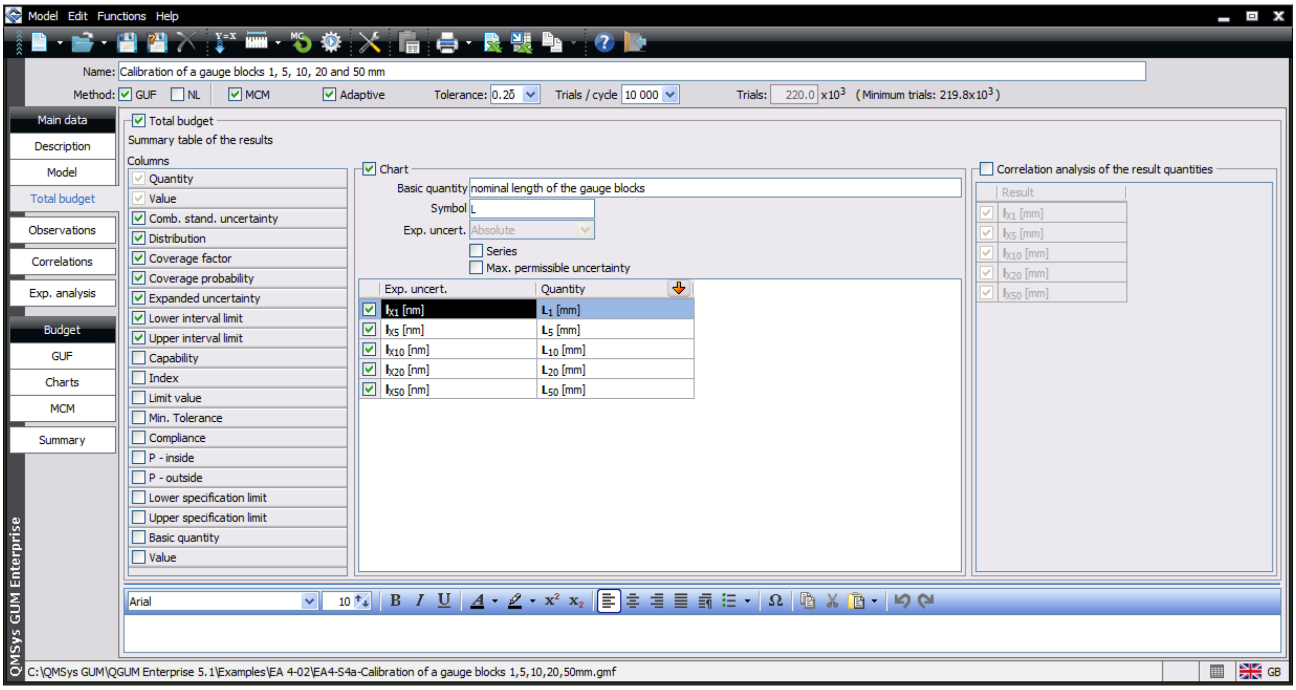
The software allows the defining of supplementary system of units for all units, used in the model equations. The supplementary system is saved in the model file, and the user can with simple click switch between the systems.

It is necessary to enter the designations of the basic and the supplementary system of units, and to select from the catalog the corresponding units of the supplementary system.



7.6. Settings in register Total budget for multiple result evaluations

In the page *Total budget,* you can activate the summary of the results in a table and set the parameters for the charts or the correlation analysis of the resulting values.

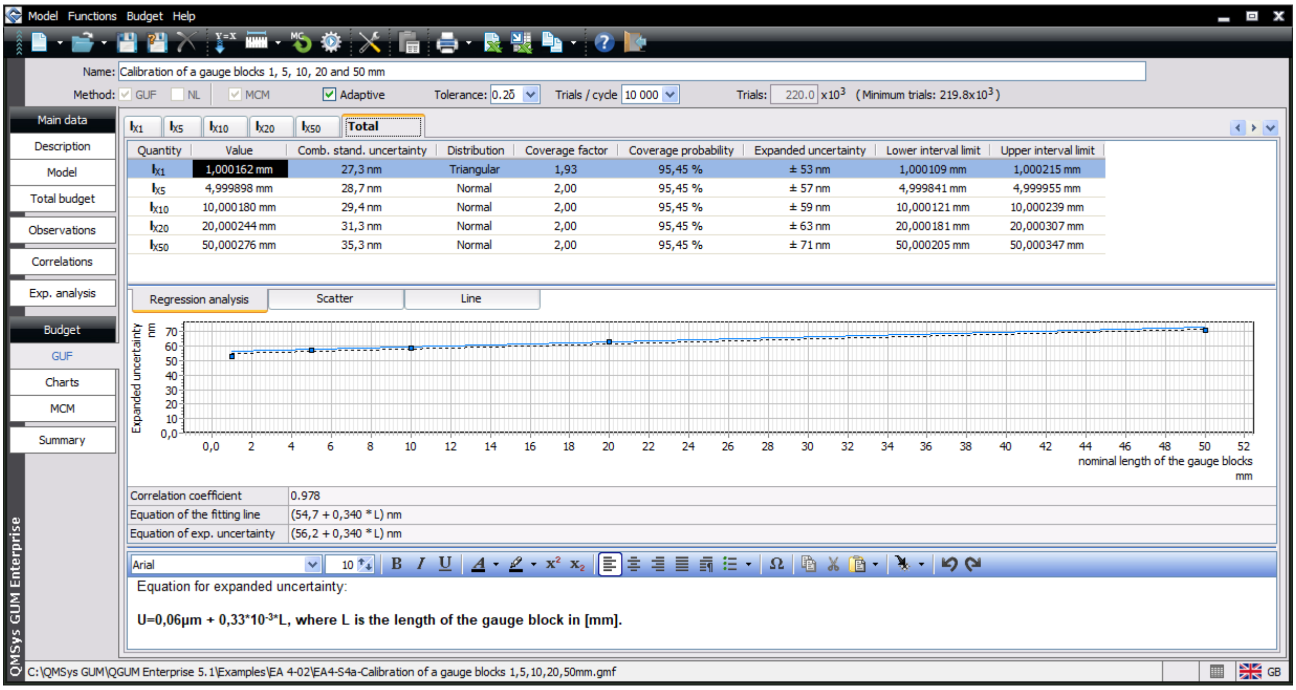


For the summary table of the results are selected the columns to be displayed in the table.

In the chart settings, you can enter the name of the basic quantity, which is used as a label for the horizontal axis of the chart. Optionally, a symbol for the basic quantity can also be entered. The results are selected in the table to be included in the chart. For each result in the second column is selected the basic quantity, the value of which will be used for the horizontal axis of the diagram. Automatic entering of the basic quantity is possible with the built-in search function.

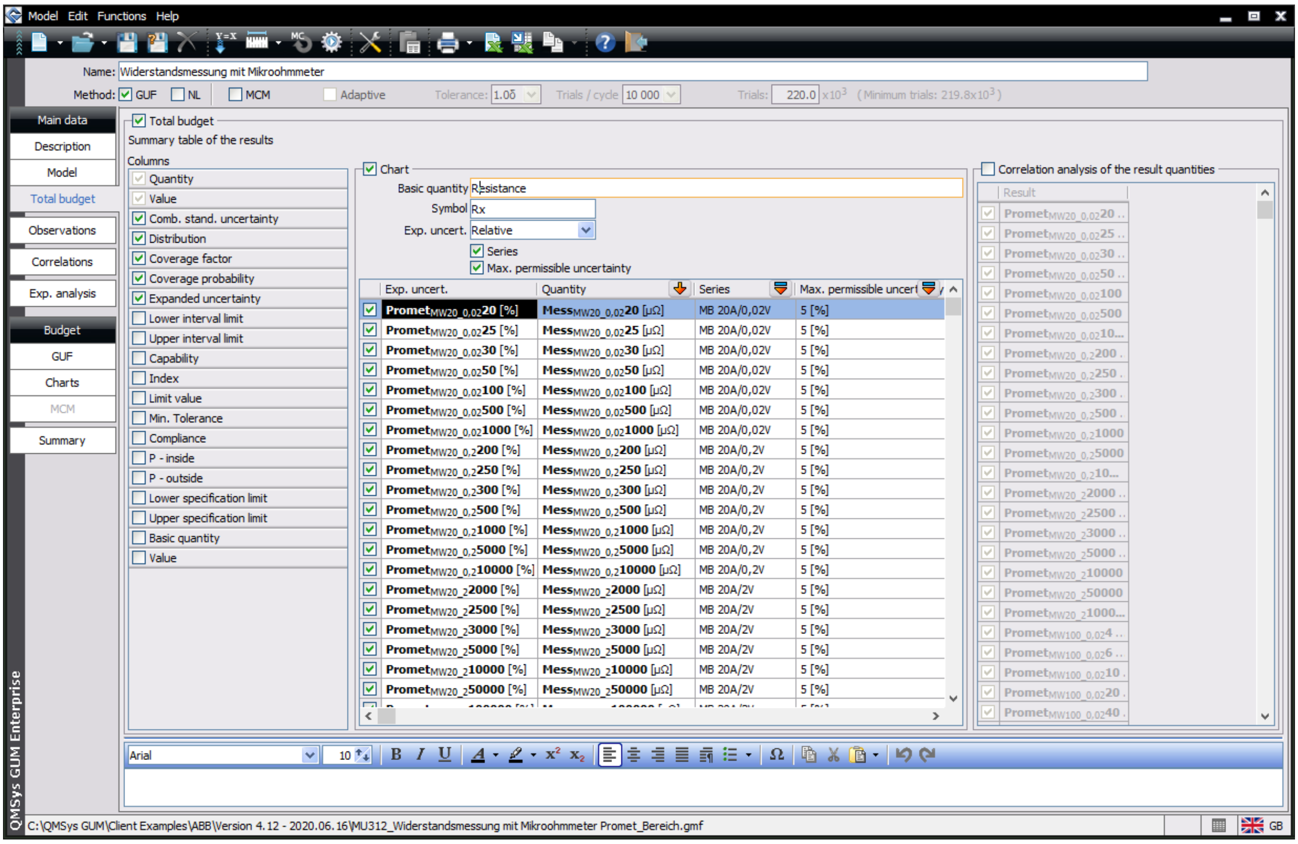
The type of chart is selected in the total budget view, and the software offers the following options:

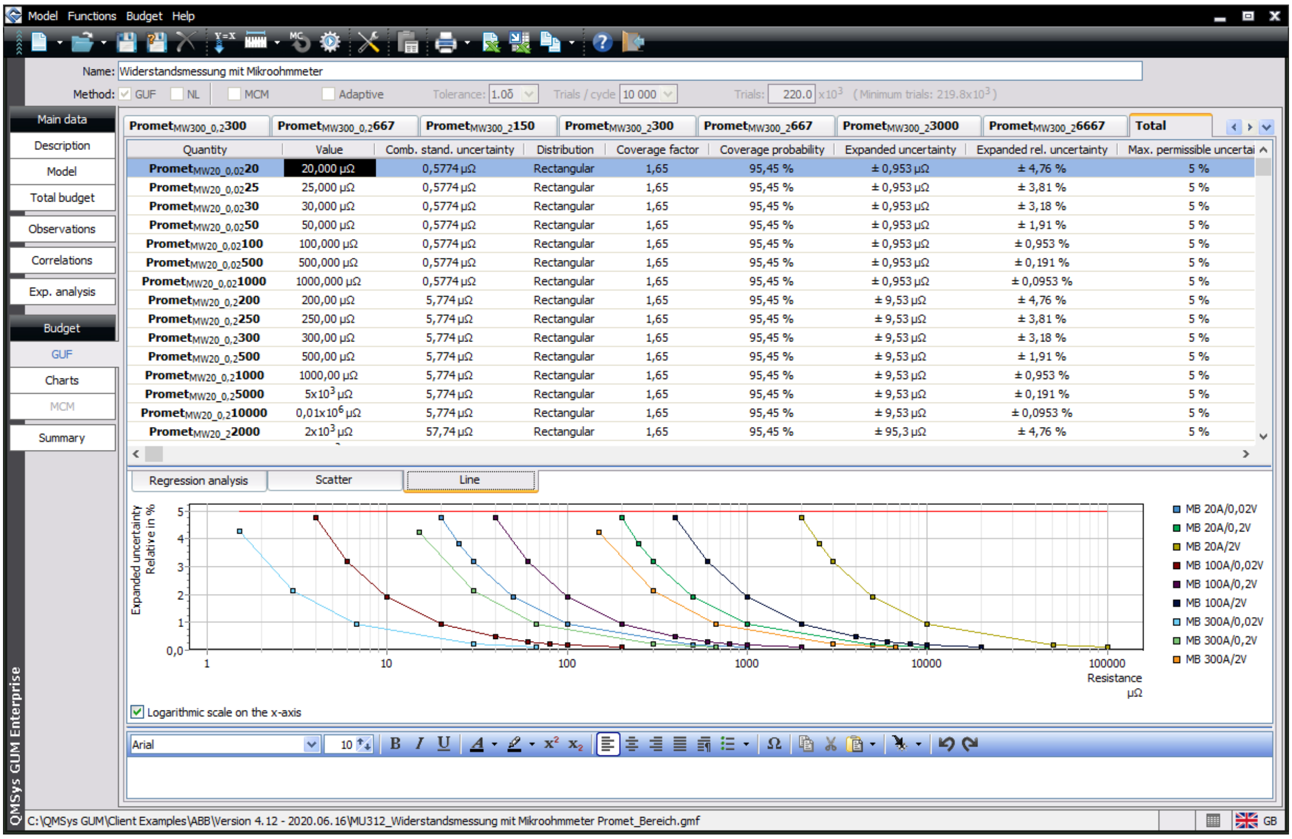
- Regression analysis and calculation of the equation of the expanded measurement uncertainty for a certain measurement range
- Scatter or line chart with option for logarithmic scaling of the horizontal axis.



A column for data series can be added to the chart settings table by selecting the respective option. Individual data series are marked with successively increasing numbers or with names and are only considered in the line or scatter chart types; regression analysis is calculated over all data. Additionally, a column for the maximum permissible uncertainty can be added and the values for each result can be entered.

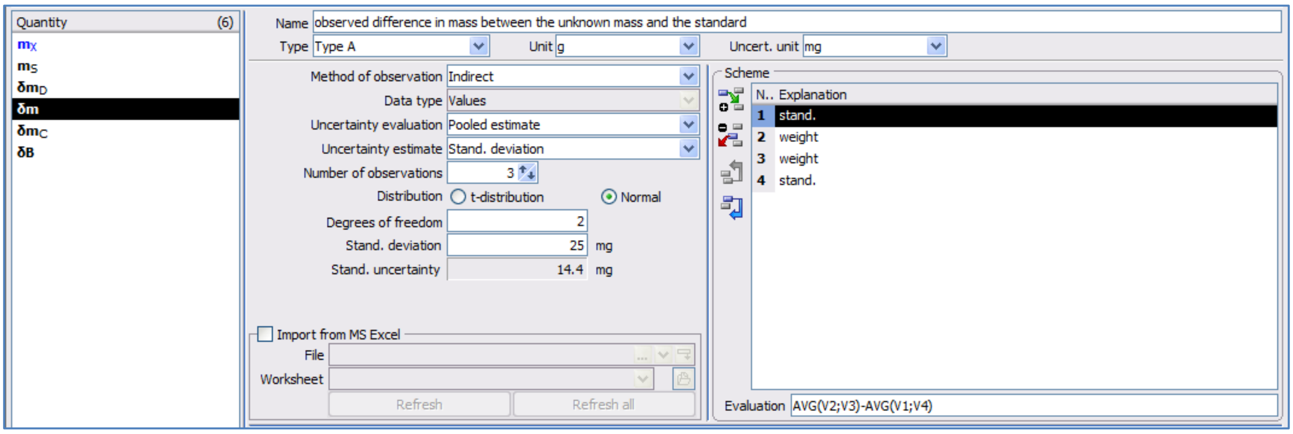
The buttons in the header row of the last two columns automatically fill in the selected value in the corresponding column in the following rows to the end of the table.

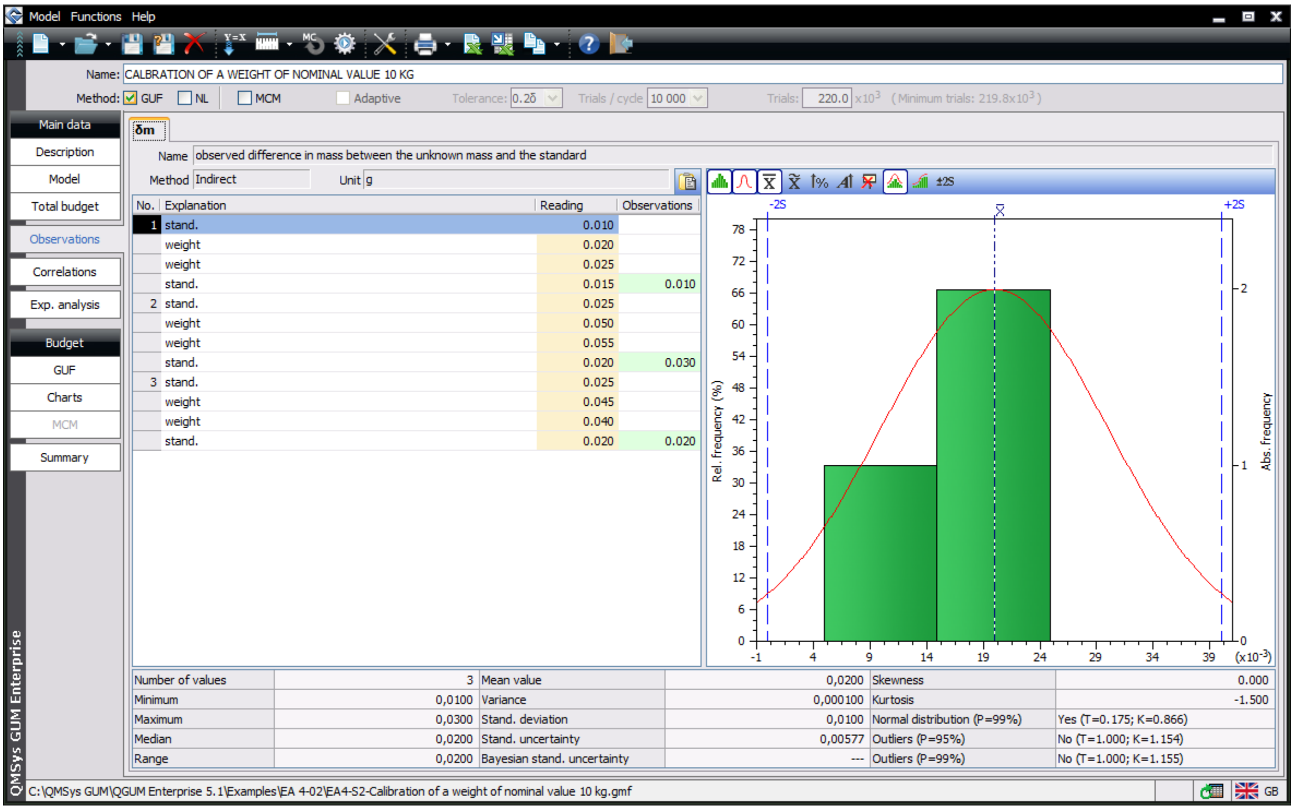




8. Observations

The view *Observation* processes the values of repeatedly observed quantities. The data is typed into a table, the structure of which depends on the method of observation. All readings and observations must be given in the same unit, as the one entered for the quantity value. If the model uses several type A quantities, the observed quantity can be selected at the upper border of the window.





When valid data for all observations (or readings) have been entered, the statistical information including the mean value, the standard deviation, the standard uncertainty, and the Histogram of the data will be displayed.

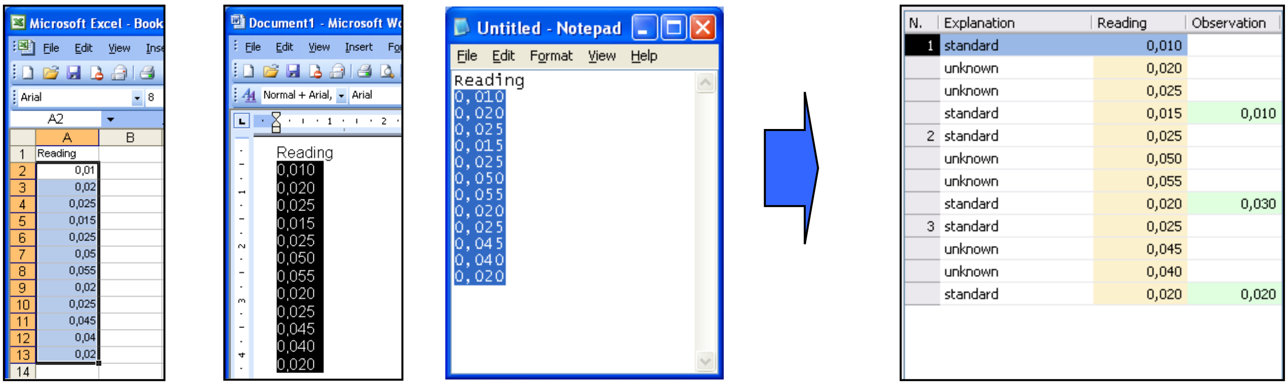
With at least 3 observations following tests are performed:
- Kolmogorov-Smirnov-Lilliefors fit test for normal distribution at 99% confidence level
- Grubb's outlier test at confidence level P = 95% and P = 99%; outliers are marked in the column “No.” accordingly by "*" or "**".

In parenthesis are shown the test statistic (T) and the critical values (K).

In the *Correlations* page, the measurands can be analyzed for possible correlation. A prerequisite for a correlation analysis is that the number of observations of the two quantities must be equal and that all of the observations are filled in and are valid.

8.1. Import of Measured Values from the Clipboard

The button *Import measured values from the clipboard* after the unit of the quantity or in the menu *Functions* imports measured values from the clipboard for an observed quantity. The data are read in, checked and inserted in the observation table. Any existing data will be replaced by the imported data.

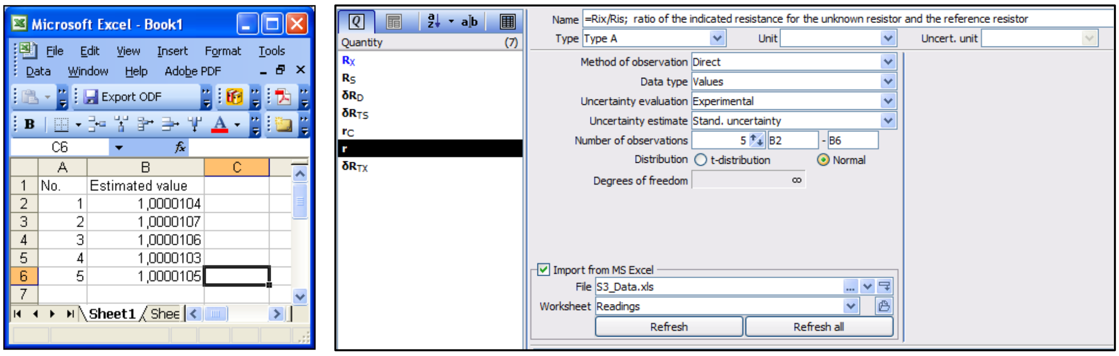


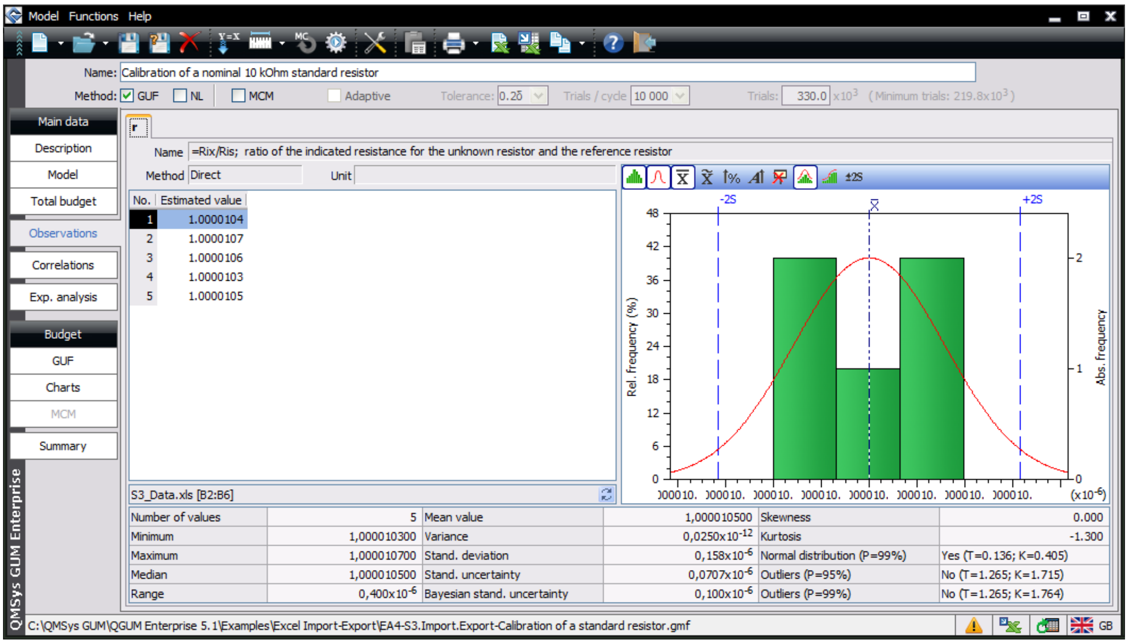
8.2. Import of Measured Values from Microsoft Excel Files

This feature in the software editions ***GUM Enterprise, GUM Professional***, and ***GUM Calculator*** allows the reading and import of measured values for an input quantity of type A from an MS Excel file. The measured values must be ordered in the Excel table in a row or column.

Using names for cell areas is also supported - in this case, the name is entered in square brackets, e.g. [values]. Upper / lower case must be considered. The area can contain multiple rows and columns, the measured values will be read line by line.

Empty cells are not allowed in the defined region. The measured values are read from the file, validated and imported into the table for the observations. Any existing data will be replaced by the imported data.





Import of measured values for type A indexed quantities

When using import from MS Excel for indexed quantities the user must define the cell references only for the main quantity and the program will automatically initialize the appropriate cells to the sub-quantities with the corresponding indexes. Using names for cell areas is also supported - in this case, the name is entered in square brackets, e.g. [values].

Following options for importing measurements from MS Excel are offered:

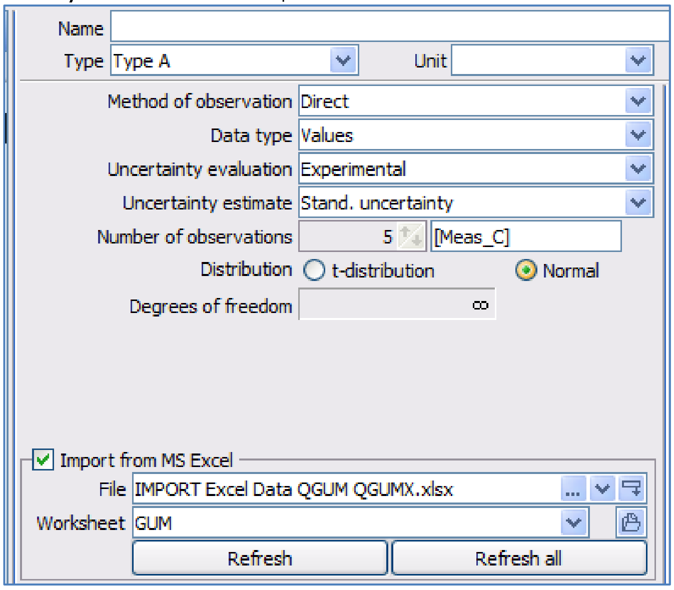
- **Measured values of each index quantity are arranged by columns**

Example
- Index definition: index n=(1:4)
- Number of observations: 5
- Cell area: "Meas_C" = $C$3:$F$7

Meas_C | fx 10.2

	B	C	D	E	F
1		Quantity index			
2	No.	Q1	Q2	Q3	Q4
3	1	10.2	25.2	35.4	50.4
4	2	10.3	25.3	35.6	50.6
5	3	10.3	25.3	35.6	50.6
6	4	10.1	25.1	35.2	50.2
7	5	10.2	25.2	35.4	50.4
8					

Definition of cell area of the main quantity:
- by name: [*Name*_C]
- or by cell references: $C3:F7



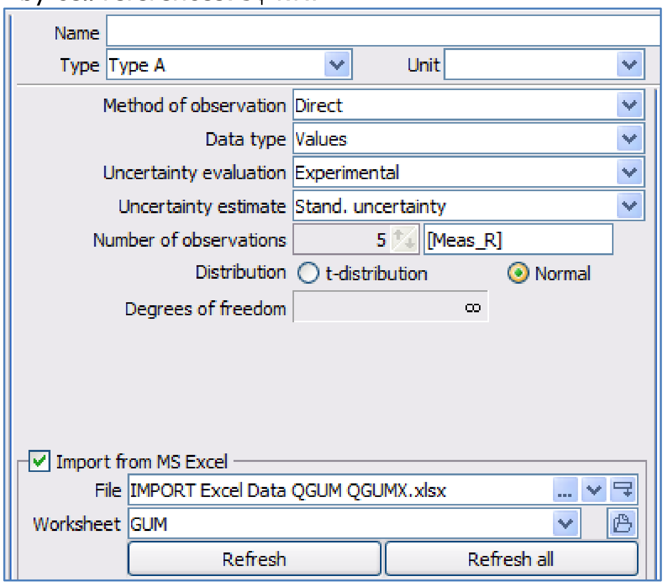
- **Measured values of each index quantity are arranged by rows**

Example
- Index definition: index n=(1:4)
- Number of observations: 5
- Cell area: "Meas_R" = $J$4:$N$7

Meas_R | fx 10.2

	H	I	J	K	L	M	N
3		No.	1	2	3	4	5
4	Quantity index	Q1	10.2	10.3	10.3	10.1	10.2
5		Q2	25.2	25.3	25.3	25.1	25.2
6		Q3	35.4	35.6	35.6	35.2	35.4
7		Q4	50.4	50.6	50.6	50.2	50.4

Definition of cell area of the main quantity:
- by name: [*Name*_R]
- by cell references: J$4:N7



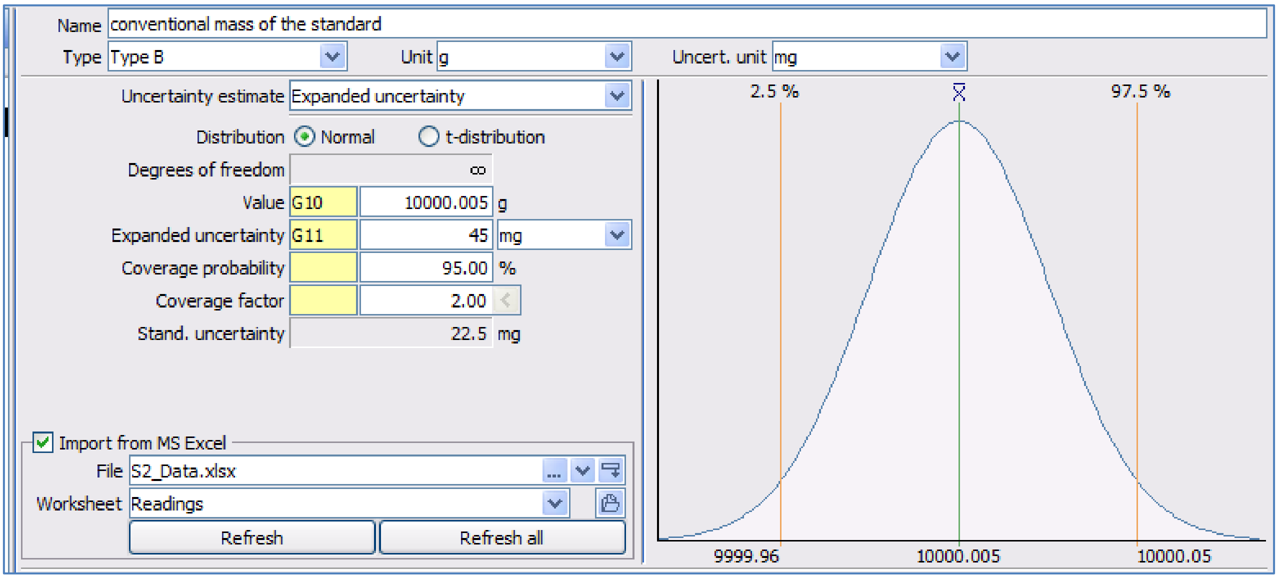
If in the defined area are empty cells, the software will automatically reduce the number of observations for the corresponding sub-quantity. At least 1 measured value must be entered in the Excel file for each sub-quantity.

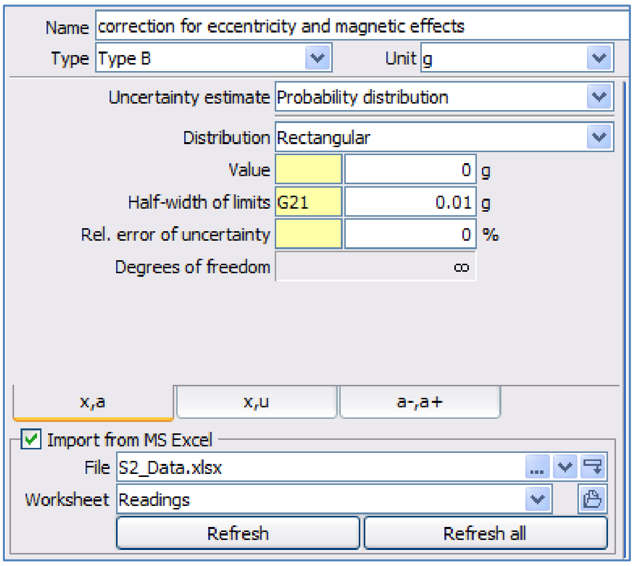
9. Import of Data from Microsoft Excel Files

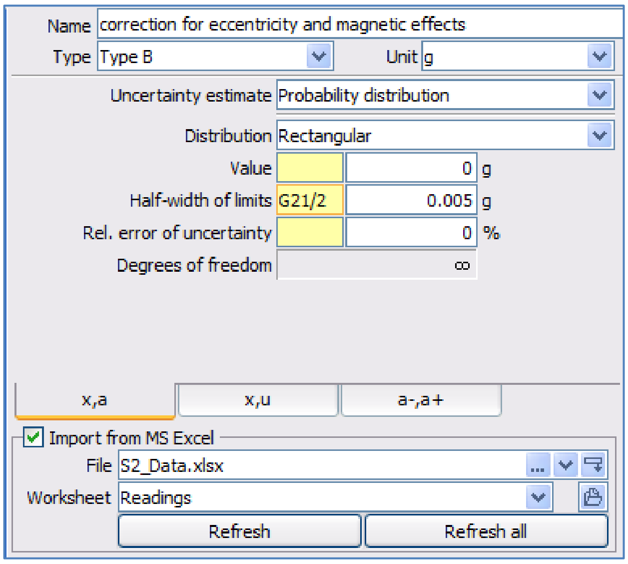
The software editions *GUM Enterprise, GUM Professional*, and *GUM Calculator* support an import of data from MS Excel files (version 2016 and newer, *.XLS, *.XLSX) to all types of input variables. To import data from Excel files the MS Excel application should be installed correctly. The import is based on the OLE automation interface. It starts the MS Excel software in the background as soon as an analysis imports data from an Excel file.

The import from MS Excel is activated by checking the option *Import from MS Excel*. In the *File* field the name of the Excel file is entered. Alternatively, the first button in the same box opens a search window for selecting the Excel file. In the second box the worksheet, which contains the data to be imported, is selected. The selected MS Excel file can be opened by clicking on the button.

Values from one Excel file can be imported to multiple input quantities and an uncertainty analysis can be linked to multiple Excel files. Already linked Excel files are displayed in the list of the *Name* field. The last linked Excel file is automatically suggested when processing the next input quantities.







In the corresponding cell fields should be entered the names of the cells in the same spelling of MS Excel, from which the *QMSys GUM software* should read the respective values. In the right field the read value is displayed. It is allowed to import only certain parameters and to the other parameters, the values are entered in the right field.

Small formulas can be entered in the cell fields, when a conversion of the read value is needed; for example, calculation of the half-width of the limits of a rectangular distribution, when the entire interval is read from the Excel file.

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With the *Refresh* button, the data of the selected input quantity will be read again and updated, and with the *Refresh all* button, the data of all input quantities will be updated.

The program automatically saves the relative file path if the Excel file is in the same folder or in a subfolder to the GMF file. Otherwise, the absolute file path is stored. When you copy or move the files, the relative position of the Excel file to the GMF file is maintained. The folder structure, in which the files are stored, can be shifted as a whole or common parent folder can be renamed, but they should not be changed relative to each other. Otherwise, the ***QMSys GUM software*** will not find the Excel file. Source Excel files should not be moved if the absolute file path is saved.

Import for indexed quantities - type B, constants, results

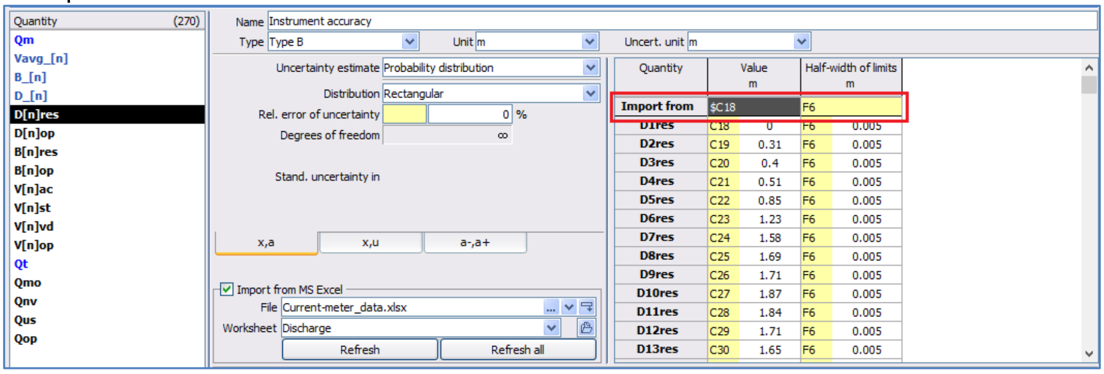
Cell references for the respective quantity parameters are entered in the same manner as in MS Excel in the row "Import from":

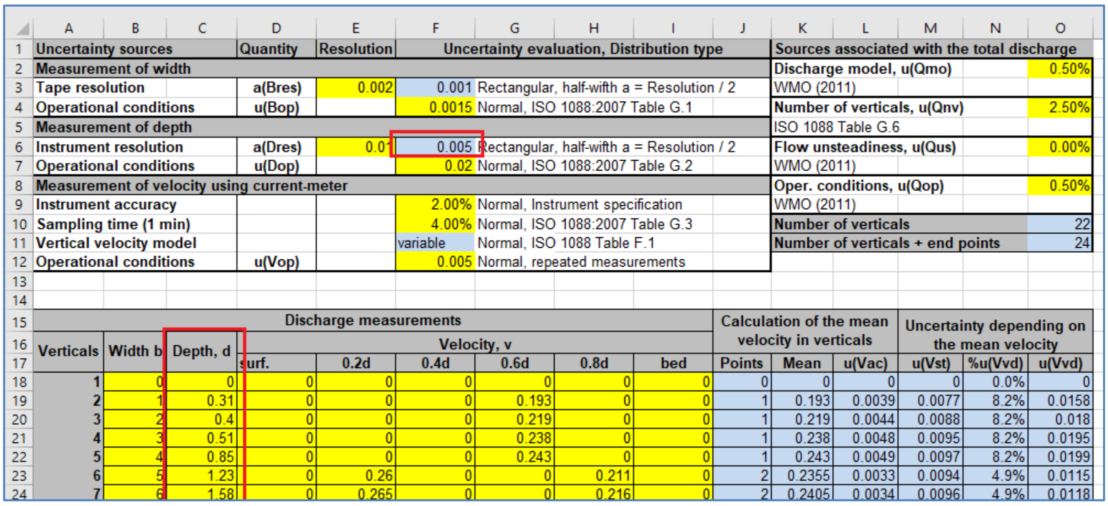
Reference type	Cell reference	Cell references for the respective index
Absolute cell reference	A1 or $A$1	Quantity_1 = A1 Quantity_2 = A1 Quantity_3 = A1
Absolute column reference	$A1	Quantity_1 = A1 Quantity_2 = A2 Quantity_3 = A3
Absolute row reference	A$1	Quantity_1 = A1 Quantity_2 = B1 Quantity_3 = C1

Entering individual cell references for each sub-quantity is also possible.

Using names for cell areas is also supported - in this case, the name is entered in square brackets, e.g. [values]. The number of cells in the area should not be less than the number of index elements, otherwise the parameters of the last sub-quantities will remain empty. If the area contains multiple rows and columns, the values are read by columns.

Example:



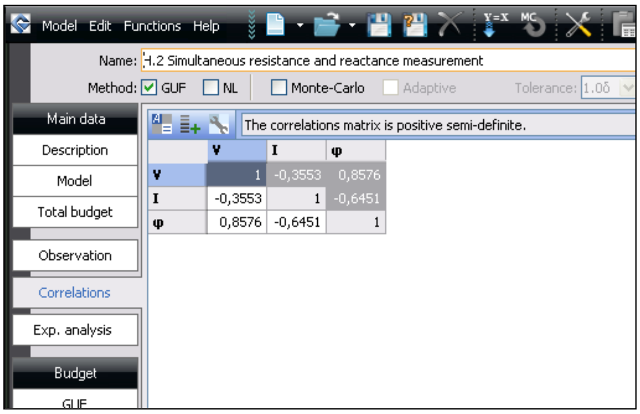


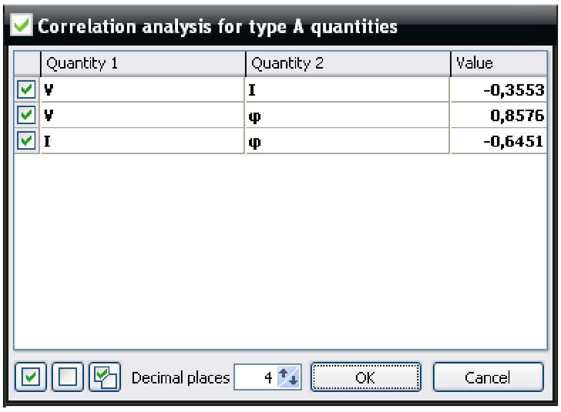
10. Correlation matrix

In the *Correlations* page, known correlations between the input quantities are entered in a matrix of correlation coefficients. To change the correlation coefficient between two input quantities, the corresponding cell that belongs to the two values is selected and the value is entered or changed. The coefficients value has to be in the range -1 <= value <= +1. The matrix value is updated as soon as change is committed by pressing the Enter key or by selecting a different matrix cell.

The *Correlation analysis for type A quantities* button starts an analysis for possible correlations between two types A quantities. If the number of observations of two measurands is the same for both of them and all of the values are valid, a correlation coefficient is computed. A correlation coefficient is not calculated if one of both standard deviations is zero. After clicking OK in the *Correlation analysis* window, the selected rows from the correlation analysis will be inserted in the relevant cells of the correlation matrix of the model.

If new data is entered for quantities of Type A at a later time, a new correlation analysis should be considered. ***QMSys GUM Software*** will not automatically adjust the correlation coefficient to the new data.

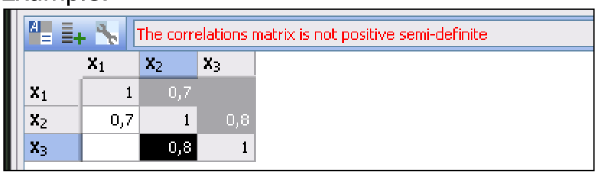


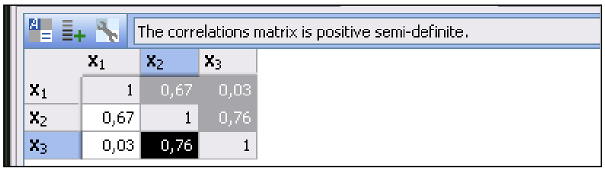


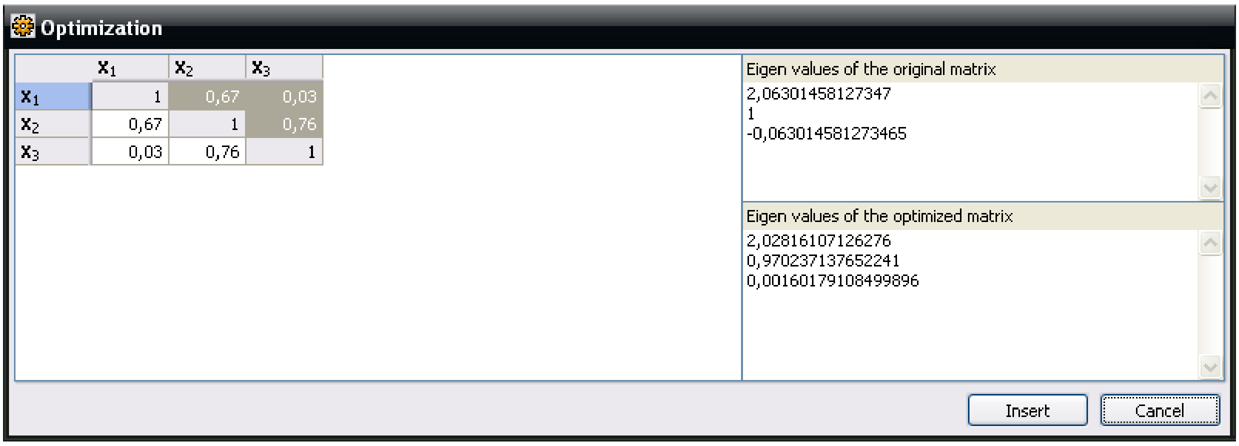
Further input quantities are inserted in the correlation matrix with the button *Select / remove quantities*.

The software checks automatically if the correlation matrix is positive semi-definite. All Eigen values of the positive semi-definite correlation matrix are non-negative (>= 0). If the correlation matrix is not positive semi-definite, it is recommended to perform an optimization with the button *Optimization*.

Example:







Only a positive semi-definite correlation matrix can be used in an uncertainty evaluation. When correlations between input values are considered, then the description field should contain the reason why, and where the correlation coefficients came from.

11. Expert Analysis of the Model

The view *Expert analysis* presents the results of the expanded analysis of the model. The software checks the conditions for the application of the different methods and determines the appropriate ones for the following calculation of the measurement uncertainty.

The following tests and calculations are performed:

11.1. Linearity of the Model

The program determines for each input quantity the nonlinearity of the model equation in the sixth areas of the distribution interval. As indicator of nonlinearity are used the relative differences between the sensitivity coefficients, calculated from the symmetric interval [x+u(x); x-u(x)] and from the one-sided intervals [x; x+0,5*u(x)], [x; x+u(x)], [x; upper limit], [x; x-0,5*u(x)], [x; x-u(x)], [x; lower limit]. The model equation is sufficiently linear, if the nonlinearity is less than a specified limit. The default limit is set to 0.05.

Additionally, the software displays all input quantities with invalid (zero) sensitivity coefficient. The reason for that problem can be a local minimum or maximum of the model function or a nonlinear relation between the result and that input quantity. Such nonlinearity should be seen in relation to the magnitude of the standard uncertainty associated with the quantity.

11.2. Validity of the Results of the Equivalent Linear Model

The software uses the method according to the GUM Uncertainty Framework for the calculation of the results of the equivalent linear model. Here, the partial derivatives (the first term of a Taylor series) are determined to calculate the sensitivity coefficients and the combined standard uncertainty. The results of the quasi-real model are calculated by using the quasi-Monte Carlo method with Sobol sequences (50000 simulations) and a special algorithm for generating correlated values while maintaining the specified probability distributions.

The software validates the results of the GUF method for linear models by comparing the values and the combined standard uncertainties. The numerical tolerance δ in this comparison is calculated as a percentage of the combined standard uncertainty of the quasi-real model. The default percentage limit is set to 5%. The results of the linear model are validated if the absolute values of the differences do not exceed the specified tolerance δ.

11.3. Symmetry and Type of the Probability Distribution of the Result quantities

The program calculates the skewness of the real distribution of the result quantities and determines the appropriate theoretical distribution. The resulting distribution is considered sufficient symmetric if the skewness is smaller than the specified limit. The default limit is set to 0.5.

If the resulting distribution is sufficient symmetric, the software selects the most suitable from the following theoretical distributions:

- Normal distribution
- Rectangular distribution
- Triangular distribution
- Trapezoidal distribution with automatic determination of the form factor
- Mixed symmetric distribution with automatic determination of the coverage factor.

11.4. Condition regarding the correlated input quantities in linear models

For the correct calculation of the expanded uncertainty in a linear model with normal or t- distribution of the result quantity the model equation may not contain correlated input quantities with a finite degree of freedom. Otherwise, the ***GUF-NL method for non-linear models*** and the ***Monte Carlo method*** should be used.

11.5. Conditions regarding the correlated input quantities and the probability distribution in nonlinear models

For models with non-linear correlated input quantities or non-linear non-normal distributed input quantities the ***GUF-NL method for non-linear models*** and the ***Monte Carlo method*** should be used. These cases are not considered in GUM and GUM Suppl.1, but they can be analyzed with the ***QMSys GUM Software***, because the software uses a special algorithm for generating correlated values for several input quantities while maintaining the specified probability distributions. This algorithm keeps the shape of the probability distribution of the input quantities unchanged, and the maximum deviation from the prescribed correlation coefficient is in the most cases smaller than 0.001.

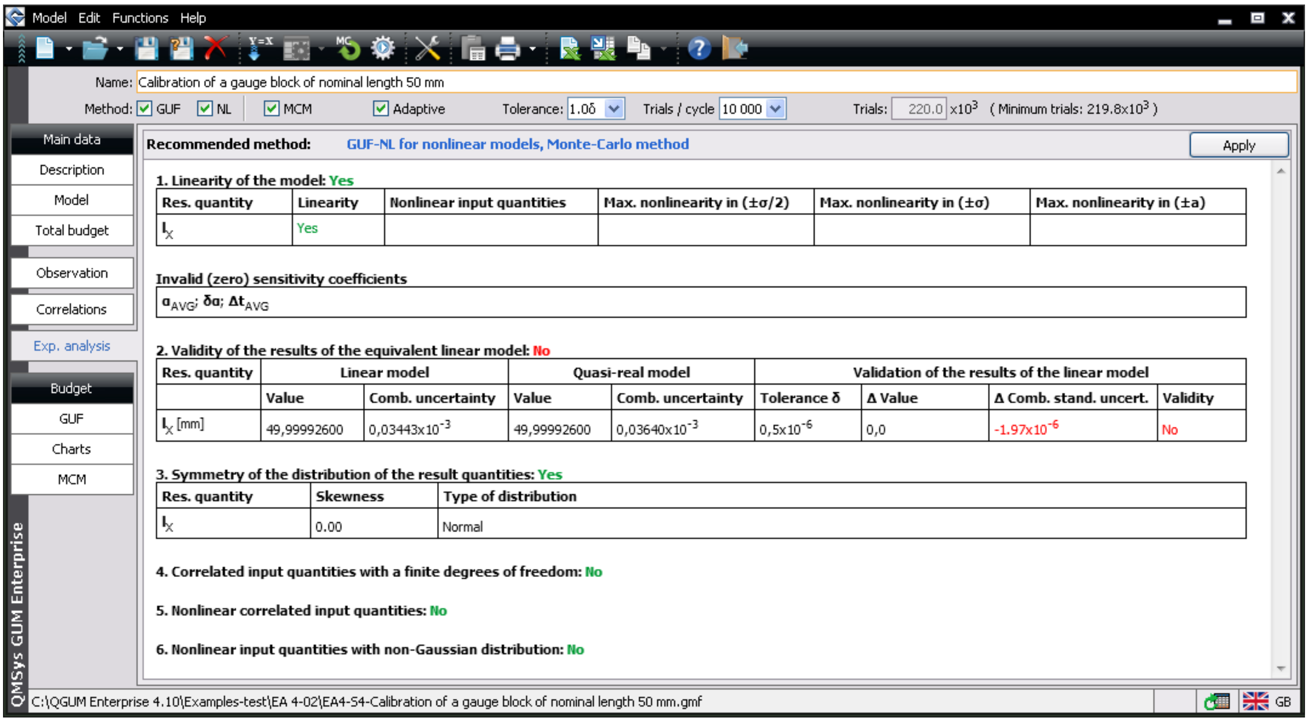
11.6. Selection of Appropriate Methods for Uncertainty Analysis

The GUM methods (GUF, GUF-NL) for analysis of the measurement uncertainty are suitable only for models with symmetric distribution of the result quantities. Moreover, additional conditions regarding covariance, degrees of freedom and probability distribution of the input quantities (test points 4 to 6) must be met. Since the range of validity of the Monte Carlo method is wider, it is recommended to use both methods (GUF / GUF-NL and MCM) and to compare the results. The following table shows the possible situations in the selection of the appropriate methods for the uncertainty analysis.

Method **Test**	**GUF**	**GUF + MCM**	**GUF-NL**	**GUF-NL + MCM**	**MCM**
1. Linearity of the Model	Yes	Yes	Yes / No	Yes / No	Yes / No
2. Validity of the results of the equivalent linear model	Yes	Yes	No	No	No
3.1. Symmetry of the distribution of the result quantity	Yes	Yes	Yes	Yes	No
3.2. Type of the probability distribution of the result quantity	Normal	All	Normal	All	All
4. Correlated input quantities with a finite degree of freedom	No	No	Yes / No	Yes / No	Yes / No
5. Non-linear correlated input quantities	No	No	No	Yes / No	Yes / No
6. Non-linear non-normally distributed input quantities	No	No	No	Yes / No	Yes / No

GUF – GUM Method for linear Models; GUF-NL - GUM Method for non-linear Models; MCM – Monte Carlo Method

The software checks the conditions for the application of each method and suggests the appropriate methods. With the *Apply* button, the methods for the following calculation of the measurement uncertainty are set automatically. Manual setting of the methods is also possible.



12. Measurement Uncertainty Budget

The result of the analysis is presented in pages *GUF*, *Charts* and *Monte Carlo* of the *Budget* view.

GUF-Budget

The page *GUF* shows a clearly structured measurement uncertainty budget in a table form. This table holds all used quantities with their quantity names and values, the associated standard uncertainty and effective degrees of freedom, the sensitivity coefficient automatically derived from the model equation and the contribution to the standard uncertainty of the result of the measurement. The *Interim results* are only shown with the value and the standard uncertainty. Additional columns can be activated in the *Budget* menu.

The result quantity is displayed in the bottom line with its value, the corresponding combined standard Uncertainty and the degrees of freedom. Finally, the complete result of the examination is presented as a value with associated expanded uncertainty and automatically or manually selected coverage factor. The results are automatically rounded and displayed in E-Format if necessary.

The relative contribution of uncorrelated input quantity is calculated by the ratio of the square of the uncertainty contribution to the square of the combined standard uncertainty.

Where input quantities are not independent, the calculation of the relative contribution takes account of the entered correlation coefficients, after that for easier result interpretation the calculated relative uncertainty contributions are normalized. The sum of all relative uncertainty contributions is always equal to 100%.

Charts

The page *Charts* summarizes the information from the GUF-budget and helps the user quickly to identify the most significant sources of uncertainty. The software offers several types of charts and adjustable limit of the cumulative relative uncertainty contribution.

The table contains all input quantities forming the sum of the uncertainty contribution not less than the selected limit. The significant quantities are sorted in descending order of the uncertainty contribution.

For indexed quantities the software calculates and draws in the chart the cumulative uncertainty contribution of all sub-quantities for the respective main quantity. The individual uncertainty contribution of the sub-quantities is displayed only in the table.

Monte Carlo-Method

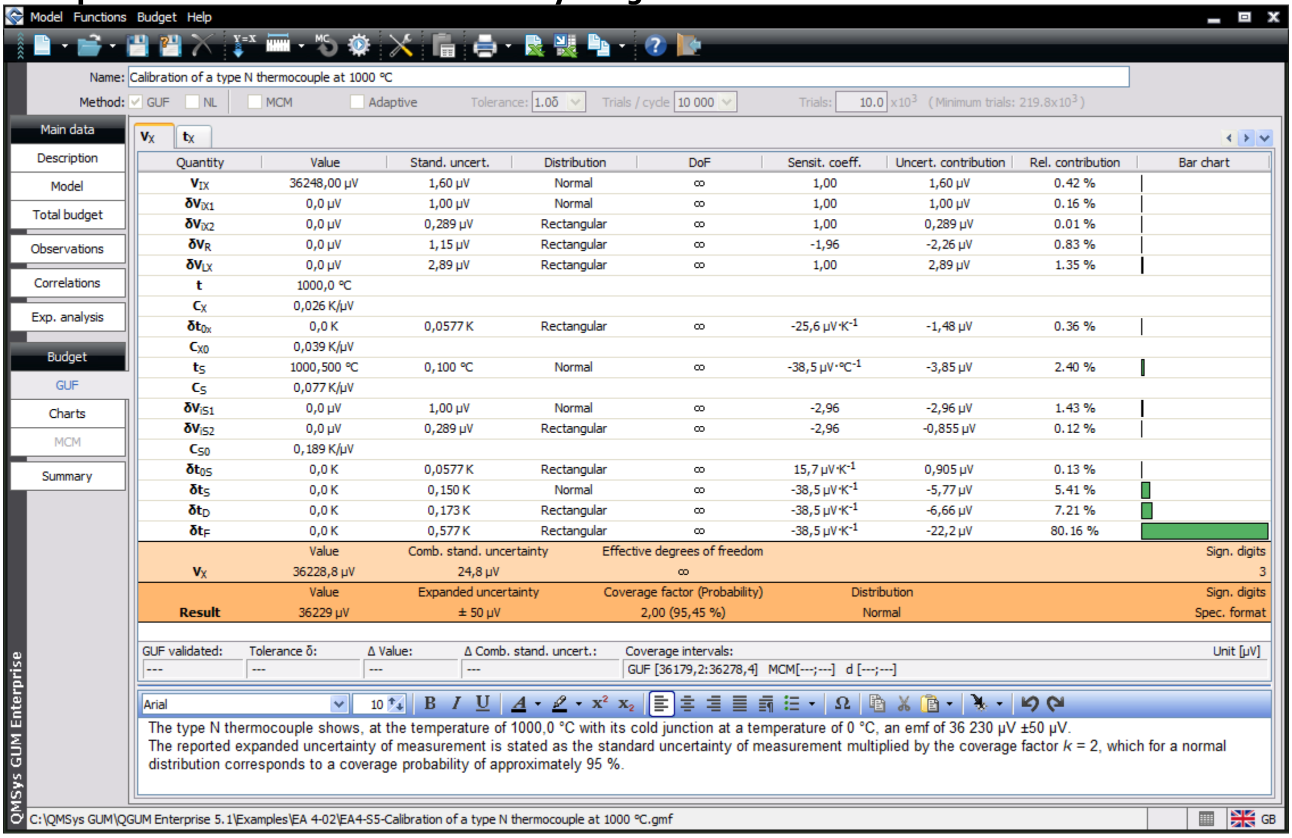
The *Monte Carlo method* displays a histogram, statistical parameters of the estimated distribution of the result quantities and validation of the results. For result quantities with asymmetric distribution the program estimates the shortest coverage interval, the asymmetric expanded measurement uncertainty and the asymmetric coverage factor.

The *Total budget* offers the following additional analyses:

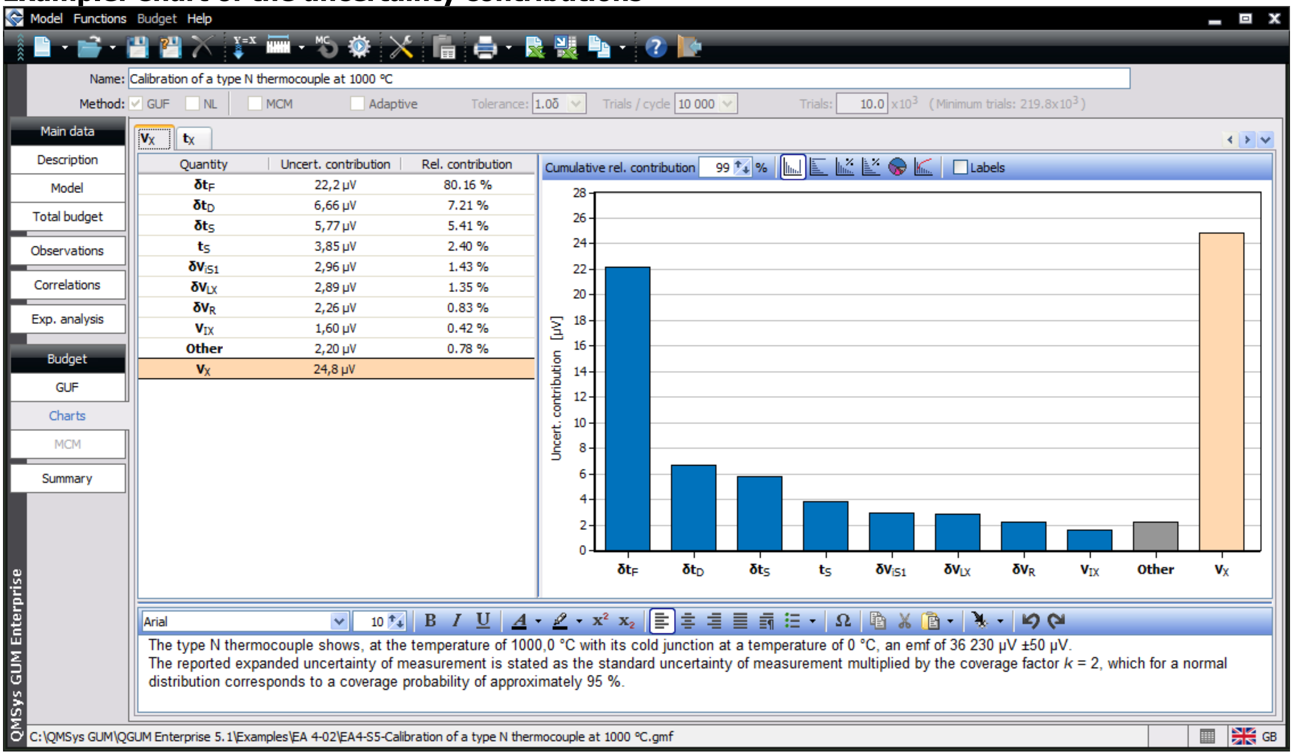
- Regression analysis and calculation of the equation of the expanded measurement uncertainty for a certain measurement range
- Diagrams of the expanded measurement uncertainty for a certain measurement range
- Correlation analysis of the result quantities.

The software automatically validates the results of the GUF Method by comparing the values, the combined standard uncertainties, and the limits of the coverage intervals. The numerical tolerance δ in this comparison is calculated based on the combined standard uncertainty and the number of significant digits. The software offers an alternative calculation of the tolerance δ as a percentage of the combined standard uncertainty. Should the comparison be positive, then the GUM uncertainty framework can be used on this occasion and for sufficiently similar models in the future. Otherwise, consideration should be given to using MCM or another appropriate method instead.

Example: GUF – Measurement uncertainty budget



Example: Chart of the uncertainty contributions



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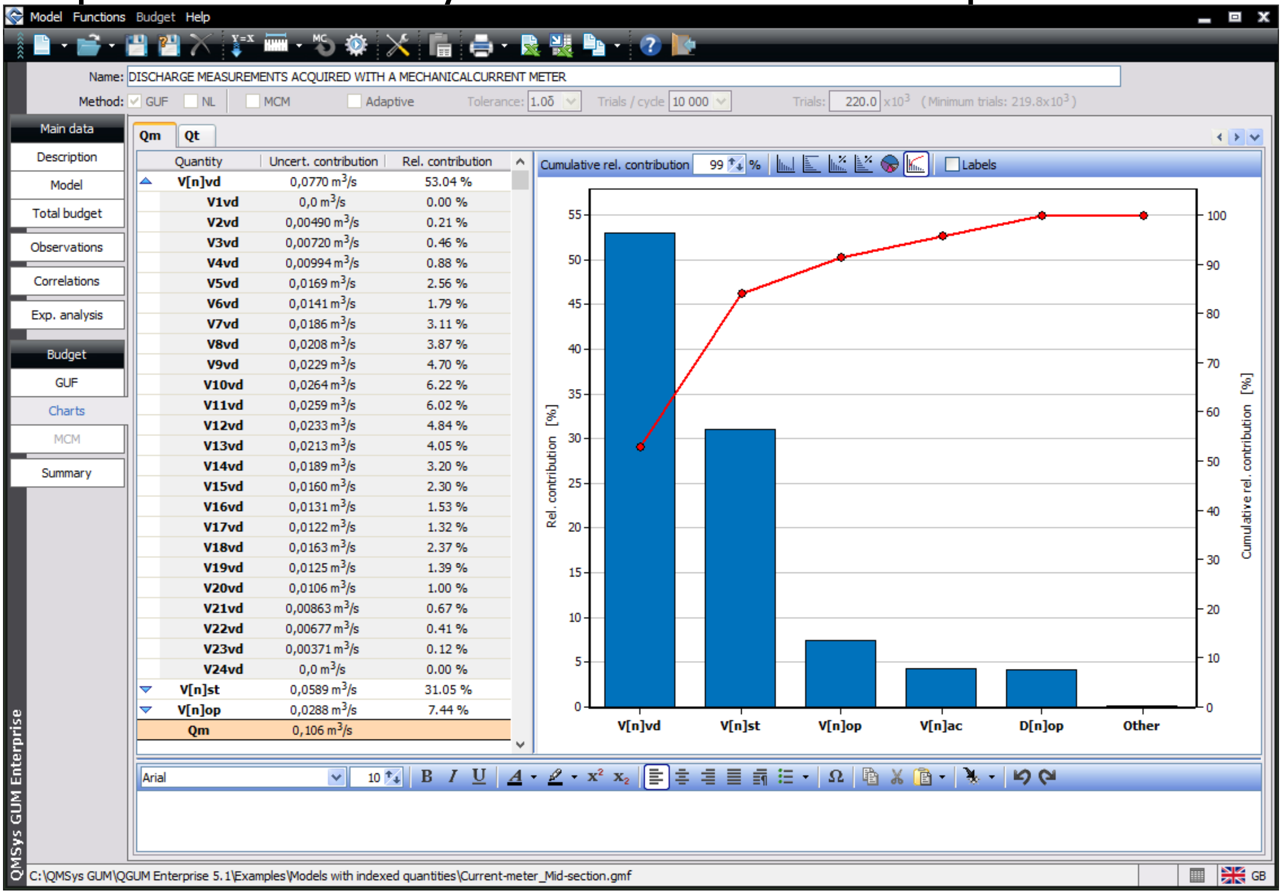
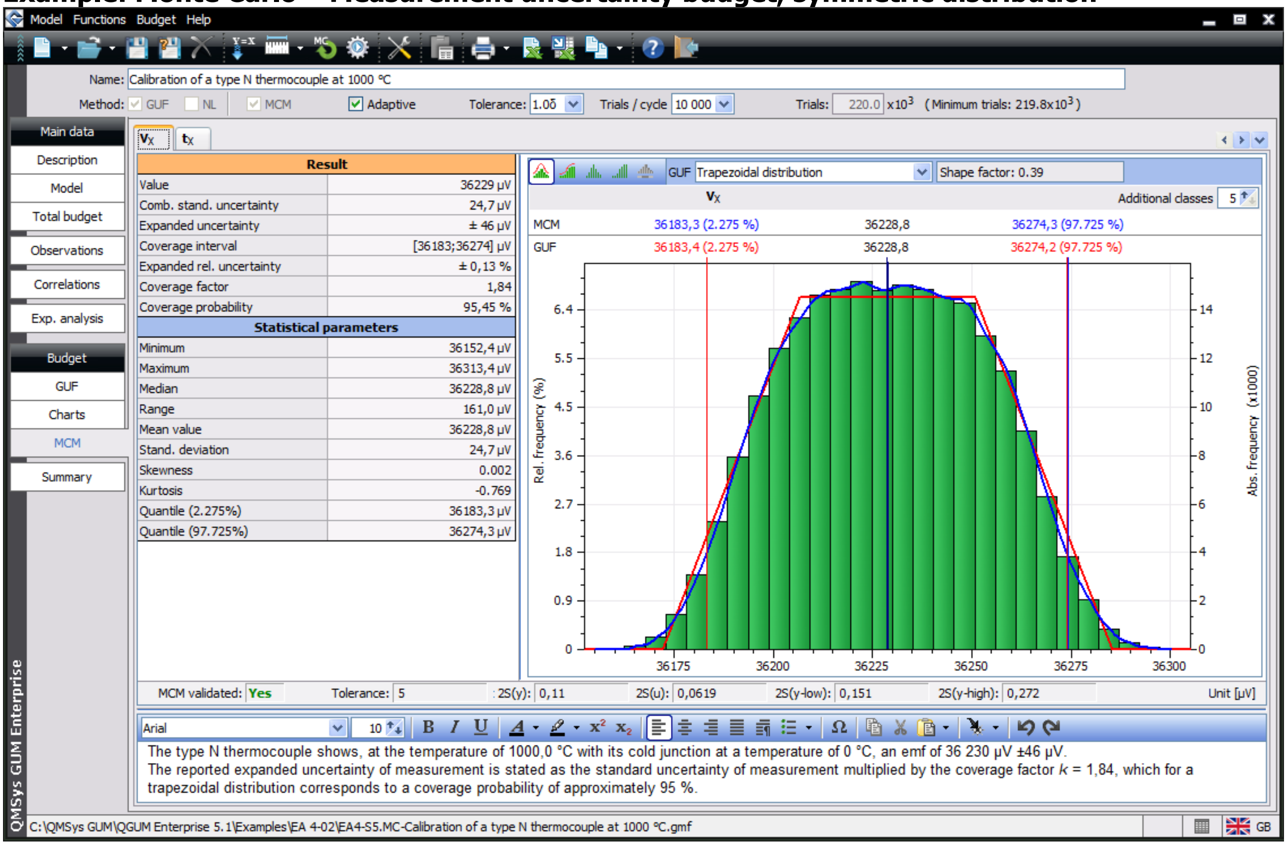
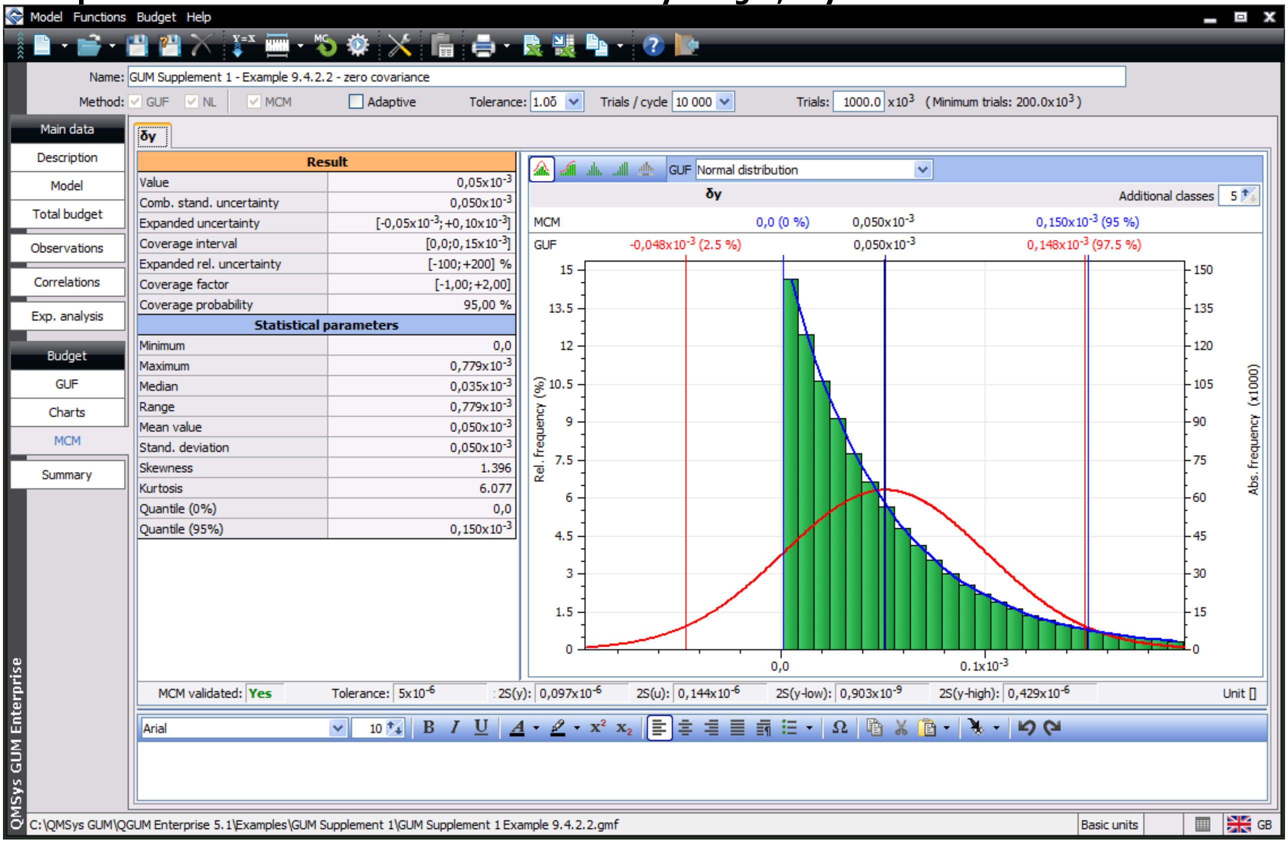
Example: Chart of the uncertainty contributions of model with indexed quantities



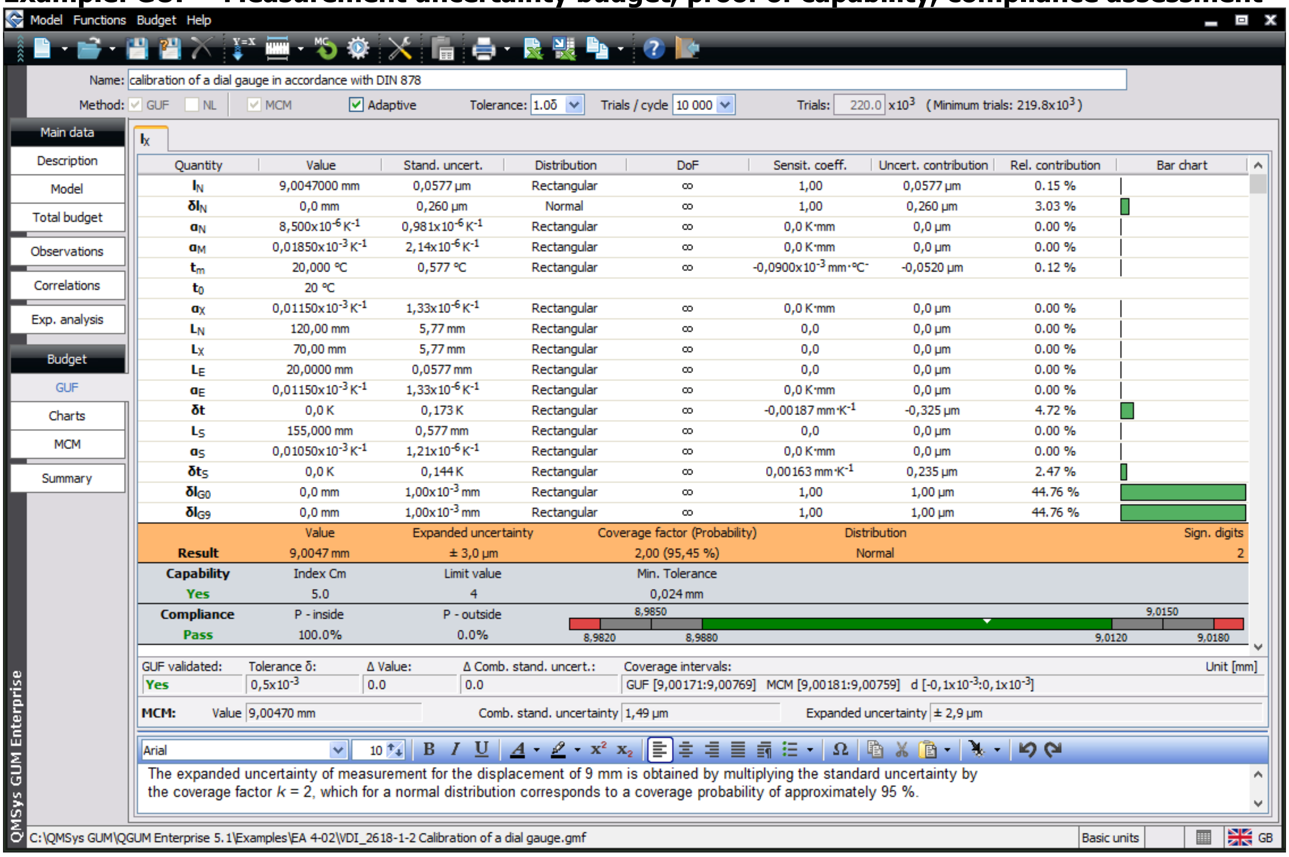
Example: Monte Carlo – Measurement uncertainty budget, symmetric distribution



Example: Monte Carlo – Measurement uncertainty budget, asymmetric distribution



Example: GUF – Measurement uncertainty budget, proof of capability, compliance assessment



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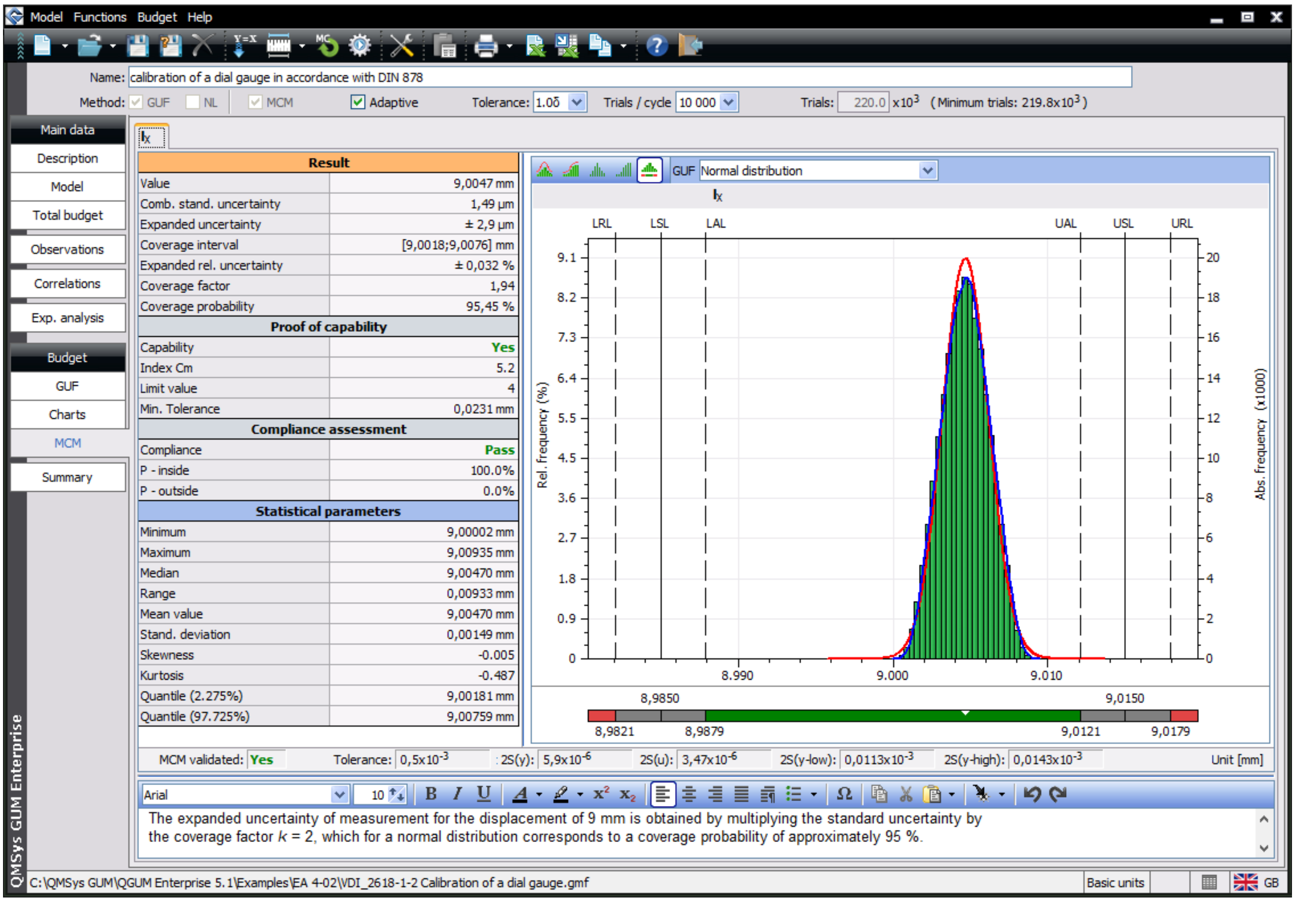
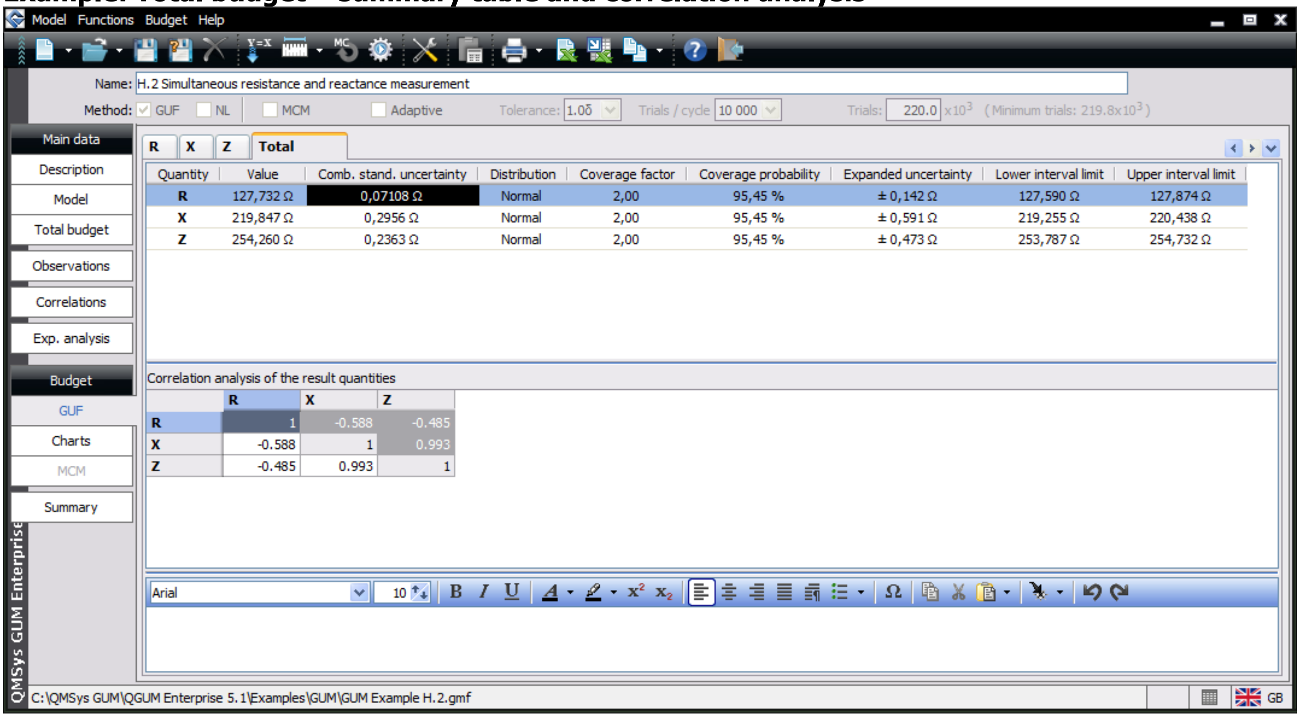
Example: Monte Carlo – Measurement uncertainty budget, proof of capability, compliance assessment



Example: Total budget – summary table and correlation analysis



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13. Printing and Export to Microsoft Excel

13.1. Printing of Measurement Uncertainty Report

Printout in the program is made by configurable templates in RTF format (*.RTF) with coded fields, e.g. *$FA01#* for individual fields and *$TB01#* for table fields. The user can provide or adapt the report by using the coding of the fields from the standard report. The sequence of the individual fields or tables can be changed. The RTF Template can contain additional texts and pictures (Company Logo).

Types of coded fields

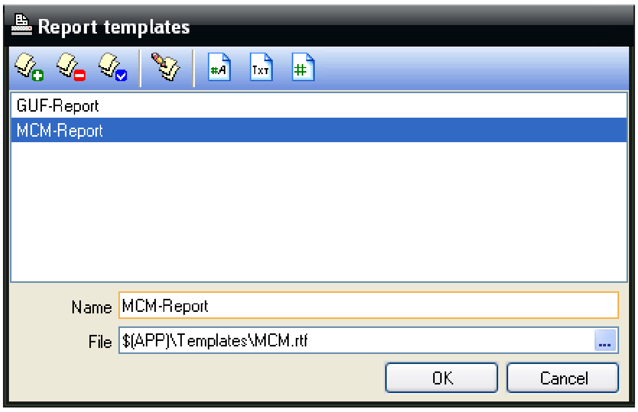
Coding	Field Content
$FA01#	Individual data fields.
$TB01#	Table data fields.
$FI07?100;50# $FI07?100;50;M# $TE20?4;2;I#	Graphic fields, the two additional numbers define the size of the graphic (width; height) in unit [mm] (M#) or [inch] (I#). If no coding symbol is entered for the unit, the numbers for width and height will be interpreted in [mm].
$C0118#	Texts from the program language file for the selected language. Language files are normal text files in the program folder *Languages* and can be open with programs like MS Notepad.
$ELOGO#	Company logo.

The following rules however must be fulfilled:

- The marking of a field consists of "$" + "Field name" + "#".
- Individual data fields ($F*) and table data fields ($T*) cannot stand mixed in one table.
- Table data fields from different sources ($TA*;$TB*) cannot stand mixed in one table.
- Coded fields must be with the same font - e.g. $FA01# is correct, $FA**01#** is wrong.

Setting of custom templates

The dialog window for selecting custom templates is called over *Function/Print/Template...* menu or over the *Print/Template...* button. Enter a designation of the report in the *Name* field and select the template file in the *File* field.



Toolbar buttons

	Add template	Selection of a file for new custom template
	Delete template	Deletes the setting for the selected custom template
	Set template as default	Marks the selected custom template as default
	Open template	Opens the selected custom template for redaction
	Standard template	Opens a template with all coded fields; program texts are shown with both code and text
	Standard template	Opens a template with all coded fields; program texts are shown only with text
	Standard template	Opens a template with all coded fields; program texts are shown only with code.

When generating a report, the software automatically selects the corresponding standard template, if there are no custom templates selected. If there are set custom templates, the software selects the default custom template. Additional custom templates are selected for printing with the arrow on the *Print* button or over the *Function/Print* menu.

Generated reports can be printed, saved in a file with a selectable name or send by email.

Standard templates

The software editions ***GUM Enterprise, GUM Professional***, and ***GUM Calculator*** include following standard templates:

Template file	Analysis method	Total budget
GUF.rtf	GUF/GUF-NL	No
GUF_CA.rtf	GUF/GUF-NL	Yes, with correlation analysis of the results
GUF_RA.rtf	GUF/GUF-NL	Yes, with regression analysis of the results
MCM.rtf	Monte Carlo	No
MCM_CA.rtf	Monte Carlo	Yes, with correlation analysis of the results
MCM_RA.rtf	Monte Carlo	Yes, with regression analysis of the results
GUF_MCM.rtf	GUF/GUF-NL and Monte Carlo	No
GUF_MCM_CA.rtf	GUF/GUF-NL and Monte Carlo	Yes, with correlation analysis of the results
GUF_MCM_RA.rtf	GUF/GUF-NL and Monte Carlo	Yes, with regression analysis of the results

The standard templates include the most important mean data, the observations with statistical evaluation and the budget data according to the selected method for analysis of the measurement uncertainty. These templates are compiled in the file "templates.data".

Copies of the standard templates are also located in the program folder *Templates* and can be adapted by entering additional coded fields, text, or pictures (Company Logo).

Creating custom templates for the measurement uncertainty reports

The following steps provide the procedure for creating your own report templates:

1. Open an appropriate standard template from the folder "Templates" with a text editor for RTF files (such as MS Word) and save it under a new name.
2. Make the desired changes. You can find the coding for field names and field contents in the standard coded fields template, which is opened with the button in the Template settings window.
3. Save the changes in the new template.
4. Open the dialog window for the selection of custom templates over *Function/Print/Template...* menu or over the *Print/Template...* button.
5. Select the new template file with the button and type a description of the template in the *Name* field.
6. Close the dialog with the OK button.

The new template is already set and can be selected for printing with the arrow on the *Print* button or over the *Function/Print/{Template name}* menu.

Appendix B shows an example of a report.

Example 1: Excerpt from the standard template with all coded fields, field names are shown with both coding and text.

$C0212# Description

$C7410# File	$FA01#
$C0173# Name	$FA02#
$C0212# Description	$FA03#
$C0204# Model	$FA04#

$C0712# Quantities description

$C0118# Quantity	$C0173# Name	$C0438# Unit	$C0126# Type	$C0709# Comment	$C0212# Description
$TB01#	$TB02#	$TB03#	$TB04#	$TB05#	$TB06#

Example 2: Excerpt from the standard template with all coded fields, field names are shown only with text.

$C0212#

$C7410#	$FA01#
$C0173#	$FA02#
$C0212#	$FA03#
$C0204#	$FA04#

$ C0712#

$C0118#	$C0173#	$C0438#	$C0126#	$C0709#	$C0212#
$TB01#	$TB02#	$TB03#	$TB04#	$TB05#	$TB06#

Example 3: Excerpt from the standard template with all coded fields, field names are shown only with coding.

Description

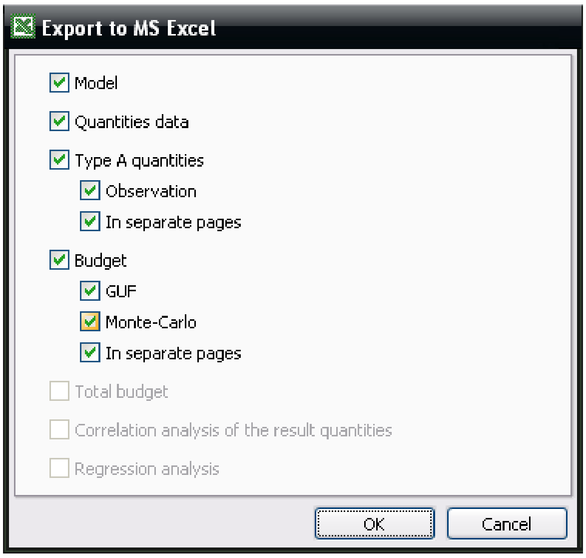
File	$FA01#
Name	$FA02#
Description	$FA03#
Model	$FA04#

Quantities description

Quantity	Name	Unit	Type	Comment	Description
$TB01#	$TB02#	$TB03#	$TB04#	$TB05#	$TB06#

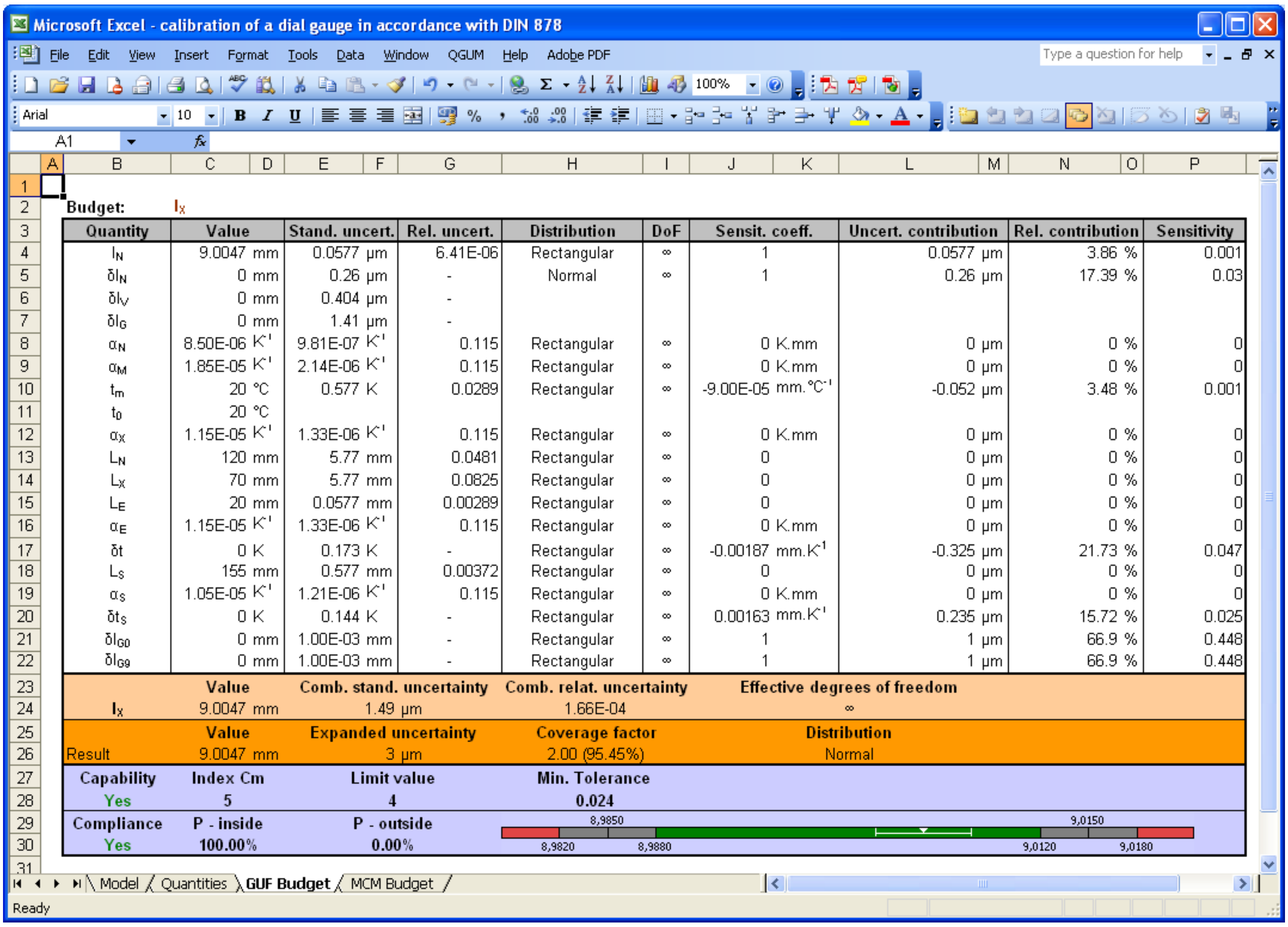
13.2. Export to Microsoft Excel

The *Export to MS Excel* is a useful feature to transfer data from an uncertainty analysis over the OLE interface to Excel files. With the command / button *Export to MS Excel*, a dialog window with the settings for the data transfer is open.



The data transfer starts when selecting the OK button. The program MS Excel is loaded in the background and according to the active options in the dialog window, the software creates a new workbook with multiple worksheets and fills them with data. Numbers and units are exported into separate cells. Appendix C shows an example of an export.

Note: Do not use the MS Excel program during the export, since otherwise the export operation can be disturbed.



Budget: l_X

Quantity	Value	Stand. uncert.	Rel. uncert.	Distribution	DoF	Sensit. coeff.	Uncert. contribution	Rel. contribution	Sensitivity
l_N	9.0047 mm	0.0577 µm	6.41E-06	Rectangular	∞	1	0.0577 µm	3.86 %	0.001
δl_N	0 mm	0.26 µm	-	Normal	∞	1	0.26 µm	17.39 %	0.03
δl_V	0 mm	0.404 µm	-						
δl_G	0 mm	1.41 µm	-						
α_N	8.50E-06 K^{-1}	9.81E-07 K^{-1}	0.115	Rectangular	∞	0 K.mm	0 µm	0 %	0
α_M	1.85E-05 K^{-1}	2.14E-06 K^{-1}	0.115	Rectangular	∞	0 K.mm	0 µm	0 %	0
t_m	20 °C	0.577 K	0.0289	Rectangular	∞	-9.00E-05 mm.°C^{-1}	-0.052 µm	3.48 %	0.001
t_0	20 °C								
α_X	1.15E-05 K^{-1}	1.33E-06 K^{-1}	0.115	Rectangular	∞	0 K.mm	0 µm	0 %	0
L_N	120 mm	5.77 mm	0.0481	Rectangular	∞	0	0 µm	0 %	0
L_X	70 mm	5.77 mm	0.0825	Rectangular	∞	0	0 µm	0 %	0
L_E	20 mm	0.0577 mm	0.00289	Rectangular	∞	0	0 µm	0 %	0
α_E	1.15E-05 K^{-1}	1.33E-06 K^{-1}	0.115	Rectangular	∞	0 K.mm	0 µm	0 %	0
δt	0 K	0.173 K	-	Rectangular	∞	-0.00187 mm.K^{-1}	-0.325 µm	21.73 %	0.047
L_S	155 mm	0.577 mm	0.00372	Rectangular	∞	0	0 µm	0 %	0
α_S	1.05E-05 K^{-1}	1.21E-06 K^{-1}	0.115	Rectangular	∞	0 K.mm	0 µm	0 %	0
δt_S	0 K	0.144 K	-	Rectangular	∞	0.00163 mm.K^{-1}	0.235 µm	15.72 %	0.025
δl_{G0}	0 mm	1.00E-03 mm	-	Rectangular	∞	1	1 µm	66.9 %	0.448
δl_{G9}	0 mm	1.00E-03 mm	-	Rectangular	∞	1	1 µm	66.9 %	0.448

	Value	Comb. stand. uncertainty	Comb. relat. uncertainty	Effective degrees of freedom
l_X	9.0047 mm	1.49 µm	1.66E-04	∞

	Value	Expanded uncertainty	Coverage factor	Distribution
Result	9.0047 mm	3 µm	2.00 (95.45%)	Normal

Capability	Index Cm	Limit value	Min. Tolerance
Yes	5	4	0.024

Compliance	P - inside	P - outside
Yes	100.00%	0.00%

14. Automatic Export of the Results in Configurable MS Excel Files

In the imported Excel file, from which are read the data of the input quantities, can be automatically exported the parameters of the statistical analysis of the measurements and the results of the uncertainty analysis. The export is started manually or automatically after the calculation of the measurement uncertainty.

Settings for the automatic export of the results in the imported MS Excel file

The codes of the data for the automatic export are entered directly into the cells or in the comment field of the cells. The order of the data fields in the worksheet can be arbitrary. When the codes are entered in the cells, it is not allowed to enter any other characters or texts. If the codes are entered in the comment fields of the cells, the comments may contain additional texts. The use of the comment fields allows the multiple use of the same Excel file.

Examples:

Model file: *EA4-S2.Import.Export-Calibration of a weight.gmf* **Import:** Entering the name of the cell area defined in the Excel file *S2_Data.xls* **Export:** Codes are entered in the cells; the results are exported in new Excel file *S2_Data_exp.xls*
Model file: *EA4-S3.Import.Export-Calibration of a standard resistor.gmf* **Import:** Entering a range of cells (B2 - B6) from the Excel file *S3_Data.xls* **Export:** Codes are entered in the comment fields; the results are exported in the same Excel file.

Following table provides an overview of the codes for the input quantities of type A:

Field Contents	Codes	Comments
Quantity	$A01I01#	
Number of values	$A02I01#	
Minimum	$A03I01#	
Maximum	$A04I01#	
Median	$A05I01#	
Mean value	$A06I01#	
Range	$A07I01#	
Variance	$A08I01#	
Stand. deviation	$A09I01#	
Stand. uncertainty	$A10I01#	
Bayesian stand. uncertainty	$A11I01#	
Skewness	$A12I01#	
Kurtosis	$A13I01#	
Normal distribution (P=99%)	$A14I01# $A14I01T#	Test result is returned with text "Yes" or "No". Inserting the letter "T" in front of the character "#" activates the export of the test value and critical value.
Outliers (P=95%)	$A15I01# $A15I01T#	
Outliers (P=99%)	$A16I01# $A16I01T#	
Normal distribution (P=99%)	$A17I01#	Exporting with number: 0 = No, 1 = Yes
Outliers (P=95%)	$A18I01#	
Outliers (P=99%)	$A19I01#	

Following table provides an overview of the codes for the result quantities, depending on the method of calculation of measurement uncertainty:

Field Contents \ Method	GUF, GUF-NL	Monte Carlo
Quantity	$G01I01#	$M01I01#
Value	$G02I01#	$M02I01#
Comb. standard uncertainty	$G03I01#	$M03I01#
Distribution	$G04I01#	$M04I01#
Coverage factor	$G05I01#	$M05I01#
Coverage probability	$G06I01#	$M06I01#
Expanded absolute uncertainty	$G07I01#	$M07I01#
Expanded relative uncertainty	$G08I01#	$M08I01#
Lower interval limit	$G09I01#	$M09I01#
Upper interval limit	$G10I01#	$M10I01#
Capability (text "Yes" or "No")	$G11I01#	$M11I01#
Index	$G12I01#	$M12I01#
Limit value	$G13I01#	$M13I01#
Min. Tolerance	$G14I01#	$M14I01#
Compliance (text "Yes", "No" or "U-Range")	$G15I01#	$M15I01#
P-inside	$G16I01#	$M16I01#
P-outside	$G17I01#	$M17I01#
Lower specification limit	$G18I01#	$M18I01#
Upper specification limit	$G19I01#	$M19I01#
Capability (0="No", 1="Yes")	$G20I01#	$M20I01#
Compliance (0="No", 1="Yes", 2=" U-Range")	$G21I01#	$M21I01#

Inserting the letter "U" in front of the character "#" will export the corresponding values with the selected unit, for example, $G02I01U#.

In case of exporting several input or result quantities, the field codes are entered for each individual quantity in the corresponding cells. The two-digit number following the letter "I" corresponds to the order number of the input quantity in the view "Observation" respectively the result quantity in the view "Budget".

Example:

Value	Comb. standard uncertainty	Expanded absolute uncertainty
$G02I01#	$G03I01#	$G07I01#
$G02I02#	$G03I02#	$G07I02#
$G02I03#	$G03I03#	$G07I03#

Exporting the results in editions *GUM Enterprise, GUM Professional*, and *GUM Calculator*

The export is started manually or automatically after the calculation of the measurement uncertainty. By default, the generated Excel file with the measurement uncertainty results is saved under a different name. At the first export of the current model, the program automatically displays a dialog window for entering the new name and selecting the folder, where the file will be saved. The new file name will be used in the following calculations to measurement uncertainty budgets until the model is closed. At the next opening of the model, the program automatically shows the window for entering the name of the export file.

In the program *Preferences* and *Properties* dialog windows are available additional setting options for exporting the results in the same import file, for automatic export after any calculation of measurement uncertainty and automatic opening the generated file for further editing.

Switching on the automatic export is displayed in the status line of the program.

15. Determination of Uncertainties in Flow Measurement in Open Channels

The edition *QMSys GUM Enterprise* is designed for the assessment of uncertainty in flow measurements made using the velocity-area method and computed by the mid-section and mean-section method described in the standards ISO 748:2007 and ISO 1088:2007.

The software offers the following methods for estimating the sources of uncertainty:

- ISO 748, ISO 1088 method
- FLow Analog UnceRtainty Estimation (FLAURE) method
- Q+ method
- Interpolation Variance Estimator (IVE) method

Several methods for determination of the mean velocity in a vertical are included:

- Reduced point method
- Velocity distribution method
- Integration method
- Surface one-point method

The table below lists the possible combinations:

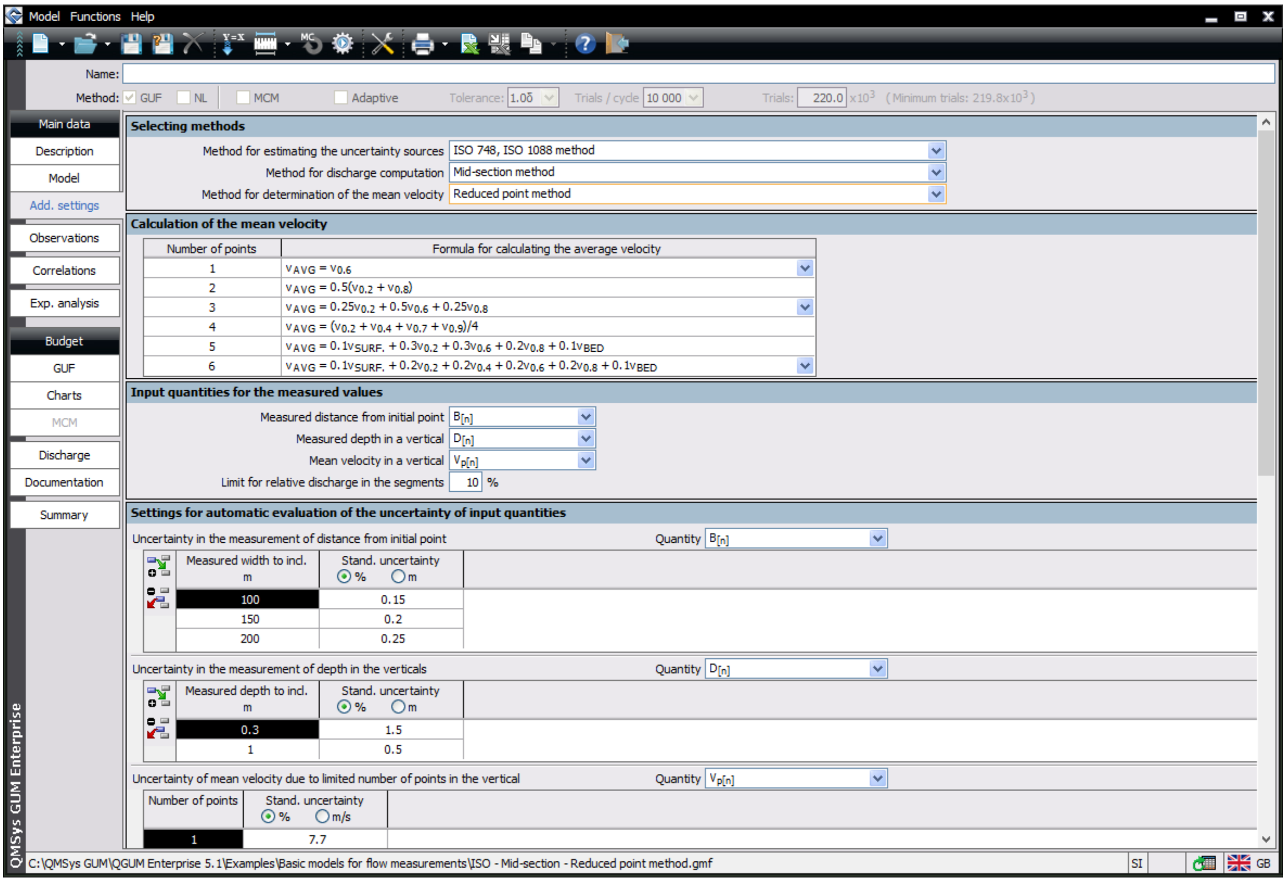
Method for estimating the sources of uncertainty	**ISO**	**FLAURE**	**Q+**	**IVE**
Methods for discharge calculation				
- Mid-section method	x	x	x	x
- Mean-section method	x	x	x	
Methods for determination of the mean velocity				
- Reduced point method	x	x	x	x
- Velocity distribution method	x	x	x	x
- Integration method	x	x	x	x
- Surface one-point method	x			

Basic quantities (sources of uncertainty) with automatic calculation of the measurement uncertainty for the respective evaluation methods:

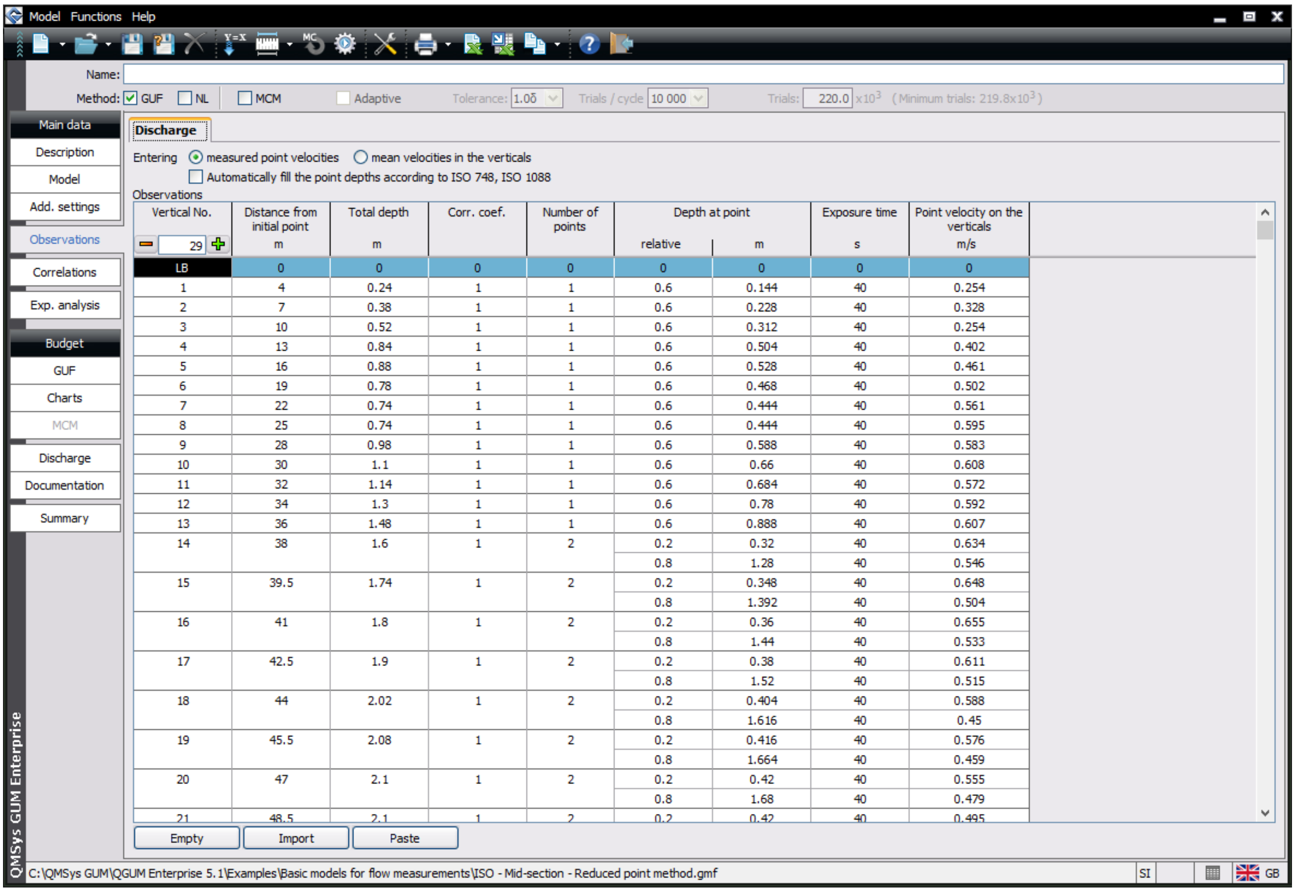
Des.	**Name**	**ISO**	**FLAURE**	**Q+**	**IVE**
B	Uncertainty in the measurement of distance from initial point	x	x	x	x
D	Uncertainty in the measurement of depth in the verticals	x	x	x	x
V	Uncertainty in the measurement of velocity in the verticals				x
V_p	Uncertainty of mean velocity due to limited number of points in the vertical	x	x	x	x
V_c	Uncertainty in point velocity measurement due to current-meter rating error	x	x	x	
V_e	Uncertainty in point velocity measurement due to limited exposure time	x	x	x	
D_m	Uncertainty of depth due to limited number of verticals (transversal integration)			x	
V_m	Uncertainty of mean velocity due to limited number of verticals (transversal integration)			x	
Q_m	Uncertainty of discharge due to limited number of verticals	x	x		

When selecting the type of *Measurement of flow in open channels*, the software shows the following additional windows:

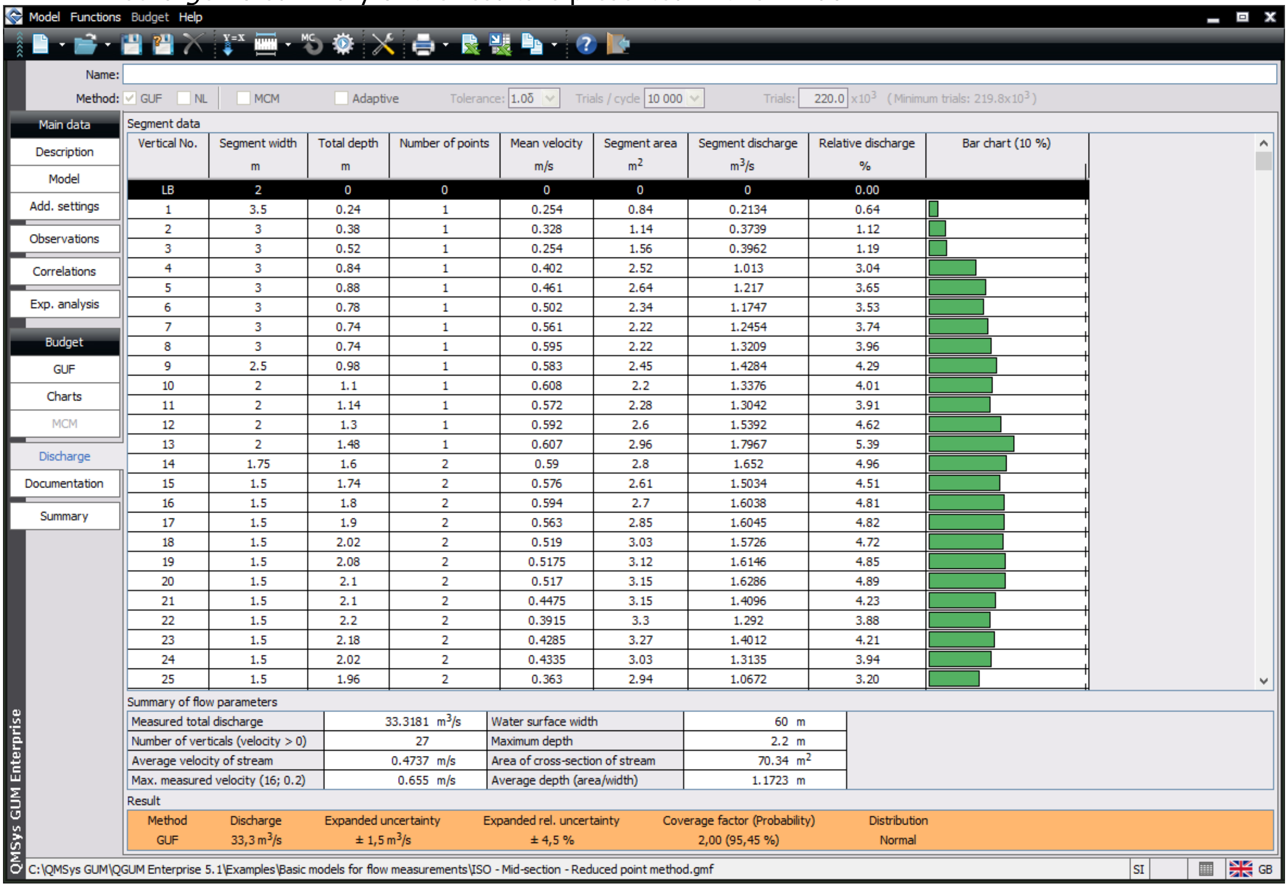
- *Additional settings* for automated calculation of uncertainty components, depending on the selected evaluation method



- *Observations / Discharge* for entering the measurements of width, depth, and point velocity in each vertical



- *Discharge* - a summary of the results is presented in this window

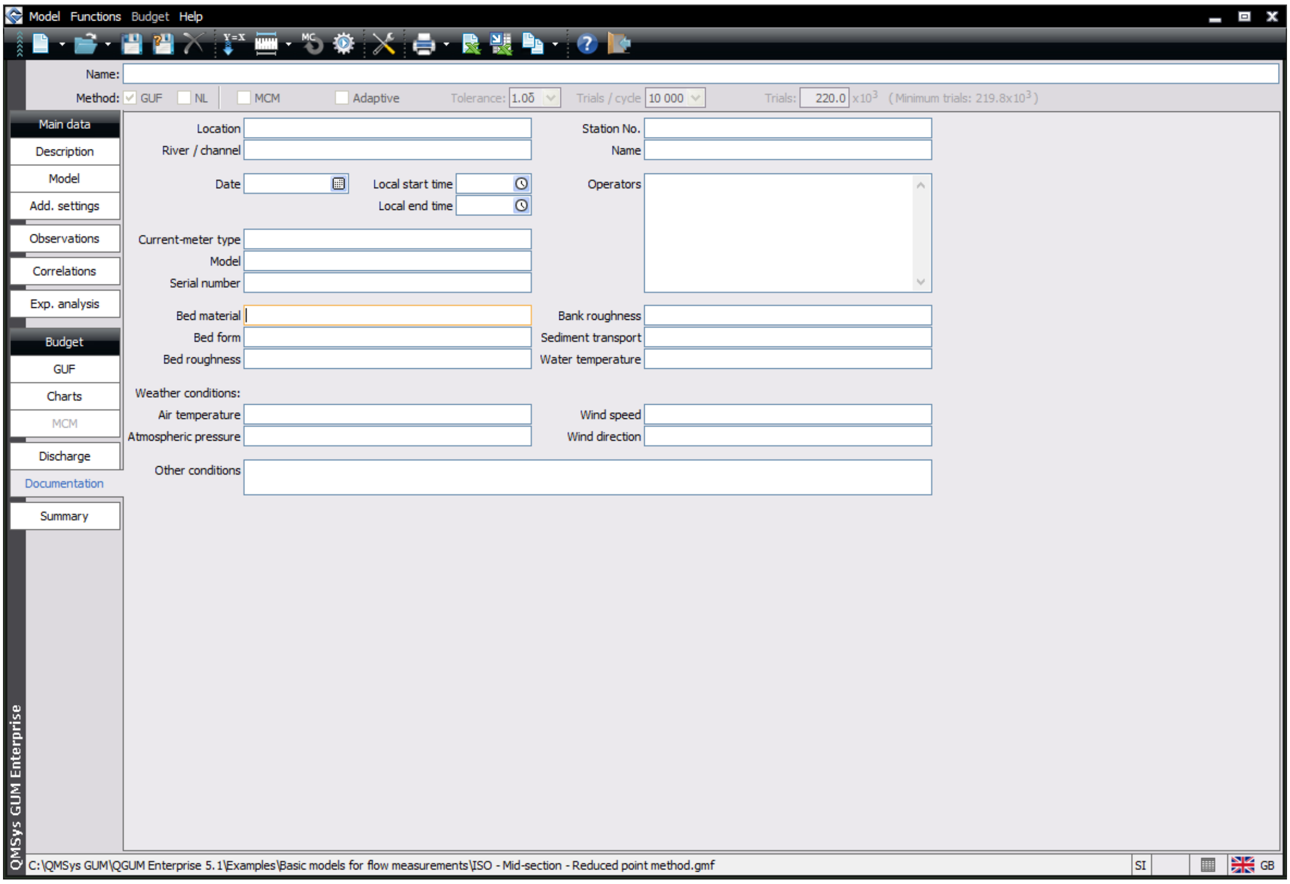


Vertical No.	Segment width m	Total depth m	Number of points	Mean velocity m/s	Segment area m²	Segment discharge m³/s	Relative discharge %	Bar chart (10 %)
LB	2	0	0	0	0	0	0.00	
1	3.5	0.24	1	0.254	0.84	0.2134	0.64	
2	3	0.38	1	0.328	1.14	0.3739	1.12	
3	3	0.52	1	0.254	1.56	0.3962	1.19	
4	3	0.84	1	0.402	2.52	1.013	3.04	
5	3	0.88	1	0.461	2.64	1.217	3.65	
6	3	0.78	1	0.502	2.34	1.1747	3.53	
7	3	0.74	1	0.561	2.22	1.2454	3.74	
8	3	0.74	1	0.595	2.22	1.3209	3.96	
9	2.5	0.98	1	0.583	2.45	1.4284	4.29	
10	2	1.1	1	0.608	2.2	1.3376	4.01	
11	2	1.14	1	0.572	2.28	1.3042	3.91	
12	2	1.3	1	0.592	2.6	1.5392	4.62	
13	2	1.48	1	0.607	2.96	1.7967	5.39	
14	1.75	1.6	2	0.59	2.8	1.652	4.96	
15	1.5	1.74	2	0.576	2.61	1.5034	4.51	
16	1.5	1.8	2	0.594	2.7	1.6038	4.81	
17	1.5	1.9	2	0.563	2.85	1.6045	4.82	
18	1.5	2.02	2	0.519	3.03	1.5726	4.72	
19	1.5	2.08	2	0.5175	3.12	1.6146	4.85	
20	1.5	2.1	2	0.517	3.15	1.6286	4.89	
21	1.5	2.1	2	0.4475	3.15	1.4096	4.23	
22	1.5	2.2	2	0.3915	3.3	1.292	3.88	
23	1.5	2.18	2	0.4285	3.27	1.4012	4.21	
24	1.5	2.02	2	0.4335	3.03	1.3135	3.94	
25	1.5	1.96	2	0.363	2.94	1.0672	3.20	

Measured total discharge	33.3181 m³/s	Water surface width	60 m
Number of verticals (velocity > 0)	27	Maximum depth	2.2 m
Average velocity of stream	0.4737 m/s	Area of cross-section of stream	70.34 m²
Max. measured velocity (16; 0.2)	0.655 m/s	Average depth (area/width)	1.1723 m

Method	Discharge	Expanded uncertainty	Expanded rel. uncertainty	Coverage factor (Probability)	Distribution
GUF	33,3 m³/s	± 1,5 m³/s	± 4,5 %	2,00 (95,45 %)	Normal

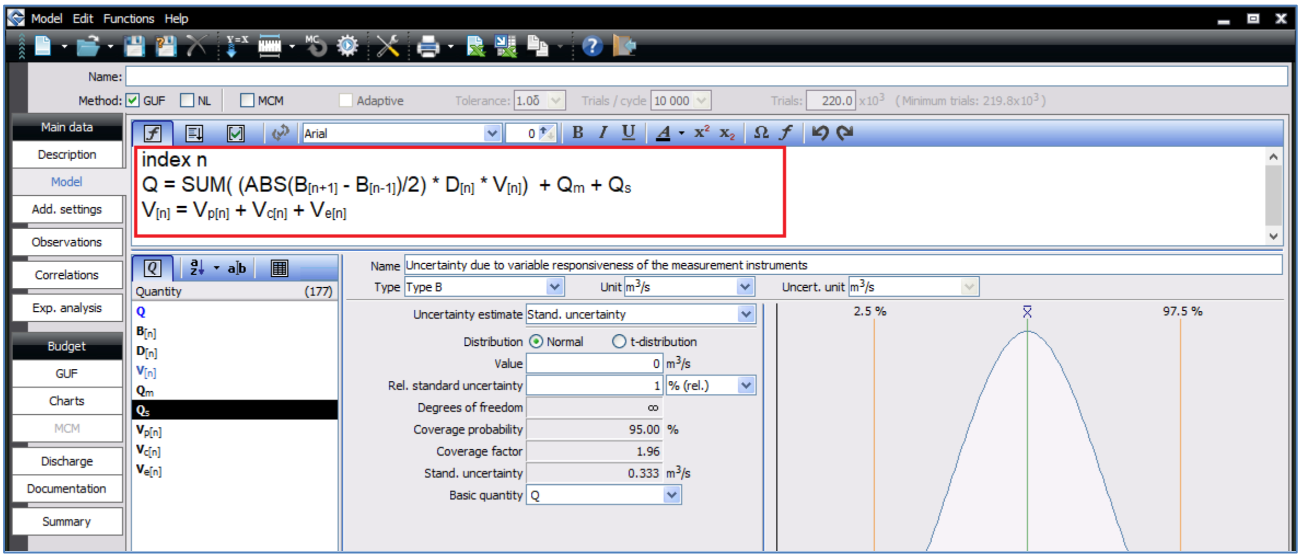
- *Documentation* - in this window are entered data about the location and measurement conditions



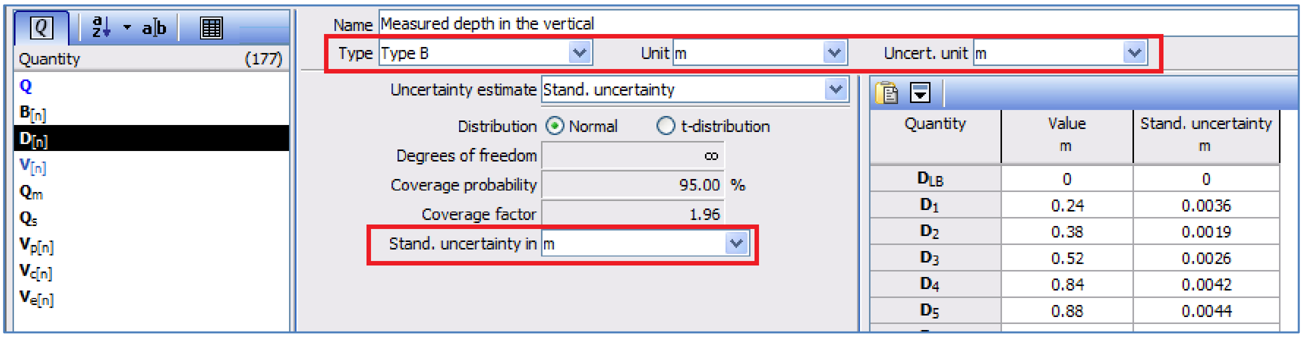
15.1. Development of the Models for Flow Measurements

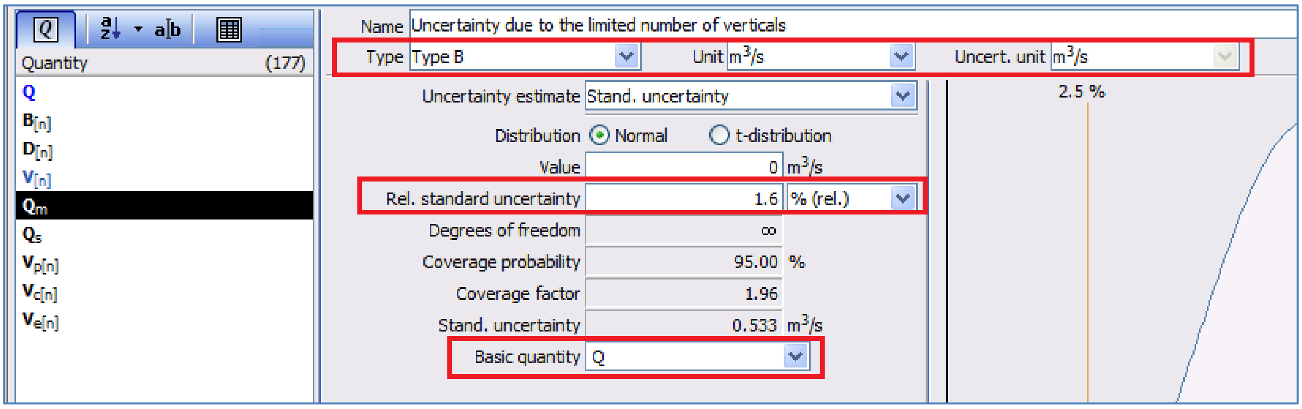
In the software folder *Examples* subfolder *Basic models for flow measurements* are located the model files for the most used evaluation methods for mean velocity, discharge, and measurement uncertainty. These models include all basic quantities for the respective uncertainty evaluation method and can be extended with quantities for additional uncertainty sources.

In the models is defined a dynamic index for the number of verticals by entering only the index designation. The dynamic index allows using the same file for measurements with different number of verticals – after entering or importing the measurements the software automatically updates the index n.



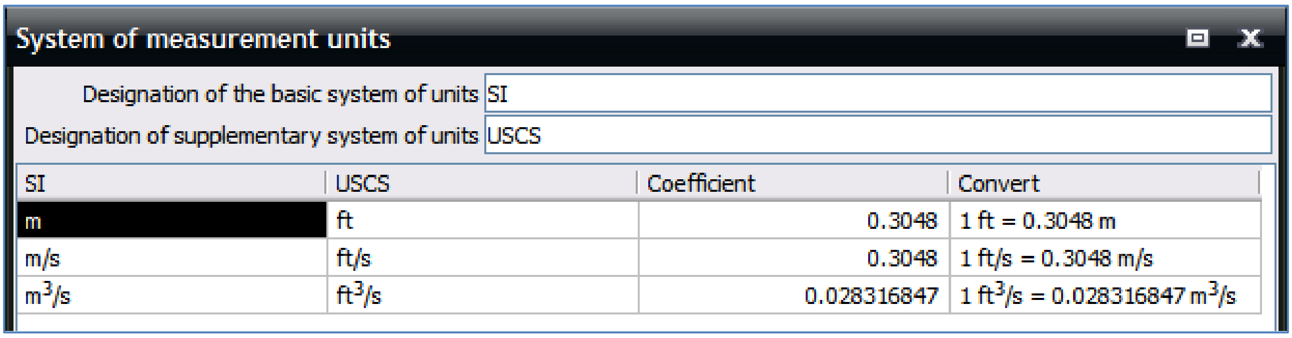
All quantities with additional settings for automated uncertainty evaluation are of type *B – Standard uncertainty* in absolute units. An exception makes only the quantity Q_m for the discharge uncertainty due to the limited number of verticals, that is of the type *B – Standard uncertainty* in absolute unit or in %, calculated based on the computed discharge *Q*.





Quantity Q_s for the discharge uncertainty, due to variable responsiveness of the current-meter, width measurement instrument and depth sounding instrument, is defined as normal quantity type *B – Standard uncertainty* in %, calculated based on the discharge *Q*.

The software allows the defining of supplementary system of units for all units, used in the model equations. The supplementary system is saved in the model file, and the user can with simple click switch between the systems.



The basic system of units of the developed models is SI, additionally are developed two models for ISO and IVE - Mid-section - Reduced point methods with USCS (English) basic system of units.

15.2. Basic Models for Flow Measurements

Basic models for ISO method and FLAURE method

- Mid-section method

index n
$Q = SUM((ABS(B_{[n+1]} - B_{[n-1]})/2) * D_{[n]} * V_{[n]}) + Q_m + Q_s$
$V_{[n]} = V_{p[n]} + V_{c[n]} + V_{e[n]}$

- Mean-section method

index n
$Q = SUM(ABS(B_{[n+1]} - B_{[n]}) * ((D_{[n+1]} + D_{[n]})/2) * ((V_{[n+1]} + V_{[n]})/2)) + Q_m + Q_s$
$V_{[n]} = V_{p[n]} + V_{c[n]} + V_{e[n]}$

Basic models for Q+ method

- Mid-section method

index n
$Q = SUM((ABS(B_{[n+1]} - B_{[n-1]})/2) * (D_{[n]} + D_{m[n]}) * V_{[n]}) + Q_s$
$V_{[n]} = V_{p[n]} + V_{c[n]} + V_{e[n]} + V_{m[n]}$

- Mean-section method

index n
$Q = SUM(ABS(B_{[n+1]} - B_{[n]}) * ((D_{[n+1]} + D_{m[n+1]} + D_{[n]} + D_{m[n]})/2) * ((V_{[n+1]} + V_{[n]})/2)) + Q_s$
$V_{[n]} = V_{p[n]} + V_{c[n]} + V_{e[n]} + V_{m[n]}$

Basic model for IVE method

- Mid-section method

index n
$Q = SUM((ABS(B_{[n+1]} - B_{[n-1]})/2) * D_{[n]} * V_{[n]}) + Q_s$

- Mid-section method, including uncertainty of mean velocity due to limited number of points in the vertical

index n
$Q = SUM((ABS(B_{[n+1]} - B_{[n-1]})/2) * D_{[n]} * (V_{[n]} + V_{p[n]}) + Q_s$

15.3. Additional Settings

This window is divided into the following areas:

- Selecting the methods for estimating the uncertainty sources, discharge computation and determination of the mean velocity in the verticals. When *Reduced point method* is selected, the software displays additionally a table of the computational rules according to ISO 1088:2007, with possibility to select the appropriate formula for 1, 3 and 6 points.

Selecting methods

Method for estimating the uncertainty sources	ISO 748, ISO 1088 method
Method for discharge computation	Mid-section method
Method for determination of the mean velocity	Reduced point method

Calculation of the mean velocity

Number of points	Formula for calculating the average velocity
1	$v_{AVG} = v_{0.6}$
2	$v_{AVG} = 0.5(v_{0.2} + v_{0.8})$
3	$v_{AVG} = 0.25v_{0.2} + 0.5v_{0.6} + 0.25v_{0.8}$
4	$v_{AVG} = (v_{0.2} + v_{0.4} + v_{0.7} + v_{0.9})/4$
5	$v_{AVG} = 0.1v_{SURF.} + 0.3v_{0.2} + 0.3v_{0.6} + 0.2v_{0.8} + 0.1v_{BED}$
6	$v_{AVG} = 0.1v_{SURF.} + 0.2v_{0.2} + 0.2v_{0.4} + 0.2v_{0.6} + 0.2v_{0.8} + 0.1v_{BED}$

- Input quantities for measured distance, depth and velocity, and limit for the relative discharge in the segments

Input quantities for the measured values

Measured distance from initial point	$B_{[n]}$
Measured depth in a vertical	$D_{[n]}$
Mean velocity in a vertical	$V_{P[n]}$
Limit for relative discharge in the segments	10 %

- Settings for automatic uncertainty evaluation of the basic input quantities depending on the selected method – for each uncertainty source is selected the respective input quantity, and the uncertainty values are entered.

15.3.1. Settings for ISO Method

➢ ***Uncertainty in the measurement of distance from initial point***

Absolute or relative standard uncertainty is entered in the table for particular range of width. Inserting new rows or deleting existing rows is done with buttons in front of the table. In the table for the standard uncertainty, instead of a number can be entered a formula with the measured width B as a parameter.
In the list box, the model quantity of type *B – Standard uncertainty* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical, the relative uncertainty is converted to absolute uncertainty and is associated to the selected input quantity. For values above the maximum range is taken the uncertainty of the last row.

Settings for automatic evaluation of the uncertainty of input quantities

Uncertainty in the measurement of distance from initial point — Quantity: $B_{[n]}$

Measured width to incl. m	Stand. uncertainty (•) % ( ) m
100	0.15
150	0.2
200	0.25

➢ *Uncertainty in the measurement of depth in the verticals*

Absolute or relative standard uncertainty is entered in the table for particular range of depth. Inserting new rows or deleting existing rows is done with buttons in front of the table. In the table for the standard uncertainty, instead of a number can be entered a formula with the measured depth D as a parameter.
In the list box, the model quantity of type *B – Standard uncertainty* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical, the relative uncertainty is converted to absolute uncertainty and is associated to the selected input quantity. For values above the maximum range is taken the uncertainty of the last row.

Settings for automatic evaluation of the uncertainty of input quantities

Uncertainty in the measurement of distance from initial point — Quantity: $B_{[n]}$

Measured width to incl. m	Stand. uncertainty (● % ○ m)
100	0.15
150	0.2
200	0.25

Uncertainty in the measurement of depth in the verticals — Quantity: $D_{[n]}$

Measured depth to incl. m	Stand. uncertainty (● % ○ m)
0.3	1.5
1	0.5

➢ *Uncertainty of mean velocity due to limited number of points in the vertical*

Reduced point method - absolute or relative standard uncertainty is entered in the table for the number of points in the vertical.
In the list box, the model quantity of type *B – Standard uncertainty* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical, the relative uncertainty is converted to absolute uncertainty and is associated to the selected input quantity.

Uncertainty of mean velocity due to limited number of points in the vertical — Quantity: $V_{p[n]}$

Number of points	Stand. uncertainty (● % ○ m/s)
1	7.7
2	4
3	4.8
4	2.4
5	2.2
6	3

Velocity distribution method – the software offers options for automatic calculation, based on the measurement data by using the method in [3] or entering absolute or relative standard uncertainty for the number of points in the vertical.
In the list box, the model quantity of type *B – Standard uncertainty* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical, the relative uncertainty is converted to absolute uncertainty and is associated to the selected input quantity.

Uncertainty of mean velocity due to limited number of points in the vertical — Quantity: $V_{p[n]}$
● Calculation based on measurement data ○ Table

Uncertainty of mean velocity due to limited number of points in the vertical — Quantity: $V_{p[n]}$
○ Calculation based on measurement data ● Table

Number of points	Stand. uncertainty (● % ○ m/s)
2	7
3	5
4	3.5
5	3
6	2.5
7	2
8	1.5
9	1
10	0.5

Integration method and *Surface one-point method* - absolute or relative standard uncertainty is entered in the field.
In the list box, the model quantity of type *B – Standard uncertainty* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical, the relative uncertainty is converted to absolute uncertainty and is associated to the selected input quantity.

Uncertainty of mean velocity due to limited number of points in the vertical — Quantity: $V_{p[n]}$ — Uncertainty: 0.5 (● % ○ m/s)

Uncertainty of mean velocity due to limited number of points in the vertical — Quantity: $V_{p[n]}$ — Uncertainty: 15 (● % ○ m/s)

➢ *Uncertainty in point velocity measurement due to current-meter rating error*

Absolute or relative standard uncertainty is entered in the table for particular range of velocity. Inserting new rows or deleting existing rows is done with buttons in front of the table. In the table for the standard uncertainty, instead of a number can be entered a formula with the measured velocity V as a parameter.
In the list box, the model quantity of type *B – Standard uncertainty* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical, the relative uncertainty is converted to absolute uncertainty and is associated to the selected input quantity. For values above the maximum range is taken the uncertainty of the last row.

Uncertainty in point velocity measurement due to current-meter rating error — Quantity: $V_{c[n]}$

Measured velocity to incl. m/s	Stand. uncertainty ● % ○ m/s
0.03	10
0.1	2.5
0.12	1.25
0.25	1
0.5	0.5

➢ *Uncertainty in point velocity measurement due to limited exposure time*

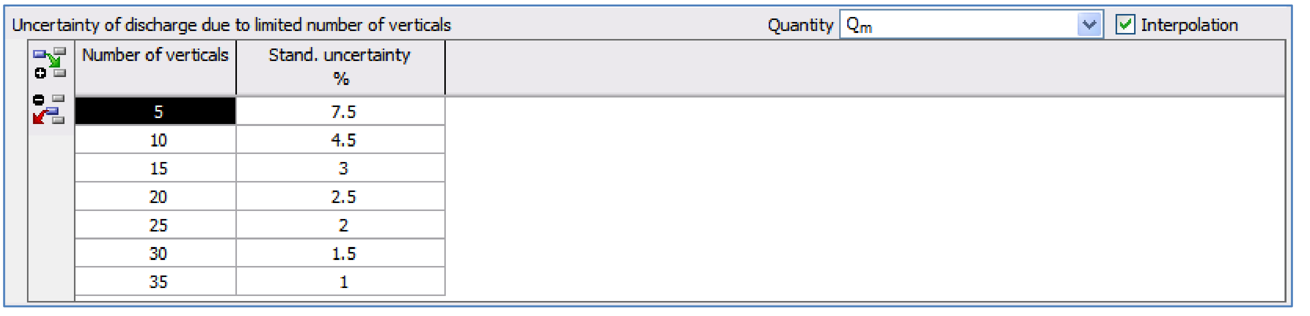
Absolute or relative standard uncertainty is entered in the table for particular range of velocity, exposure time and depth. Inserting new rows or deleting existing rows is done with buttons in front of the table.
In the list box, the model quantity of type *B – Standard uncertainty* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical, the relative uncertainty is converted to absolute uncertainty and associates to the selected input quantity. For values above the maximum range is taken the uncertainty of the last row. When activating the interpolation option, the uncertainty value is interpolated linearly for the specific velocity and exposure time.

Uncertainty in point velocity measurement due to limited exposure time — Quantity: $V_{e[n]}$ — ☑ Interpolation

Measured velocity to incl. m/s	Depth at point < 0.8D				Depth at point >= 0.8D			
	Exposure time s				Exposure time s			
	30	60	120	180	30	60	120	180
	Stand. uncertainty ● % ○ m/s				Stand. uncertainty ● % ○ m/s			
0.05	25	20	15	10	40	30	25	20
0.1	14	11	8	7	17	14	10	8
0.2	8	6	5	4	9	7	5	4
0.3	5	4	3	3	5	4	3	3
0.4	4	3	3	3	4	3	3	3
0.5	4	3	3	2	4	3	3	2

➢ ***<u>Uncertainty of discharge due to limited number of verticals</u>***

Relative standard uncertainty in % is entered in the table for particular range of number of verticals. Inserting new rows or deleting existing rows is done with buttons in front of the table. In the table for the standard uncertainty, instead of a number can be entered a formula with the number of verticals *m* as a parameter.
In the list box, the model quantity of type *B – Standard uncertainty in %* is selected, associated with this source of uncertainty.
During the calculation for the particular vertical the relative uncertainty is converted to absolute uncertainty and is associated with the selected input quantity. For values above the maximum range is taken the uncertainty of the last row. When activating the interpolation option, the uncertainty value is interpolated linearly for the specific number of verticals.



Number of verticals	Stand. uncertainty %
5	7.5
10	4.5
15	3
20	2.5
25	2
30	1.5
35	1

15.3.2. Settings for FLAURE Method

Settings for following sources are identical with the ISO method:

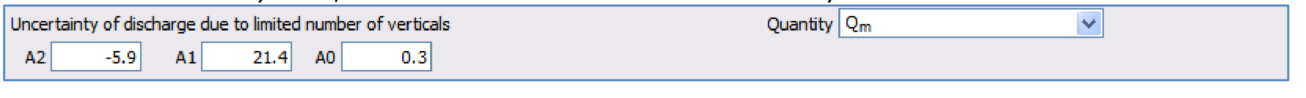
- Uncertainty in the measurement of distance from initial point
- Uncertainty in the measurement of depth in the verticals
- Uncertainty of mean velocity due to limited number of points in the vertical
- Uncertainty in point velocity measurement due to current-meter rating error
- Uncertainty in point velocity measurement due to limited exposure time

➢ ***<u>Uncertainty of discharge due to limited number of verticals</u>***

The software calculates the sampling quality index (SQI) and then the relative standard uncertainty in % by using following equation:

$U_m\% = A_2*SQI^2 + A_1*SQI + A_0$

In the settings are entered the equation coefficients, and in the list-box is selected the model quantity of type *B – Standard uncertainty in %*, associated with this source of uncertainty.

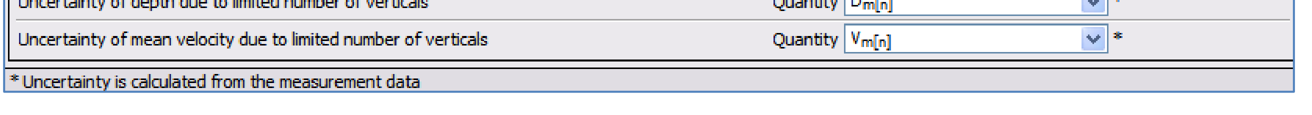


15.3.3. Settings for Q+ Method

Settings for the following sources are identical with the ISO method:

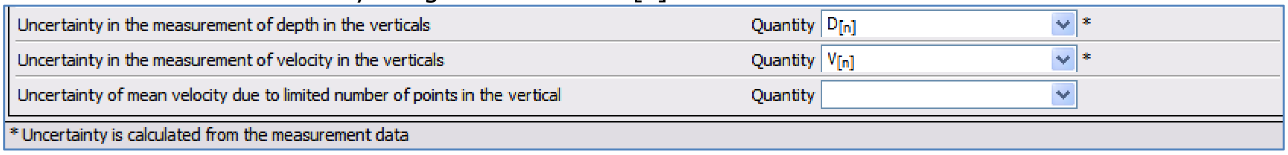
- Uncertainty in the measurement of distance from initial point
- Uncertainty in the measurement of depth in the verticals
- Uncertainty of mean velocity due to limited number of points in the vertical
- Uncertainty in point velocity measurement due to current-meter rating error
- Uncertainty in point velocity measurement due to limited exposure time

For uncertainties of depth and mean velocity due to limited number of verticals (transversal integration) are selected in the list boxes the model quantities of type *B – Standard uncertainty*, associated with these sources of uncertainty. Uncertainty is calculated from the measurement data by using the formulas in [3].



15.3.4. Settings for IVE Method

The settings for uncertainty of distance from initial point and uncertainty of mean velocity due to limited number of points are identical with the ISO method.
For uncertainties in the measurement of depth and velocity in the verticals are selected in the list boxes the model quantities of type *B – Standard uncertainty*, associated with these sources of uncertainty. Uncertainty is calculated from the measurement data by using the formulas in [3].



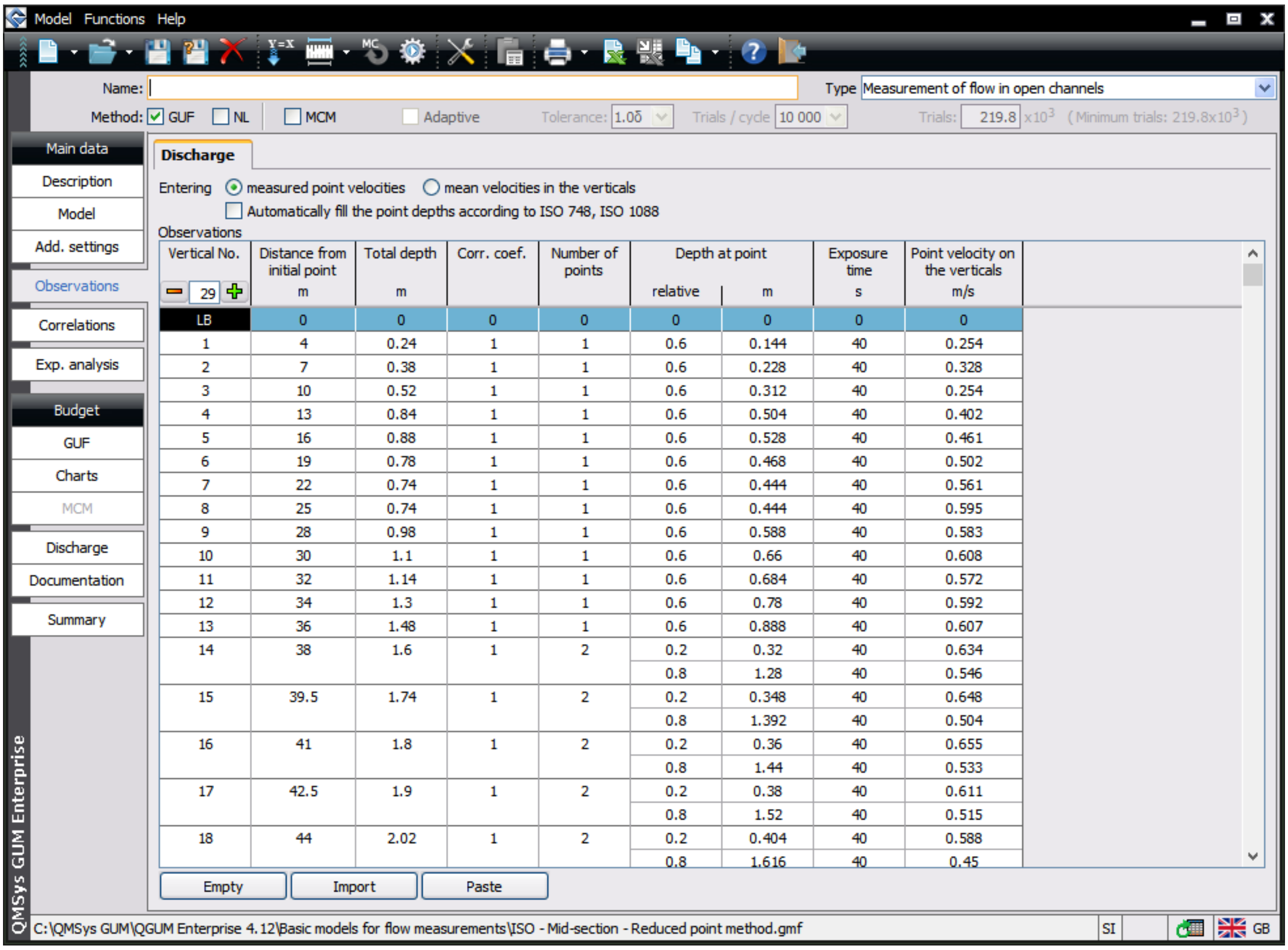
15.4. Discharge Measurements

In the window *Observations / Discharge* are entered or imported the measurements of width, depth and point velocity in each vertical. Depending on the selected method for determination of the mean velocity, the software opens the corresponding table for entering the measurements:

➢ ***Reduced point method***

The software offers additional options for entering point or mean velocity, and automatic filling the point depth according to ISO 748, ISO 1088.

The number of verticals is entered in the header cell of the first column. After entering the number of points in the vertical, the software generates the corresponding number of rows to input point depth, exposure time and point velocity.



Vertical No. (29)	Distance from initial point m	Total depth m	Corr. coef.	Number of points	Depth at point relative	Depth at point m	Exposure time s	Point velocity on the verticals m/s
LB	0	0	0	0	0	0	0	0
1	4	0.24	1	1	0.6	0.144	40	0.254
2	7	0.38	1	1	0.6	0.228	40	0.328
3	10	0.52	1	1	0.6	0.312	40	0.254
4	13	0.84	1	1	0.6	0.504	40	0.402
5	16	0.88	1	1	0.6	0.528	40	0.461
6	19	0.78	1	1	0.6	0.468	40	0.502
7	22	0.74	1	1	0.6	0.444	40	0.561
8	25	0.74	1	1	0.6	0.444	40	0.595
9	28	0.98	1	1	0.6	0.588	40	0.583
10	30	1.1	1	1	0.6	0.66	40	0.608
11	32	1.14	1	1	0.6	0.684	40	0.572
12	34	1.3	1	1	0.6	0.78	40	0.592
13	36	1.48	1	1	0.6	0.888	40	0.607
14	38	1.6	1	2	0.2	0.32	40	0.634
					0.8	1.28	40	0.546
15	39.5	1.74	1	2	0.2	0.348	40	0.648
					0.8	1.392	40	0.504
16	41	1.8	1	2	0.2	0.36	40	0.655
					0.8	1.44	40	0.533
17	42.5	1.9	1	2	0.2	0.38	40	0.611
					0.8	1.52	40	0.515
18	44	2.02	1	2	0.2	0.404	40	0.588
					0.8	1.616	40	0.45

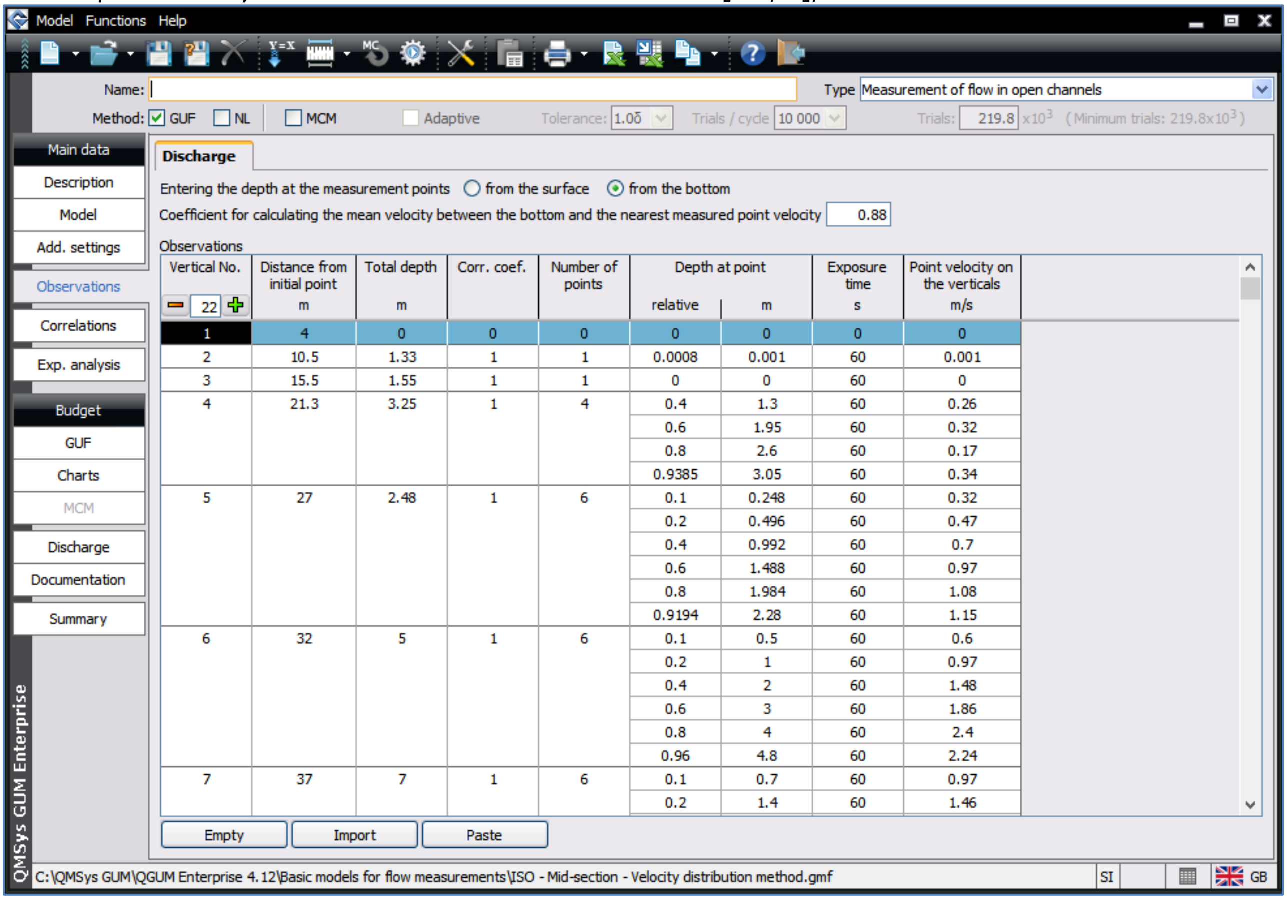
- With the Empty button are deleted the current discharge measurements and calculated uncertainty estimations of the basic quantities.

- The Import button opens a dialog window for selecting the data file. Currently the software supports Aquacalc (CSV), Flow Tracker 1 (DIS), Flow Tracker 2 (CSV), OTT-ADC (TXT) and OTT-MFPro (TSV) data formats. In the software folder *Examples* subfolder *Examples for import of flow measurements* are presented practical examples of importing data files into models for the four methods of uncertainty estimation.
- With the Paste button are imported discharge measurements from the clipboard, the file *Examples of flow measurements.xlsx* contains practical examples of the data formats that can be imported from the clipboard.

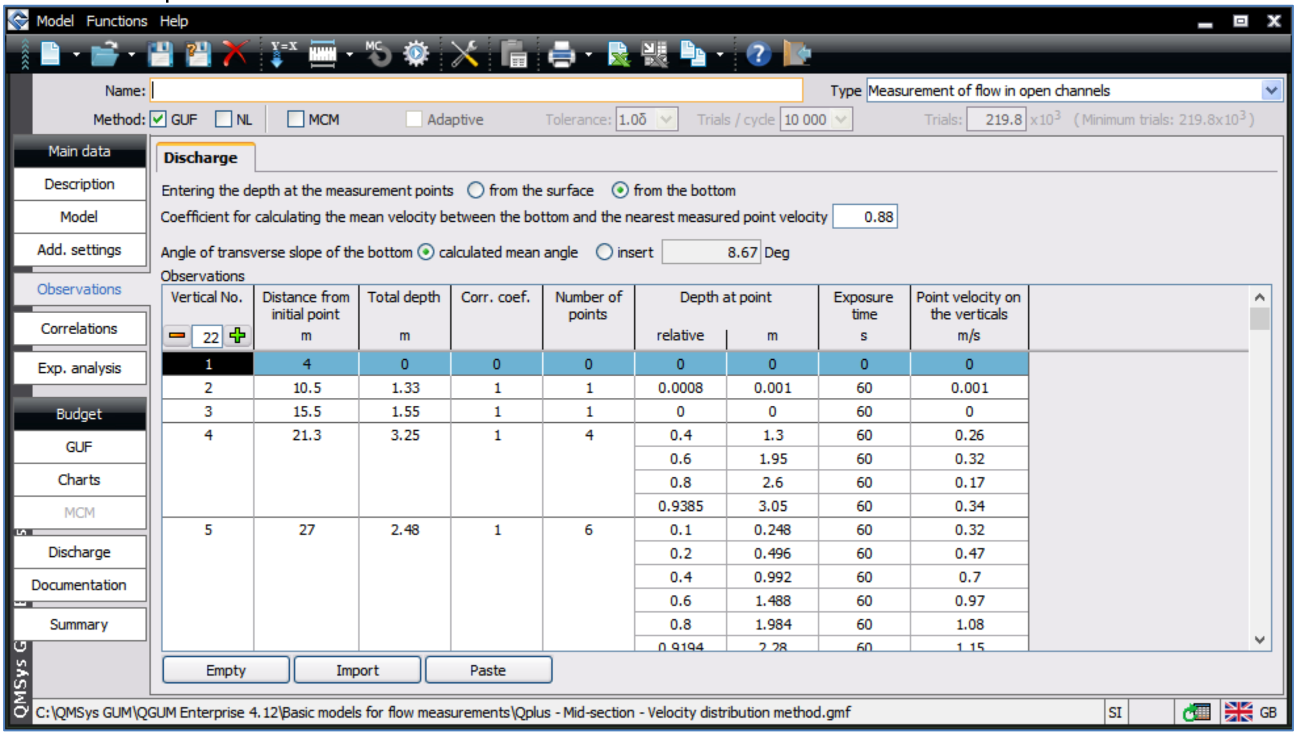
➢ ***Velocity distribution method***

The software offers the following additional options:

- entering the depth at the measurement points from the surface or from the bottom
- entering the coefficient for calculating the mean velocity between the bottom and the nearest measured point velocity - the coefficient value is in the interval [0.5; 1], default value is 0.88

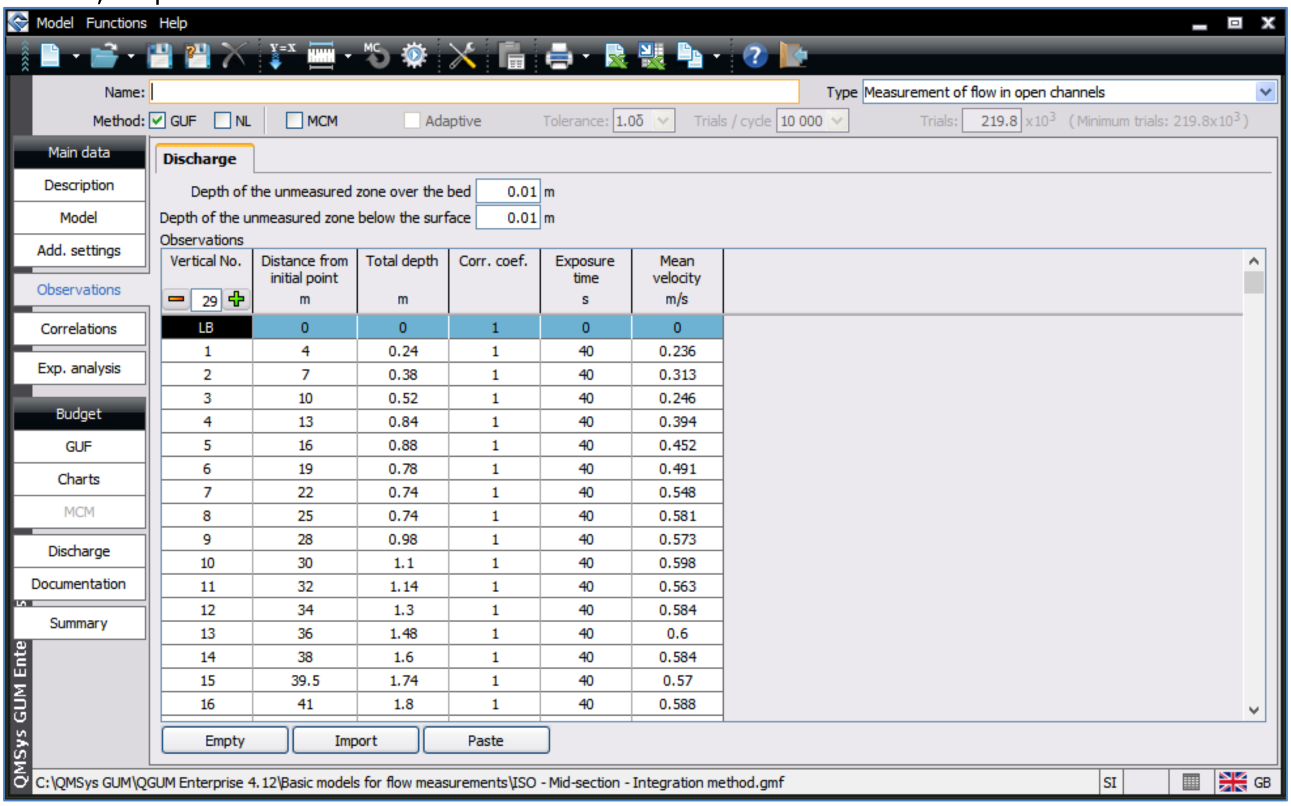


When Q+ method is selected, the software displays additional option for calculating or inserting the angle of transverse slope of the bottom:



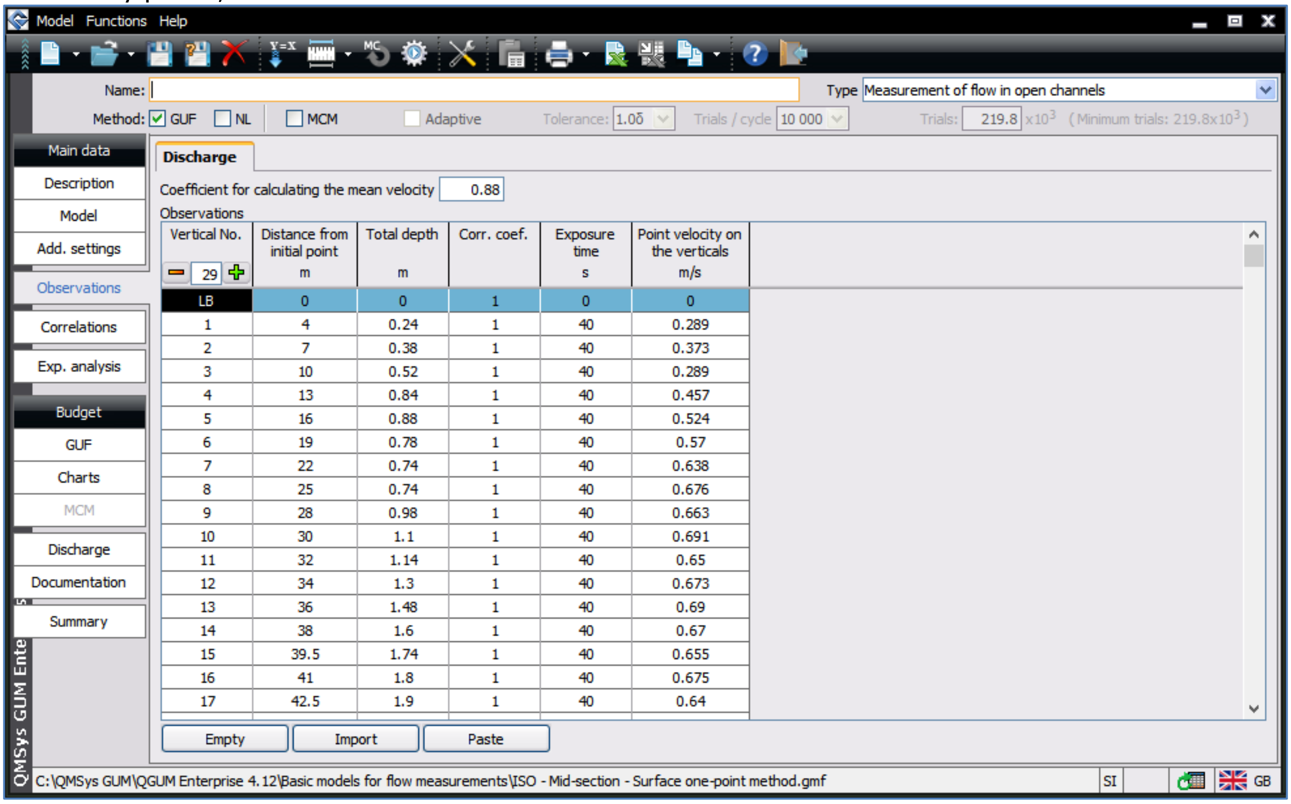
➢ ***Integration method***

The software displays two fields for entering the depth of the unmeasured zones over the bed and below the surface. The mean velocity in the vertical is recalculated taking into account the unmetered zones according to ISO 748, chapter 7.1.5.4.



➢ ***Surface one-point method***

The coefficient for calculating the mean velocity is entered between 0,84 and 0,90 depending upon the shape of the velocity profile, default value is 0.88



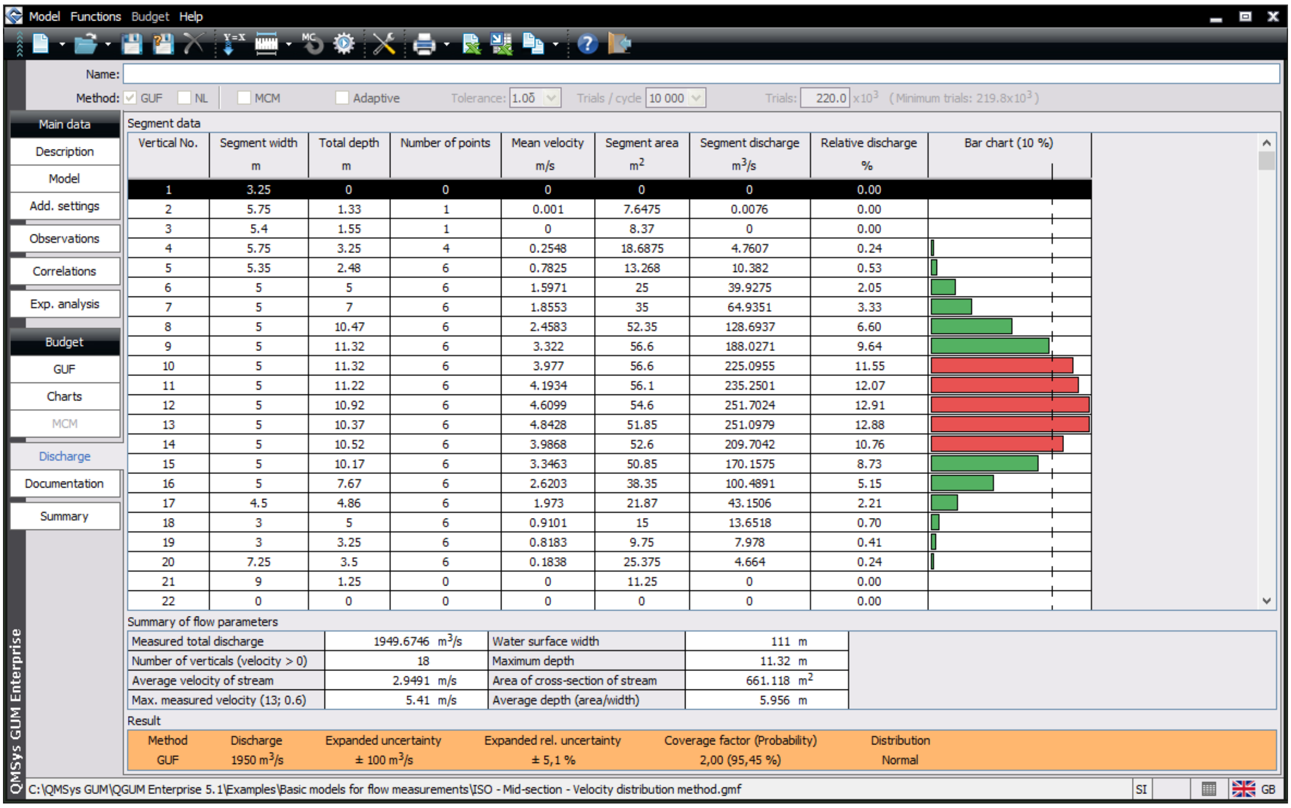
15.5. Discharge Results

The window *Discharge* presents a summary of the results:

Segment data - the table displays for each segment the vertical number, segment width, total depth, number of points, mean velocity, segment area, segment discharge and relative discharge in %. In the bar chart diagram the segments with relative discharge above the allowable limit are plotted in red.

Summary of flow parameters – software calculates additionally the measured total discharge, number of verticals with velocity > 0, average velocity of stream, maximum measured velocity, water surface width, maximum depth, area of cross-section of stream and average depth (area/width). For the maximum measured velocity, in brackets are displayed the vertical number and the point depth.

Result – includes discharge, expanded uncertainty, expanded relative uncertainty in %, coverage factor and type of distribution. According to GUM, uncertainties are rounded to the selected number of significant digits and the discharge value is rounded to be consistent with the expanded uncertainty. The results for each selected uncertainty calculation method (GUF, GUF-NL, MCM) are displayed in separate rows.



Vertical No.	Segment width m	Total depth m	Number of points	Mean velocity m/s	Segment area m²	Segment discharge m³/s	Relative discharge %
1	3.25	0	0	0	0	0	0.00
2	5.75	1.33	1	0.001	7.6475	0.0076	0.00
3	5.4	1.55	1	0	8.37	0	0.00
4	5.75	3.25	4	0.2548	18.6875	4.7607	0.24
5	5.35	2.48	6	0.7825	13.268	10.382	0.53
6	5	5	6	1.5971	25	39.9275	2.05
7	5	7	6	1.8553	35	64.9351	3.33
8	5	10.47	6	2.4583	52.35	128.6937	6.60
9	5	11.32	6	3.322	56.6	188.0271	9.64
10	5	11.32	6	3.977	56.6	225.0955	11.55
11	5	11.22	6	4.1934	56.1	235.2501	12.07
12	5	10.92	6	4.6099	54.6	251.7024	12.91
13	5	10.37	6	4.8428	51.85	251.0979	12.88
14	5	10.52	6	3.9868	52.6	209.7042	10.76
15	5	10.17	6	3.3463	50.85	170.1575	8.73
16	5	7.67	6	2.6203	38.35	100.4891	5.15
17	4.5	4.86	6	1.973	21.87	43.1506	2.21
18	3	5	6	0.9101	15	13.6518	0.70
19	3	3.25	6	0.8183	9.75	7.978	0.41
20	7.25	3.5	6	0.1838	25.375	4.664	0.24
21	9	1.25	0	0	11.25	0	0.00
22	0	0	0	0	0	0	0.00

Measured total discharge	1949.6746 m³/s	Water surface width	111 m
Number of verticals (velocity > 0)	18	Maximum depth	11.32 m
Average velocity of stream	2.9491 m/s	Area of cross-section of stream	661.118 m²
Max. measured velocity (13; 0.6)	5.41 m/s	Average depth (area/width)	5.956 m

Method	Discharge	Expanded uncertainty	Expanded rel. uncertainty	Coverage factor (Probability)	Distribution
GUF	1950 m³/s	± 100 m³/s	± 5,1 %	2,00 (95,45 %)	Normal

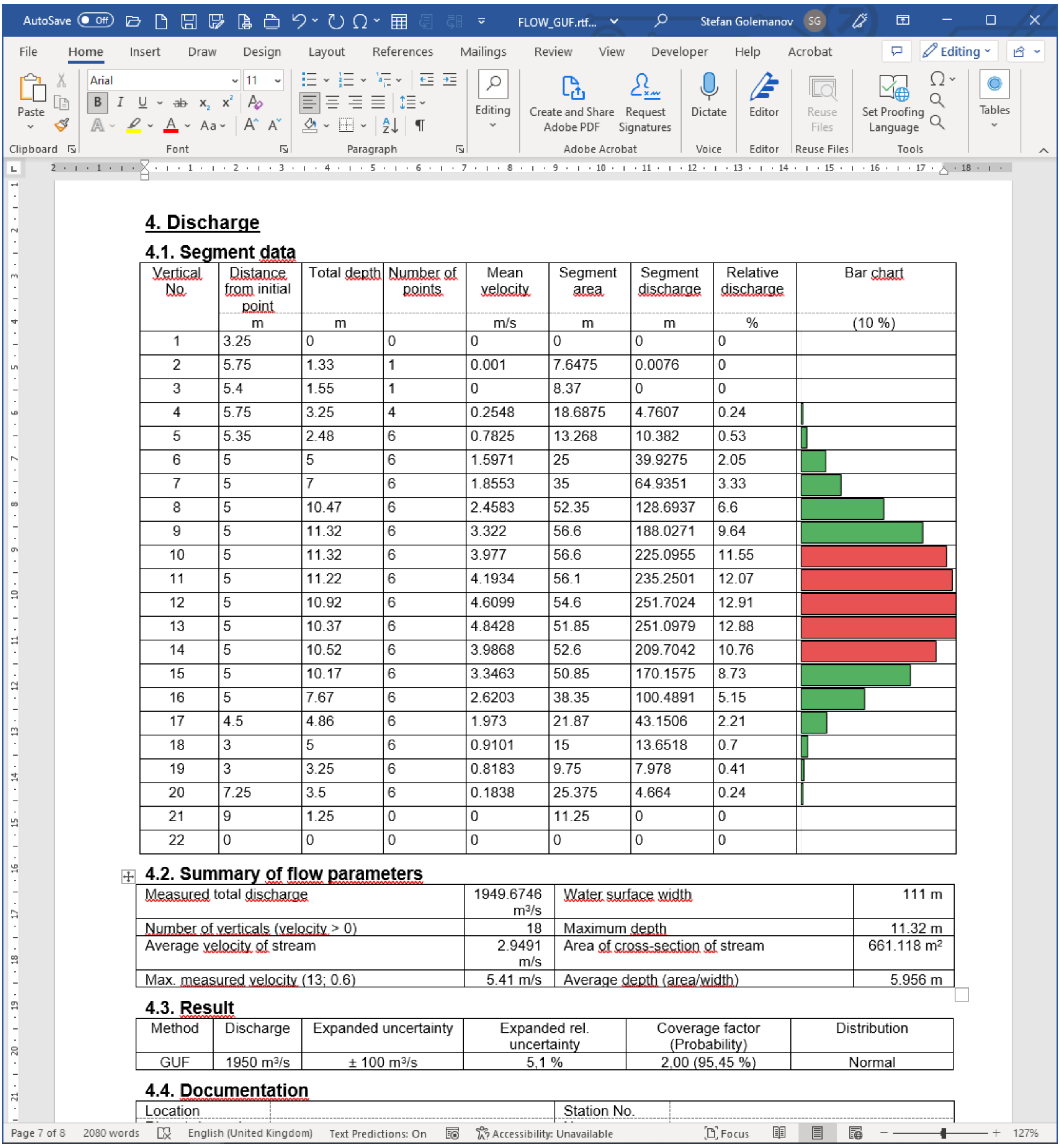
Each part of the discharge overview can be copied to the clipboard and imported into other applications.

15.6. Printing and Export

Printing of Measurement Uncertainty Report

Printout in the program is made by configurable templates in RTF format (*.RTF) with coded fields. The user can provide or adapt the report by using the coding of the fields from the standard report. The sequence of the individual fields or tables can be changed. The RTF Template can contain additional texts and pictures (Company Logo).

When generating a report, the software automatically selects the corresponding standard template, if there are no custom templates selected. If custom templates are set, the software selects the default custom template. Additional custom templates are selected for printing with the arrow on the Print button or over the Function/Print menu. Generated reports can be printed, saved in a file with a selectable name or sent by email.



4. Discharge

4.1. Segment data

Vertical No.	Distance from initial point	Total depth	Number of points	Mean velocity	Segment area	Segment discharge	Relative discharge	Bar chart
	m	m		m/s	m	m	%	(10 %)
1	3.25	0	0	0	0	0	0	
2	5.75	1.33	1	0.001	7.6475	0.0076	0	
3	5.4	1.55	1	0	8.37	0	0	
4	5.75	3.25	4	0.2548	18.6875	4.7607	0.24	
5	5.35	2.48	6	0.7825	13.268	10.382	0.53	
6	5	5	6	1.5971	25	39.9275	2.05	
7	5	7	6	1.8553	35	64.9351	3.33	
8	5	10.47	6	2.4583	52.35	128.6937	6.6	
9	5	11.32	6	3.322	56.6	188.0271	9.64	
10	5	11.32	6	3.977	56.6	225.0955	11.55	
11	5	11.22	6	4.1934	56.1	235.2501	12.07	
12	5	10.92	6	4.6099	54.6	251.7024	12.91	
13	5	10.37	6	4.8428	51.85	251.0979	12.88	
14	5	10.52	6	3.9868	52.6	209.7042	10.76	
15	5	10.17	6	3.3463	50.85	170.1575	8.73	
16	5	7.67	6	2.6203	38.35	100.4891	5.15	
17	4.5	4.86	6	1.973	21.87	43.1506	2.21	
18	3	5	6	0.9101	15	13.6518	0.7	
19	3	3.25	6	0.8183	9.75	7.978	0.41	
20	7.25	3.5	6	0.1838	25.375	4.664	0.24	
21	9	1.25	0	0	11.25	0	0	
22	0	0	0	0	0	0	0	

4.2. Summary of flow parameters

Measured total discharge	1949.6746 m³/s	Water surface width	111 m
Number of verticals (velocity > 0)	18	Maximum depth	11.32 m
Average velocity of stream	2.9491 m/s	Area of cross-section of stream	661.118 m²
Max. measured velocity (13; 0.6)	5.41 m/s	Average depth (area/width)	5.956 m

4.3. Result

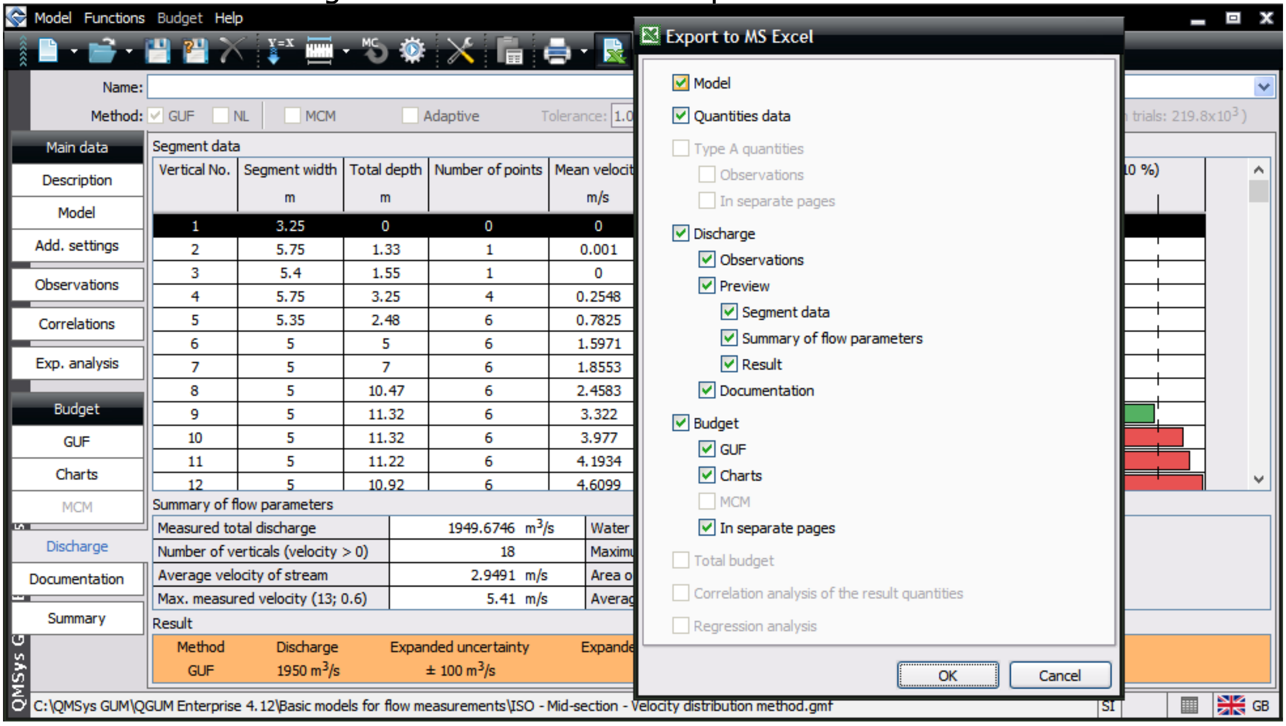
Method	Discharge	Expanded uncertainty	Expanded rel. uncertainty	Coverage factor (Probability)	Distribution
GUF	1950 m³/s	± 100 m³/s	5,1 %	2,00 (95,45 %)	Normal

4.4. Documentation

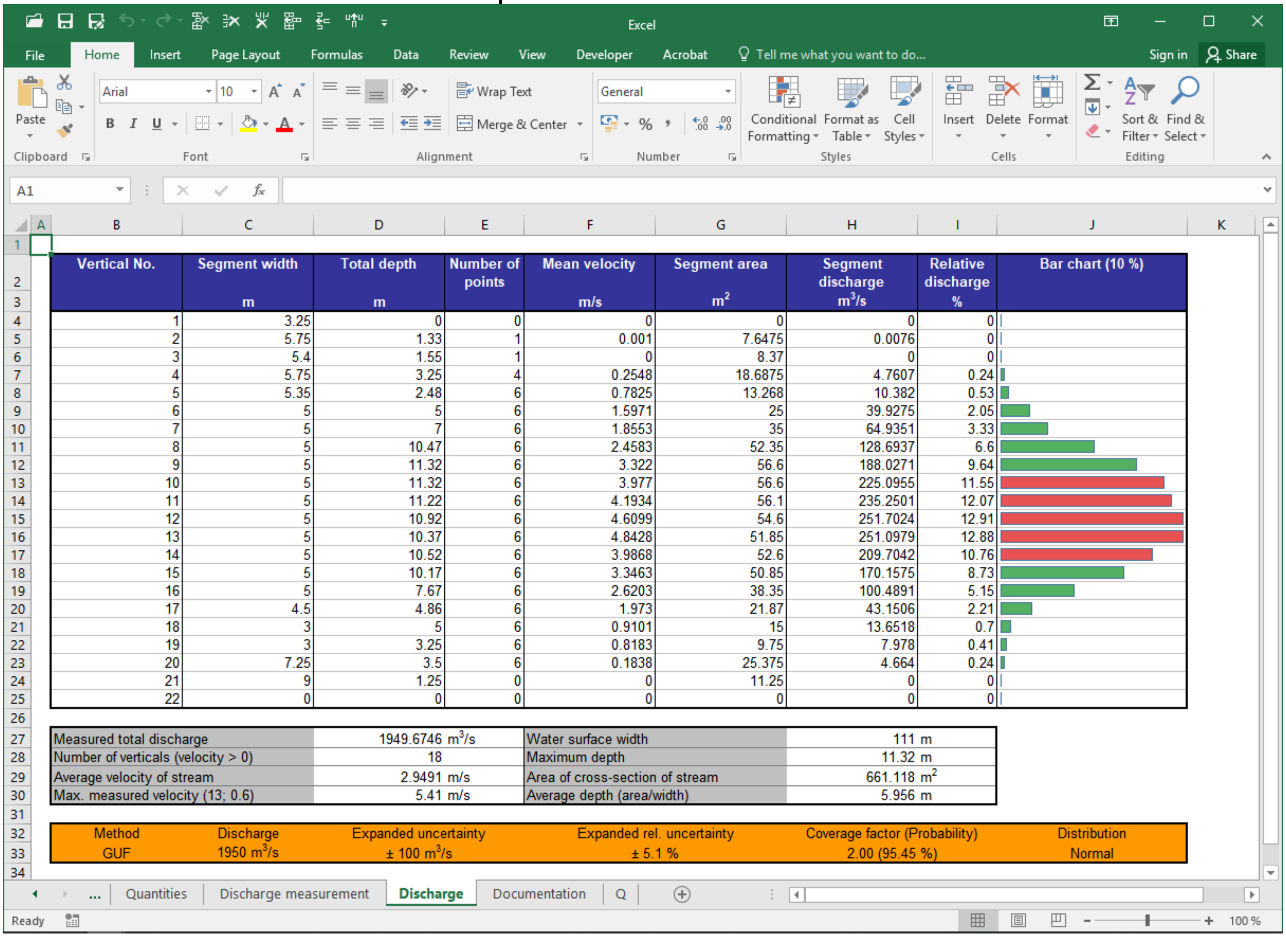
Location		Station No.	

Export to Microsoft Excel

The Export to MS Excel is a useful feature to transfer data from an uncertainty analysis over the OLE interface to Excel files. With the command / button Export to MS Excel, a dialog window with the settings for the data transfer is open.



The data transfer starts with the selection of the OK button. The program MS Excel is loaded in the background and according to the active options in the dialog window, the software creates a new workbook with multiple worksheets and fills them with data.



15.7. References for Determination of Uncertainties in Flow Measurement

[1] ISO 748:2007 - Hydrometry - Measurement of liquid flow in open channels using current-meters or floats

[2] ISO 1088:2007 - Hydrometry - Velocity-area methods using current-meters - Collection and processing of data for determination of uncertainties in flow measurement

[3] J. Le Coz*, B. Camenen, X. Peyrard, G. Dramais, 2012 - Uncertainty in open-channel discharges measured with the velocity–area method

[4] Aurélien Despax, Christian Perret, Rémy Garçon, Alexandre Hauet, Arnaud Belleville, Jérôme Le Coz, Anne-Catherine Favre, 2017 - Prise en compte de la qualité de l'échantillonnage dans l'estimation de l'incertitude des jaugeages par exploration du champ des vitesses (méthode Flaure)

[5] Aurélien Despax, Christian Perret, Rémy Garçon, Alexandre Hauet, Arnaud Belleville, Jérôme Le Coz, Anne-Catherine Favre, 2016 Considering sampling strategy and cross-section complexity for estimating the uncertainty of discharge measurements using the velocity-area method

[6] Timothy A. Cohn, Julie E. Kiang, Robert R. Mason Jr., 2013 - Estimating Discharge Measurement Uncertainty Using the Interpolated Variance Estimator

16. Uncertainty Calculation in MS Excel by the QMSys GUM Excel Add-In

The full integration of calculating the measurement uncertainties in MS Excel is implemented by using the additional software *QMSys GUMX* (Excel Add-In). This program fully supports the functions for importing data to the input quantities from a MS Excel file, the calculation of measurement uncertainty and the export of the results in the same Excel file. The model files are prepared with the software editions **QMSys GUM Enterprise / Professional** and saved in the special format for the Excel add-in *QMSys GUMX* with extension ".gxl".

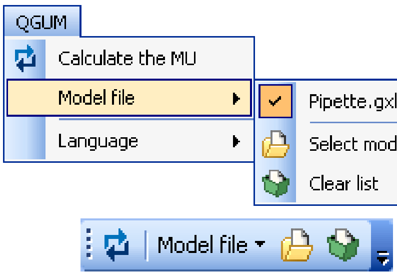
The Excel add-in *QMSys GUMX* is included in the installation files of the editions *QMSys GUM Enterprise / Professional*. The installation file "qgumx_inst.exe" of the software *QMSys GUMX* located in the subfolder ".. \ QGUMX_Inst". During the installation, the software *QMSys GUMX* will be automatically registered in MS Excel software, therefore the programs MS Excel and *QMSys GUM Enterprise / Professional* must be closed. Under operating systems MS Windows Vista / 7 / 8 are also needed administrator rights.

The program *QMSys GUMX* uses two methods to calculate the measurement uncertainty:

- **GUF Method** for linear models - when only the GUF is selected in the model file
- **Monte Carlo Method** with Sobol sequences - when GUF-NL or MCM are selected in the model.

Program Menu

Under the menu "QGUM" and in the menu bar are provided following functions:

<table>
<tr><th>MS Excel XP, 2003</th><th>MS Excel 2007, 2010, 2013 (32 and 64 Bit)</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

- *Calculate the MU* - this function will start the calculation of the measurement uncertainty. The imported data of the input quantities are read from the current Excel file and after the calculation of the measurement uncertainty, the results are written in the same file. This function is only accessible if a model file has been selected for the current Excel file.
- *Model file* - with this function, a model file for the calculation of measurement uncertainty is selected from a list with model files. Up to three different model files can be assigned to the opened Excel file with the function "*Select model*". The program automatically saves the relative file path if the model file is in the same folder or in a subfolder to the Excel file. Otherwise, the absolute file path is stored. Using the function „*Clear list*" will remove all model files from the list.
- *Language* - selecting the program language.
- *Register* - opens a window with the license key of the software and a field for entering the unlock key and the activation of the software.

Calculation of measurement uncertainty

The following steps represent the basic procedure for calculating the uncertainty from an Excel file:

1. Create a model file for calculating the measurement uncertainty with the software editions *QMSys GUM Enterprise* or *Professional*.
2. Create an Excel file containing the data to be imported to the input quantities and codes for the export of the results of the uncertainty calculation. It is recommended to save the Excel file and the model file in the same folder.
3. Select the appropriate cells or cell ranges of the input quantities in the model file, save the adjusted model file and close it.
4. Associate the model file with the Excel file over the menu "QGUM -> Model file -> Select model" and save the Excel file.
5. You can now enter in this Excel file different values for the input quantities and start the calculation of the measurement uncertainty over the menu "Calculate the MU" - the calculated results are automatically updated in the cells that contain the corresponding codes.
6. You can copy the Excel file and run the calculation of the measurement uncertainty from the new file - the software reads the imported data from the new file and writes the results in the same file.

The model file can be used in several Excel files with the same cell structure of the imported data for the input quantities. The areas with the codes for the export of the results can be structured differently in the individual files. When calculating the measurement uncertainty, the imported data is read from the current file and the results are exported in the same file.

Examples for one measurement set:

Excel File	Model File
EA4-S2.Calibration of a weight_AddIn.xlsx	EA4-S2.Calibration of a weight.gxl
EA4-S3.Calibration of a standard resistor_AddIn.xlsx	EA4-S3.Calibration of a standard resistor.gxl

Example of model with indexed quantities:

Excel File	Model Files
Discharge measurement by current meter_AddIn.xlsx	Mean-section-method.gxl
	Mid-section-method.gxl

This example shows the possibility to use two methods for calculating the measurement results with the associated uncertainty for the same measurement data set.

Calculation of measurement uncertainty for several measurement series

The software *QMSys GUMX* also offers calculation of measurement uncertainty in MS Excel for unlimited number of measurement series (result quantities with identical measurement model) using a model file for only one set of measurements. This functionality is particularly useful when calibrating in several points of the measuring range, in addition, it simplifies the modelling of the measurement process.

In this case in the model file are used the defined in MS Excel names of cell areas for the input quantities, which parameter values are read from the Excel file.

The observations values of all measurement series of one input quantity of type A are defined with one cell area. With the symbols "H" or "V" is specified, how the individual series are ordered in the cell area:

- Area name "Mess_H" - individual series of measurements are defined in columns:

Mess_H fx 10.1

	A	B	C	D
1	No.	Set 1	Set 2	Set 3
2	1	10.10	25.00	50.20
3	2	10.20	25.10	50.30
4	3	10.30	25.20	50.00
5	4	10.20	25.00	50.20
6	5	10.20	25.00	50.30
7				

- Area name "Mess_V" - individual series of measurements are defined in rows:

Mess_V fx 10.1

	A	B	C	D	E	F
1	No.	1	2	3	4	5
2	Set 1	10.10	10.20	10.30	10.20	10.20
3	Set 2	25.00	25.10	25.20	25.00	25.00
4	Set 3	50.20	50.30	50.00	50.20	50.30
5						

For the remaining parameters of the input quantities are defined separate cells areas - one area for each parameter. The cells in an area may be assigned by column or by row. Empty cells or mixed assignments are not allowed.

The codes for the statistical analysis of the type A input quantities and the result quantities are entered in the appropriate cells or comment fields only once for the first series of measurements. By inserting the characters "H" or "V" in front of the "#" symbol is defined how will be imported the results for the next series of measurements:

- Coding with "H" (for example, $G02I01H# or $G02I01UH#) - the results of a series of measurements are output by columns:

	A	B	C	D
1		Set 1	Set 2	Set 3
2	Mean value	10.200	25.060	50.200
3	Uncertainty	0.032	0.040	0.055
4				

- Coding with "V" (for example, $G02I01V# or $G02I01UV#) - the results of a series of measurements are output by rows:

	A	B	C
1		Mean value	Uncertainty
2	Set 1	10.200	0.032
3	Set 2	25.060	0.040
4	Set 3	50.200	0.055
5			

Example:

Excel File	Model File
EA4-S2.Calibration of a weight set_AddIn.xlsx	EA4-S2.Calibration of a weight set.gxl

Import of several series of measured values for type A indexed quantities

The observations values of all measurement series of one indexed quantity of type A are defined with one cell area. With the symbols "H" or "V" is specified, how the individual series are ordered in the cell area. Following options for definition of the cell area with the measurement sets in MS Excel are offered:

- **Measured values and series are arranged by columns – "*Name*_C_V"**

Example
- Index definition: index n=(1:4)
- Number of observations: 5
- Number of series: 3

Definition of cell area of the main quantity:
- Cell area: "Meas_C_V" = $C$3:$F$17

Automatic cell areas of the indexed quantities:
- Set_1 - Q_1 = C3:C7
- Set_1 - Q_2 = D3:D7
- Set_1 - Q_3 = E3:E7
- Set_1 - Q_4 = F3:F7
- Set_2 - Q_1 = C8:C12
- Set_2 - Q_2 = D8:D12

…
- Set_3 - Q_3 = E13:E17
- Set_3 - Q_4 = F13:F17

Meas_C_V   fx 10.2

	A	B	C	D	E	F
1			Quantity index			
2		No.	Q1	Q2	Q3	Q4
3	Set 1	1	10.2	25.2	35.4	50.4
4		2	10.3	25.3	35.6	50.6
5		3	10.3	25.3	35.6	50.6
6		4	10.1	25.1	35.2	50.2
7		5	10.2	25.2	35.4	50.4
8	Set 2	1	10.3	25.3	35.6	50.6
9		2	10.4	25.4	35.8	50.8
10		3	10.3	25.3	35.6	50.6
11		4	10.2	25.2	35.4	50.4
12		5	10.2	25.2	35.4	50.4
13	Set 3	1	10.4	25.4	35.8	50.8
14		2	10.1	25.1	35.2	50.2
15		3	10.3	25.3	35.6	50.6
16		4	10.3	25.3	35.6	50.6
17		5	10.2	25.2	35.4	50.4

- **Measured values and series are arranged by rows – "*Name*_R_H"**

Example
- Index definition: index n=(1:4)
- Number of observations: 5
- Number of series: 3

Definition of cell area of the main quantity:
- Cell area: "Meas_R_H" = $J$4:$X$7

Automatic cell areas of the indexed quantities:
- Set_1 - Q_1 = J4:N4
- Set_1 - Q_2 = J5:N5
- Set_1 - Q_3 = J6:N6
- Set_1 - Q_4 = J7:N7
- Set_2 - Q_1 = O4:S4
- Set_2 - Q_2 = O5:S5

…
- Set_3 - Q_3 = T4:X4
- Set_3 - Q_4 = T5:X5

Meas_R_H   fx 10.2

	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1																	
2			Set 1					Set 2					Set 3				
3		No.	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
4	Quantity index	Q1	10.2	10.3	10.3	10.1	10.2	10.3	10.4	10.3	10.2	10.2	10.4	10.1	10.3	10.3	10.2
5		Q2	25.2	25.3	25.3	25.1	25.2	25.3	25.4	25.3	25.2	25.2	25.4	25.1	25.3	25.3	25.2
6		Q3	35.4	35.6	35.6	35.2	35.4	35.6	35.8	35.6	35.4	35.4	35.8	35.2	35.6	35.6	35.4
7		Q4	50.4	50.6	50.6	50.2	50.4	50.6	50.8	50.6	50.4	50.4	50.8	50.2	50.6	50.6	50.4

If in the defined area are empty cells, the software will automatically reduce the number of observations for the corresponding sub-quantity. At least 1 measured value must be entered in the Excel file for each sub-quantity.

Appendix A: Validation of the QMSys GUM Software

ISO/IEC Guide 98-3:2008 (GUM:1995) Guide to the expression of uncertainty in measurement

Example	GUM				QMSys GUM, GUF Method				Distribution
	Estimate	u	k	U	Estimate	u	k	U	
H.1	50,000838 mm	32 nm	2,92	±92 nm	50,000838 mm	32 nm	2,92	±92 nm	t-distr., f=16
H.2 (R)	127,732 Ω	0,071 Ω	2,00	±0,142 Ω	127,732 Ω	0,0711 Ω	2,00	±0,142 Ω	Normal
H.2 (X)	219,847 Ω	0,295 Ω	2,00	±0,590 Ω	219,847 Ω	0,2956 Ω	2,00	±0,591 Ω	Normal
H.2 (Z)	254,260 Ω	0,236 Ω	2,00	±0,472 Ω	254,260 Ω	0,2363 Ω	2,00	±0,473 Ω	Normal
H.2 Corr. coeff.	r(V,I) = -0,36 ; r(V,φ) = 0,86 ; r(I,φ) = -0,65 r(R,X) = -0,588 ; r(R,Z) = -0,485 ; r(X,Z) = 0,993				r(V,I) = -0,355 ; r(V,φ) = 0,858 ; r(I,φ) = -0,645 r(R,X) = -0,588 ; r(R,Z) = -0,485 ; r(X,Z) = 0,993				Input quant. Result quant.
H.3	-0,1494 °C	0,0041 °C	2,00	±0,0082 °C	-0,1494 °C	0,00414 °C	2,00	±0,0083 °C	Normal
H.4	0,430 Bq/g	0,0083 Bq/g	2,00	±0,017 Bq/g	0,430 Bq/g	0,00833 Bq/g	2,00	±0,017 Bq/g	Normal

ISO Guide 98-3/S.1, JCGM 101 Suppl. 1 to the "GUM" - Propagation of distributions using a Monte Carlo method

Example	JCGM 101:2008				QMSys GUM				Comment / GUF-Distribution
	Method	Estimate	u	Cover. interval	Method	Estimate	u	Cover. interval	
9.2.2	MCM	0,00	2.00	[-3,92; 3,92]	MCM	0,00	2.00	[-3,92; 3,92]	
	GUF	0,00	2.00	[-3,92; 3,92]	MCM	0,00	2.00	[-3,92; 3,92]	Normal distribution
9.2.3	MCM	0,00	2.00	[-3,88; 3,88]	MCM	0,00	2.00	[-3,88; 3,88]	
	GUF	0,00	2.00	[-3,92; 3,92]	MCM	0,00	2.00	[-3,92; 3,92]	Normal distribution
9.2.4	MCM	0,00	10.1	[-17,0; 17,0]	MCM	0,00	10.1	[-17,0; 17,0]	
	GUF	0,00	10.1	[-19,9; 19,9]	GUF	0,00	10.1	[-19,9; 19,9]	Normal distribution
9.3	MCM	1,2341	0,0754	[1,0834; 1,3825]	MCM	1,2340	0,0754	[1,0845; 1,3835]	
	GUF2	1,2340	0,0750	[1,0870; 1,3810]	GUF-NL	1,2340	0,0754	[1,0862; 1,3818]	Normal distribution
9.4.2.2	MCM	50	50	[0; 150]	MCM	50	50	[0; 150]	x 10^{-6}
	Analytical	50	50	-	GUF-NL	50	50	[-48; 148]	x 10^{-6}, not validated
9.4.2.3	MCM	150	112	[0; 367]	MCM	150	112	[0; 366]	x 10^{-6}
	Analytical	150	112	-	GUF-NL	150	112	[-69; 369]	x 10^{-6}, not validated
9.4.2.4	MCM	2551	502	[1590; 3543]	MCM	2551	503	[1591; 3547]	x 10^{-6}
	Analytical	2550	502	-	GUF-NL	2550	502	[1565; 3535]	x 10^{-6}, validated
9.4.3.2.1	MCM	50	67	[0; 185]	MCM	50	67	[0; 185]	x 10^{-6}
	Analytical	50	67	-	GUF-NL	50	67	[-81; 181]	x 10^{-6}, not validated
9.4.3.2.2	MCM	150	121	[13; 398]	MCM	150	120	[13; 398]	x 10^{-6}
	Analytical	150	121	-	GUF-NL	150	120	[-86; 386]	x 10^{-6}, not validated
9.4.3.2.3	MCM	2551	504	[1628; 3555]	MCM	2550	505	[1629; 3561]	x 10^{-6}
	Analytical	2550	505	-	GUF-NL	2550	505	[1561; 3539]	x 10^{-6}, not validated
9.5	MCM	838 nm	36 nm	[745; 932] nm	MCM	838 nm	36 nm	[744; 932] nm	
	Analytical	-	-	-	GUF-NL	838 nm	36 nm	[746; 930] nm	Normal distribution
	GUF	838 nm	32 nm	[745; 931] nm	GUF	838 nm	32 nm	[745; 931] nm	t-distribution, f=16

EA-4/02 , DAkkS-DKD-3 Expression of the Uncertainty of Measurement in Calibration

Example	DKD-3, EA-4/02				QMSys GUM, GUF Method				Distribution
	Estimate	u	k	U	Estimate	u	k	U	
S.2	10000,025 g	29,3 mg	2,00	±59 mg	10000,025 g	29,3 mg	2,00	±59 mg	Normal
S.3	10000,178 Ω	8,33 mΩ	2,00	±17 mΩ	10000,178 Ω	8,33 mΩ	2,00	±17 mΩ	Normal
S.4 (EA)	49,999926 mm	36,4 nm	2,00	±73 nm	49,999926 mm	36,4 nm	2,00	±73 nm	Normal
S.4 (DAkkS)	49,999926 mm	34,3 nm	2,00	±69 nm	49,999926 mm	34,3 nm	2,00	±69 nm	Normal
S.5 (t_x)	1000,5 °C	0,641 K	2,00	±1,3 K	1000,5 °C	0,641 K	2,00	±1,3 K	Normal
S.5 (V_x)	36229 µV	25.0 µV	2,00	±50 µV	36229 µV	24.8 µV	2,00	±50 µV	Normal
S.6	0,933	0,0162	2,00	±0,032	0,933	0,0162	2,00	±0,032	Normal
S.7	30,043 dB	0,0224 dB	2,00	±0,045 dB	30,043 dB	0,0224 dB	2,00	±0,045 dB	Normal
S.9	0,10 V	0,030 V	1,65	±0,05 V	0,10 V	0,030 V	1,65	±0,05 V	Rectangular
S.10	0,10 mm	0,033 mm	1,83	±0,06 mm	0,100 mm	0,0323 mm	1,84	±0,060 mm	Trapez., ß=0,33
S.11	180,1 °C	0,164 K	1,81	±0,3 K	180,10 °C	0,164 K	1,83	±0,30 K	Trapez., ß=0,34
S.12 (V_x)	199,95 l	0,109 l	-	-	199,95 l	0,109 l	2,00	±0,22 l	Normal
S.12 (e_x)	0,3 x10^{-3}	0,68 x10^{-3}	-	-	0,24 x10^{-3}	0,68 x10^{-3}	2,00	±0,0014 l	Normal
S.12 (e_{xav})	0,001	0,91 x10^{-3}	2,28	±0,002	0,001	0,91 x10^{-3}	2.28	±0,002	t-distribution, f=10
S.13	90,00025 mm	0,414 µm	2,00	±0,9 µm	90,00024 mm	0,411 µm	2,00	±0,82 µm	Normal

EURACHEM/CITAC Guide CG 4 Quantifying Uncertainty in Analytical Measurement

Example	EURACHEM/CITAC Guide CG 4			QMSys GUM, GUF Method, Normal distribution, k = 2.00		
	Estimate	u	U	Estimate	u	U
A.1	1002,7 mg.l^{-1}	0,9 mg.l^{-1}	±1,8 mg.l^{-1}	1002,7 mg.l^{-1}	0,835 mg.l^{-1}	±1,7 mg.l^{-1}
A.2	0,1021 mol.l^{-1}	0,00010 mol.l^{-1}	±0,0002 mol.l^{-1}	0,1021 mol.l^{-1}	0,00010 mol.l^{-1}	±0,0002 mol.l^{-1}
A.3	0,1014 mol.l^{-1}	0,00018 mol.l^{-1}	±0,0004 mol.l^{-1}	0,1014 mol.l^{-1}	0,00018 mol.l^{-1}	±0,0004 mol.l^{-1}
A.3 (Repl.)	0,1014 mol.l^{-1}	0,00016 mol.l^{-1}	±0,0003 mol.l^{-1}	0,1014 mol.l^{-1}	0,00016 mol.l^{-1}	±0,0003 mol.l^{-1}
A.4	1,11	0.377 (0,34 rel.)	±0.75 (0,68 rel.)	1,11	0.377 (0,339 rel.)	±0.75 (0,68 rel.)
A.5	0,015 mg.dm^{-2}	0,0015 mg.dm^{-2}	±0,003 mg.dm^{-2}	0,0151 mg.dm^{-2}	0,00142 mg.dm^{-2}	±0,0028 mg.dm^{-2}
A.7	0,05374 µmol.q^{-1}	0,00018 µmol.q^{-1}	0,00036 µmol.q^{-1}	0,05374 µmol.q^{-1}	0,000180 µmol.q^{-1}	0,00036 µmol.q^{-1}

u - Combined standard uncertainty; U - Expanded uncertainty; k - Coverage factor; f - Degrees of freedom; ß - Shape factor

Appendix B: Example of a Report

Page 1

GUF_MCM.rtf - Microsoft Word

QUALISYST

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MEASUREMENT UNCERTAINTY REPORT

1. Main data

Name	**Calibration of a weight of nominal value 10 Kg**
File	EA4-S2-Calibration of a weight of nominal value 10 kg.gmf
Created	13.11.2011 20:01 :
Modified	22.11.2011 11:50 : Stefan Golemanov
File version	4.8 (11.11.21) / QMSys GUM Enterprise

Monte-Carlo	Adaptive procedure	Tolerance	Number of trials
Yes	Yes	0,2δ	220×10^{3}

1.1. Description

The calibration of a weight of nominal value 10 kg of OIML class MI is carried out by comparison to a reference standard (OIML class F2) of the same nominal value using a mass comparator whose performance characteristics have previously been determined.

Correlation: None of the input quantities are considered to be correlated to any significant extent.

1.2. Model

$$m_X = m_S + \delta m_D + \delta m + \delta m_C + \delta B$$

1.3. List of Quantities

Quantity	Name
m_X	conventional mass of the unknown
m_S	conventional mass of the standard
δm_D	drift of value of the standard since its last calibration
δm	observed difference in mass between the unknown mass and the standard
δm_C	correction for eccentricity and magnetic effects
δB	correction for air buoyancy

1.4. Quantities description

Quantity	Description	Comment
m_X	Type: Result Unit: g Uncert. unit: mg Factor: 10E-3 Format: Absolute Distribution: Normal distribution Coverage probability (%): 95,45 Proof of capability: Yes Tolerance or distribution interval: 1 g Capability index-Limit value: Cm = 4 Compliance assessment: Two-sided Lower specification limit: 9999,5 g Upper specification limit: 10000,5 g Decision rule: Stringent Acceptance - Stringent Rejection	Maximum Permissible Error (MPE) for Class M1 in accordance with OIML R111 is 500 mg.

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QMSys Metrology and measurement software

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QUALISYST web: http://www.qsyst.com e-mail: qualisyst@qsyst.com

m_S	Type: Type B Unit: g Uncert. unit: mg Factor: 10E-3 Uncertainty estimate: Expanded uncertainty Distribution: Normal Degrees of freedom: ∞ Value: 10000.005g Rel. uncertainty: 45mg Coverage probability (%): 95,00 Coverage factor: 2,00 Stand. uncertainty: 22,50g	**Reference standard (m_s):** The calibration certificate for the reference standard gives a value of 10 000,005 g with an associated expanded uncertainty of 45 mg (coverage factor *k* = 2).
δm_D	Type: Type B Unit: g Uncert. unit: mg Factor: 10E-3 Uncertainty estimate: Probability distribution Distribution: Rectangular Value: g Half-width of limits: 0.015g Rel. uncertainty error: 0	**Drift of the value of the standard (δm_D):** The drift of the value of the reference standard is estimated from previous calibrations to be zero within ±15 mg.
δm	Type: Type A Unit: g Uncert. unit: mg Factor: 10E-3 Method of observation: Indirect Number of observations: 3 Uncertainty evaluation: Pooled estimate Uncertainty estimate: Stand. deviation Distribution: Normal Degrees of freedom: ∞ Stand. deviation: 25mg Stand. uncertainty: 14,43mg	**Comparator (δm):** A previous evaluation of the repeatability of the mass difference between two weights of the same nominal value gives a pooled estimate of standard deviation of 25 mg. **Measurements:** Three observations of the difference in mass between the unknown mass and the standard are obtained using the substitution method and the substitution scheme ABBA ABBA ABBA:
δm_C	Type: Type B Unit: g Uncert. unit: mg Factor: 10E-3 Uncertainty estimate: Probability distribution Distribution: Rectangular Value: g Half-width of limits: 0.01g Rel. uncertainty error: 0	**Comparator (δm_C):** No correction is applied for the comparator, whereas variations due to eccentricity and magnetic effects are estimated to have rectangular limits of ±10 mg.
δB	Type: Type B Unit: g Uncert. unit: mg Factor: 10E-3 Uncertainty estimate: Probability distribution Distribution: Rectangular Value: g Half-width of limits: 0.01g Rel. uncertainty error: 0	**Air buoyancy (δB):** No correction is made for the effects of air buoyancy, the limits of deviation are estimated to be ±1x10^{-6} of the nominal value.

1.5. Correlation matrix (Input quantities)

Quantity 1	Quantity 2	Correlation coefficient
		0

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GUF_MCM.rtf - Microsoft Word

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QUALISYST

web: http://www.qsyst.com
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2. Observations

Quantity	Unit	Number of observations	Observations	Mean value	Stand. uncertainty	Bayesian stand. uncertainty
δm	g	3	0,010 0,030 0,020	0,0200	0,00577	---

3. GUF - Uncertainty budget

3.1. Budget

Quantity	Value	Stand. uncertainty	Distribution	DoF	Sensitivity coefficient	Sensitivity (Graph.)
m_S	10000,0050 g	22,5 mg	Normal	∞	1,00	
$δm_D$	0,0 g	8,66 mg	Rectangular	∞	1,00	
δm	0,0200 g	14,4 mg	Normal	∞	1,00	
$δm_C$	0,0 g	5,77 mg	Rectangular	∞	1,00	
δB	0,0 g	5,77 mg	Rectangular	∞	1,00	

3.2. Combined Standard Uncertainty

Quantity	Comb. stand. uncertainty	Comb. relat. uncertainty	Effective degrees of freedom
m_X	29,3 mg	$2,93x10^{-6}$	∞

3.3. Result

Quantity	Value	Expanded uncertainty	Expanded rel. uncertainty	Coverage factor	Coverage probability	Distribution
m_X	10000,025 g	± 59 mg	± $0,59x10^{-6}$ %	2,00	95,45 %	Normal

3.4. GUF - Validation

Quantity	Validation	Tolerance δ	Δ Coverage interval	Δ Result	Δ Comb. stand. uncert.
m_X	Yes	0,005000	[-0,00010:0,00010]	0,0	-0,000030

3.5. Proof of capability and compliance assessment

Quantity	Capability	Index	Limit value	Compliance	P-inside	P-outside
m_X	Yes	Cm = 8,5	4	Yes	100,000%	0,000%

Quantity	Graphic
m_X	9999,500 10000,500 9999,441 9999,559 10000,441 10000,559

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GUF_MCM.rtf - Microsoft Word

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QUALISYST

web: http://www.qsyst.com
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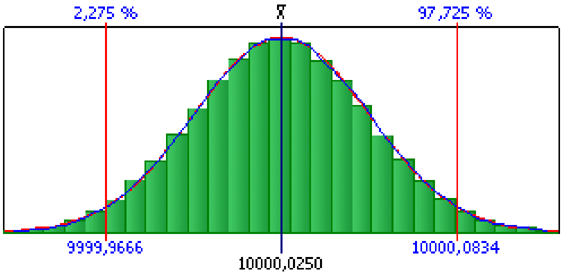
4. MCM - Uncertainty budget

4.1. Statistical parameters

Quantity	Maximum	Minimum	Mean value	Stand. deviation	Quantile	Quantile
mx	10000,1465	9999,8771	10000,0250	0,0293	9999,9666 (2,275%)	10000,0834 (97,725%)

Quantity	Skewness	Kurtosis	Histogram
mx	-0,005	-0,013	

4.2. MCM validation

Validation	Tolerance	2S(y)	2S(u)	2S(y-low)	2S(y-high)
Yes	$1x10^{-6}$	$0,121x10^{-6}$	$0,0777x10^{-6}$	$0,331x10^{-6}$	$0,356x10^{-6}$

4.3. Combined Standard Uncertainty

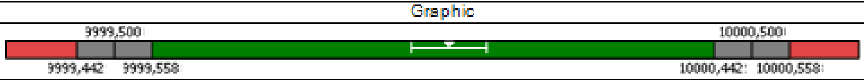
Quantity	Comb. stand. uncertainty	Comb. relat. uncertainty	Distribution
mx	29,3 mg	$2,93x10^{-6}$	Normal

4.4. Result

Quantity	Result	Expanded uncertainty	Expanded rel. uncertainty	Coverage factor	Probability
mx	10000,0250 g	± 58 mg	$\pm 0,58x10^{-6}$ %	1,99	95,45 %

4.5. Proof of capability and compliance assessment

Quantity	Capability	Index	Limit value	Compliance	P-inside	P-outside
mx	Yes	Cm = 8,6	4	Yes	100,000%	0,000%

Quantity	Graphic
mx	

5. Comment

mx	The measured mass of the nominal 10 kg weight is 10,000 025 kg ±59 mg. The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor *k* = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

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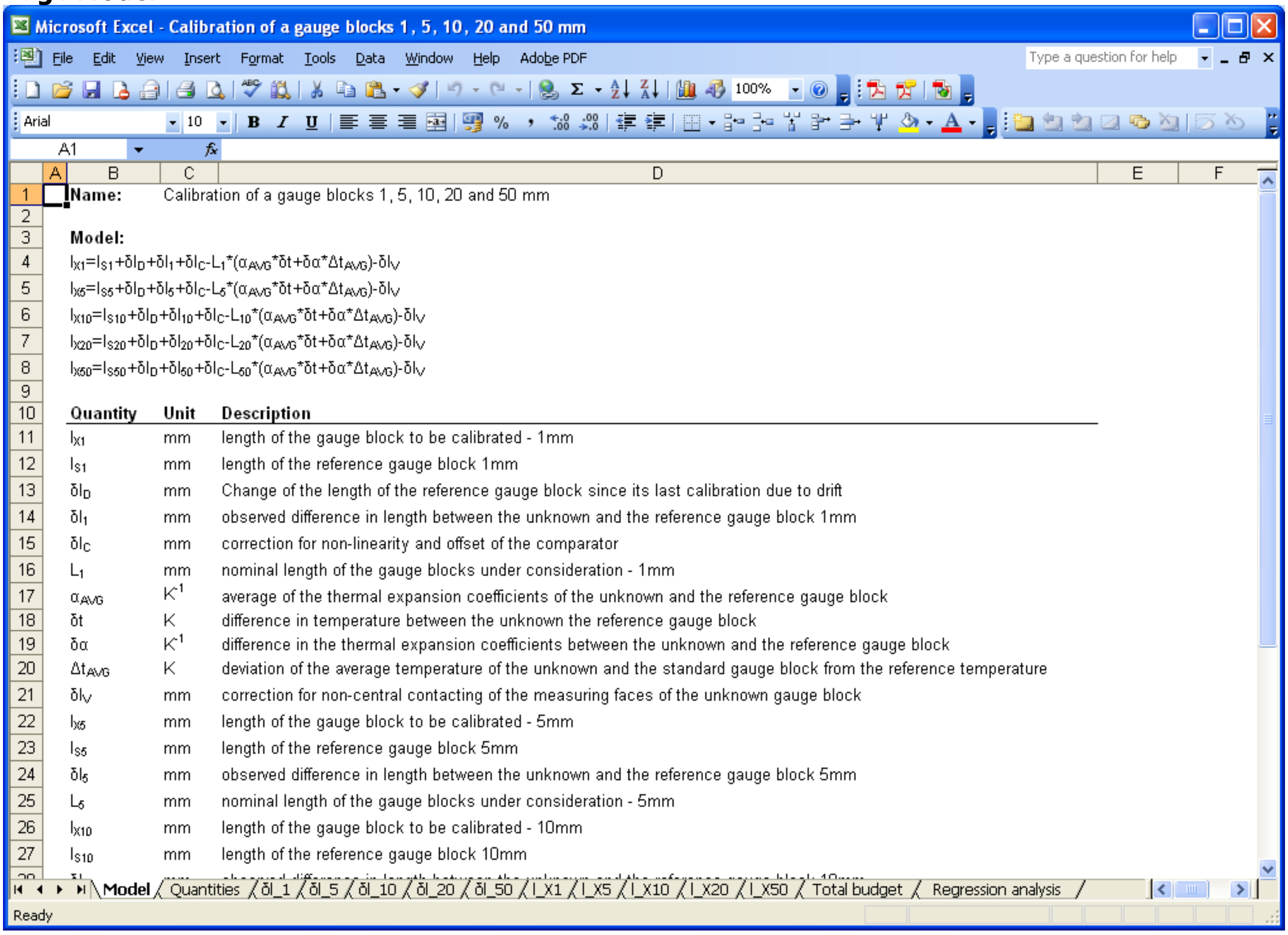
QMSys Metrology and measurement software

Appendix C: Example of an Export to MS Excel

Page *Model*



Microsoft Excel - Calibration of a gauge blocks 1, 5, 10, 20 and 50 mm

Name: Calibration of a gauge blocks 1, 5, 10, 20 and 50 mm

Model:

$l_{X1}=l_{S1}+\delta l_D+\delta l_1+\delta l_C-L_1*(\alpha_{AVG}*\delta t+\delta\alpha*\Delta t_{AVG})-\delta l_V$

$l_{X5}=l_{S5}+\delta l_D+\delta l_5+\delta l_C-L_5*(\alpha_{AVG}*\delta t+\delta\alpha*\Delta t_{AVG})-\delta l_V$

$l_{X10}=l_{S10}+\delta l_D+\delta l_{10}+\delta l_C-L_{10}*(\alpha_{AVG}*\delta t+\delta\alpha*\Delta t_{AVG})-\delta l_V$

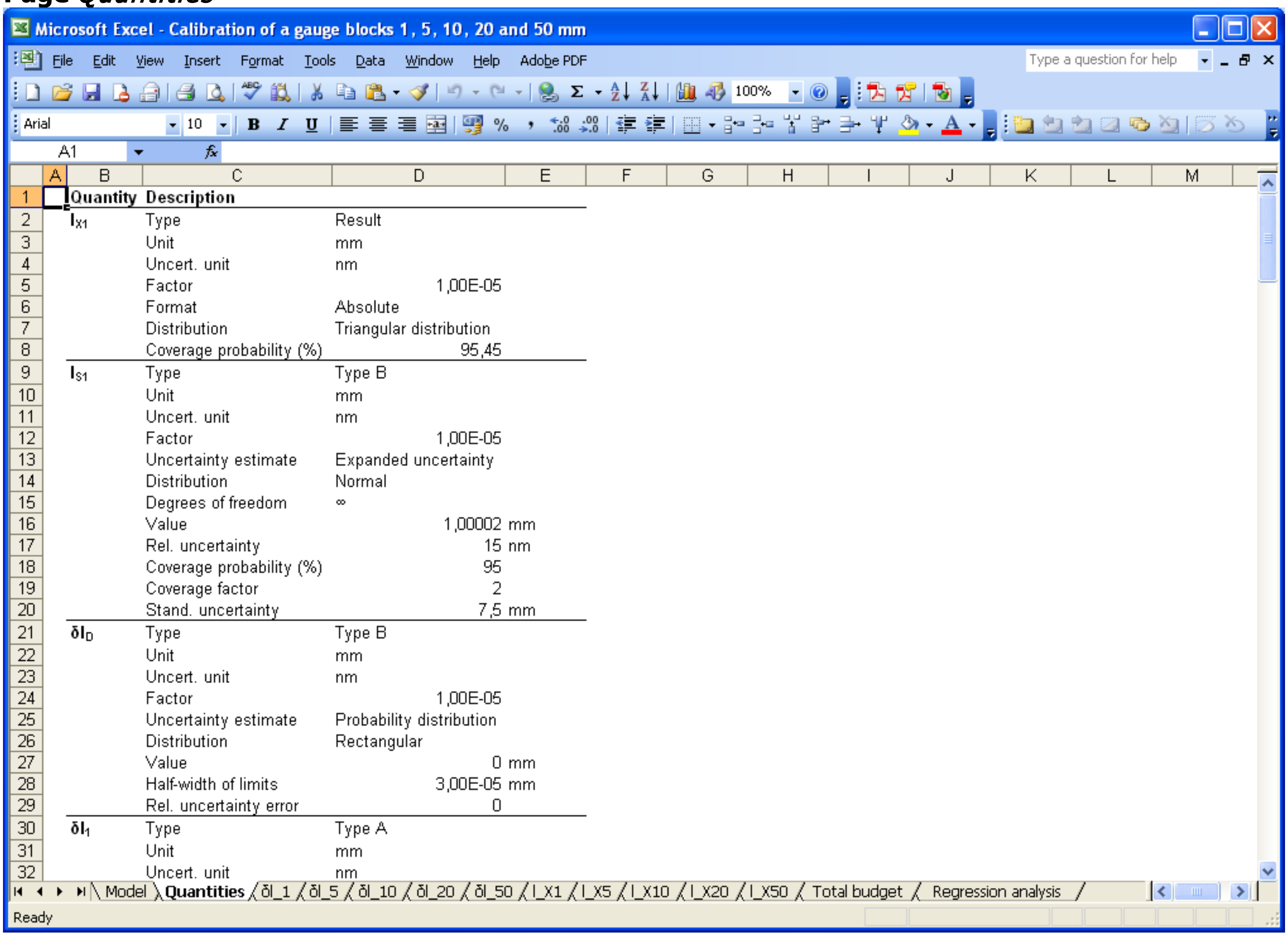
$l_{X20}=l_{S20}+\delta l_D+\delta l_{20}+\delta l_C-L_{20}*(\alpha_{AVG}*\delta t+\delta\alpha*\Delta t_{AVG})-\delta l_V$

$l_{X50}=l_{S50}+\delta l_D+\delta l_{50}+\delta l_C-L_{50}*(\alpha_{AVG}*\delta t+\delta\alpha*\Delta t_{AVG})-\delta l_V$

Quantity	Unit	Description
l_{X1}	mm	length of the gauge block to be calibrated - 1mm
l_{S1}	mm	length of the reference gauge block 1mm
δl_D	mm	Change of the length of the reference gauge block since its last calibration due to drift
δl_1	mm	observed difference in length between the unknown and the reference gauge block 1mm
δl_C	mm	correction for non-linearity and offset of the comparator
L_1	mm	nominal length of the gauge blocks under consideration - 1mm
α_{AVG}	K^{-1}	average of the thermal expansion coefficients of the unknown and the reference gauge block
δt	K	difference in temperature between the unknown the reference gauge block
$\delta\alpha$	K^{-1}	difference in the thermal expansion coefficients between the unknown and the reference gauge block
Δt_{AVG}	K	deviation of the average temperature of the unknown and the standard gauge block from the reference temperature
δl_V	mm	correction for non-central contacting of the measuring faces of the unknown gauge block
l_{X5}	mm	length of the gauge block to be calibrated - 5mm
l_{S5}	mm	length of the reference gauge block 5mm
δl_5	mm	observed difference in length between the unknown and the reference gauge block 5mm
L_5	mm	nominal length of the gauge blocks under consideration - 5mm
l_{X10}	mm	length of the gauge block to be calibrated - 10mm
l_{S10}	mm	length of the reference gauge block 10mm

Model / Quantities / δl_1 / δl_5 / δl_10 / δl_20 / δl_50 / l_X1 / l_X5 / l_X10 / l_X20 / l_X50 / Total budget / Regression analysis

Page *Quantities*

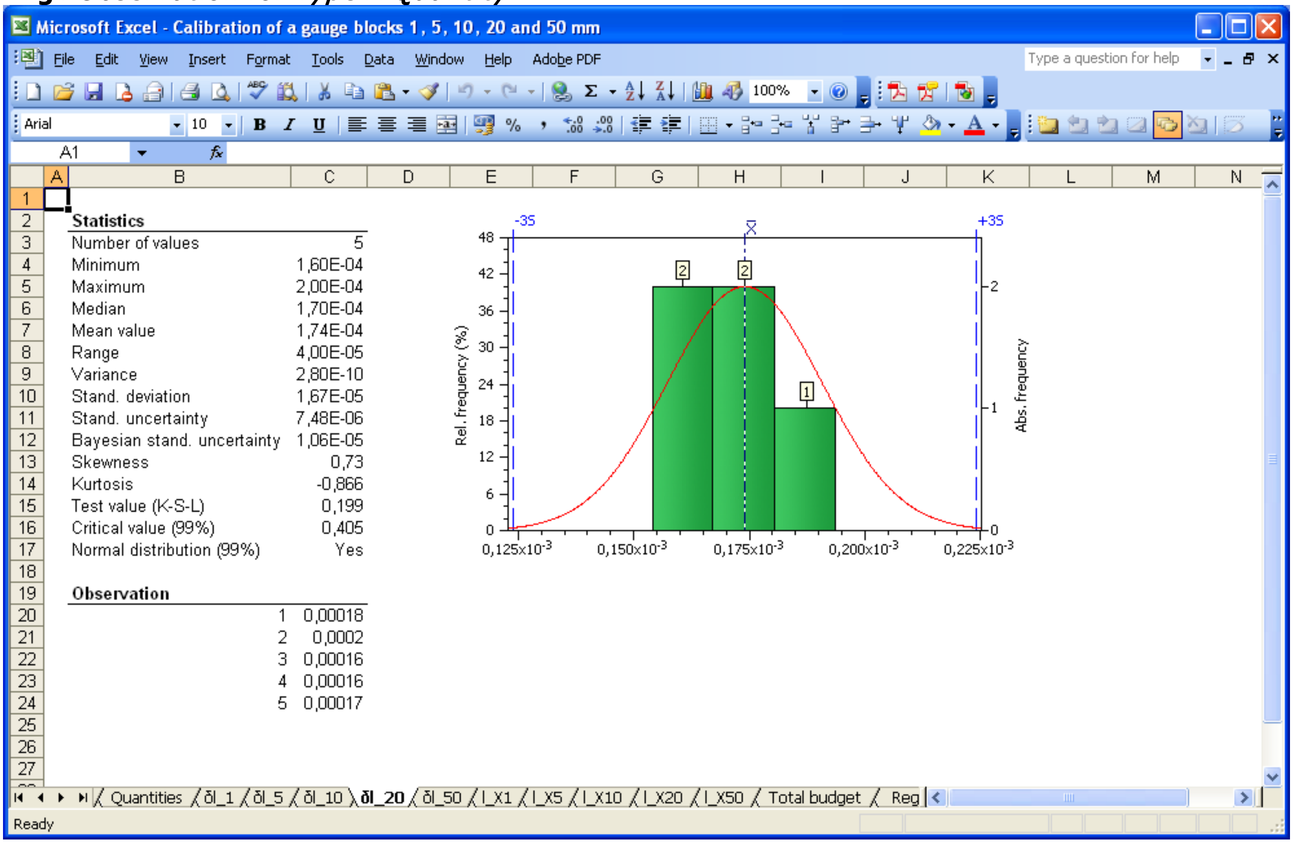


Microsoft Excel - Calibration of a gauge blocks 1, 5, 10, 20 and 50 mm

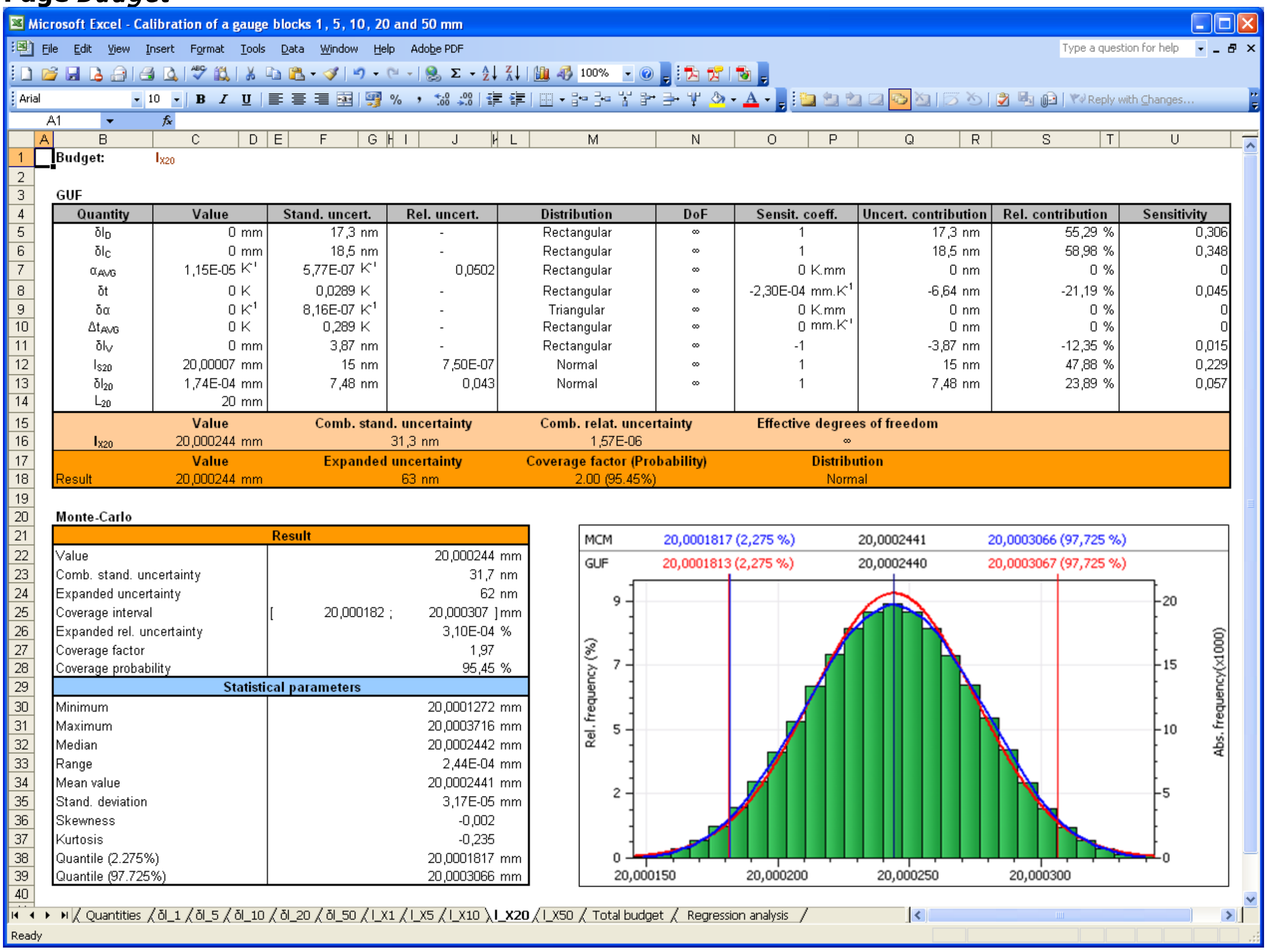
Quantity	Description			
l_{X1}	Type	Result		
	Unit	mm		
	Uncert. unit	nm		
	Factor	1,00E-05		
	Format	Absolute		
	Distribution	Triangular distribution		
	Coverage probability (%)	95,45		
l_{S1}	Type	Type B		
	Unit	mm		
	Uncert. unit	nm		
	Factor	1,00E-05		
	Uncertainty estimate	Expanded uncertainty		
	Distribution	Normal		
	Degrees of freedom	∞		
	Value		1,00002	mm
	Rel. uncertainty		15	nm
	Coverage probability (%)		95	
	Coverage factor		2	
	Stand. uncertainty		7,5	mm
δl_D	Type	Type B		
	Unit	mm		
	Uncert. unit	nm		
	Factor	1,00E-05		
	Uncertainty estimate	Probability distribution		
	Distribution	Rectangular		
	Value		0	mm
	Half-width of limits		3,00E-05	mm
	Rel. uncertainty error		0	
δl_1	Type	Type A		
	Unit	mm		
	Uncert. unit	nm		

Model / Quantities / δl_1 / δl_5 / δl_10 / δl_20 / δl_50 / l_X1 / l_X5 / l_X10 / l_X20 / l_X50 / Total budget / Regression analysis

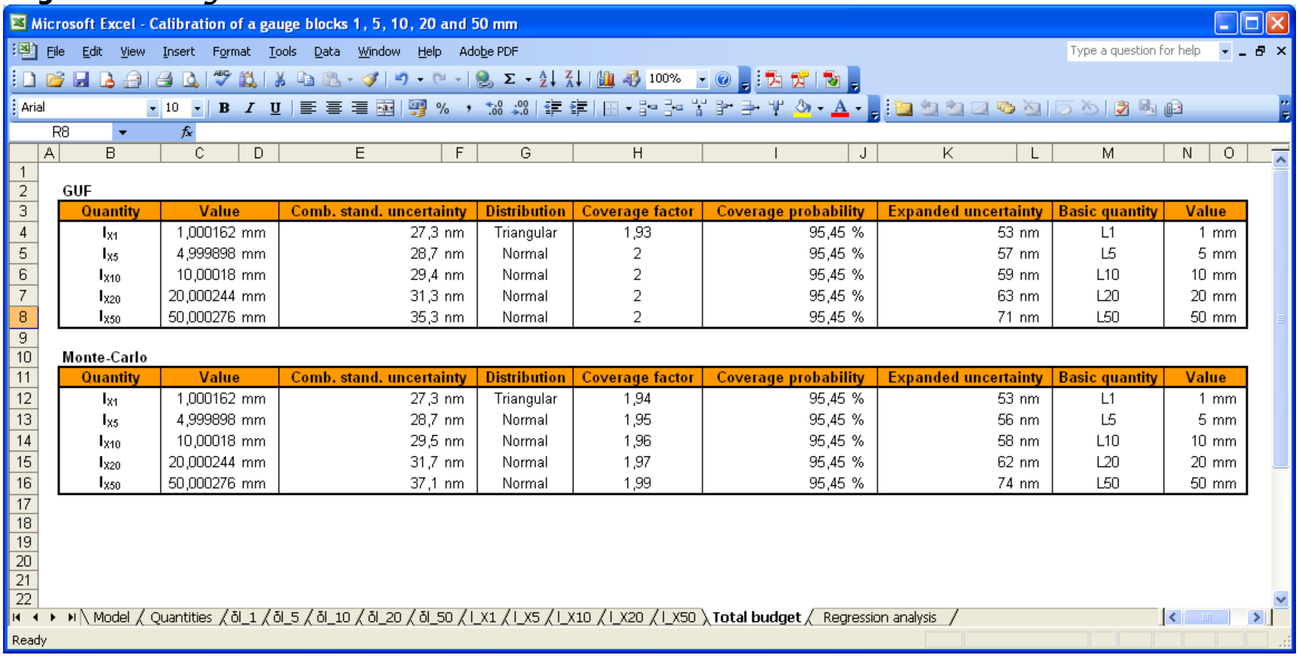
Page ***Observation for Type A Quantity***



Page ***Budget***



Page *Total budget*



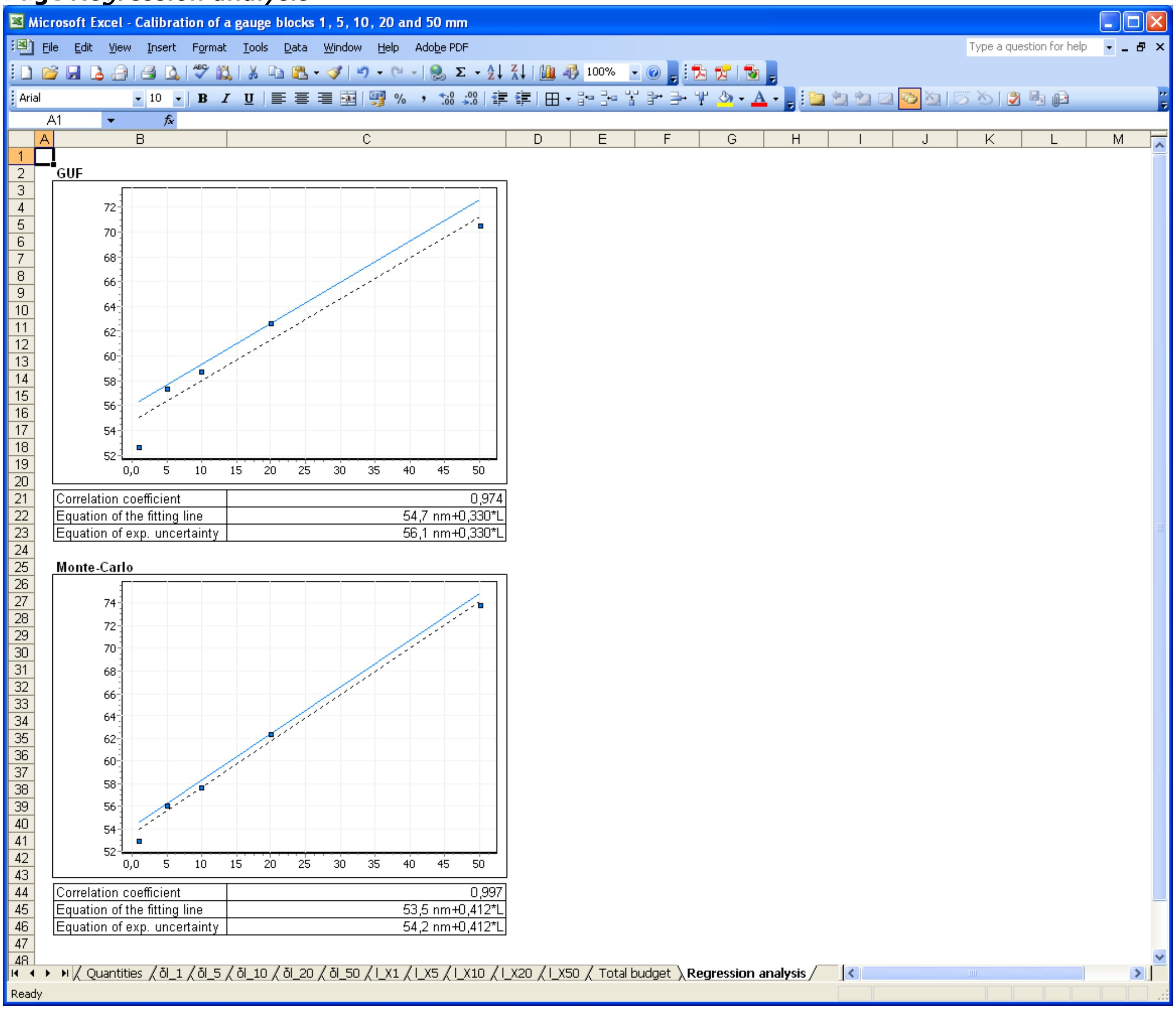
GUF

Quantity	Value	Comb. stand. uncertainty	Distribution	Coverage factor	Coverage probability	Expanded uncertainty	Basic quantity	Value
l_{x1}	1,000162 mm	27,3 nm	Triangular	1,93	95,45 %	53 nm	L1	1 mm
l_{x5}	4,999898 mm	28,7 nm	Normal	2	95,45 %	57 nm	L5	5 mm
l_{x10}	10,00018 mm	29,4 nm	Normal	2	95,45 %	59 nm	L10	10 mm
l_{x20}	20,000244 mm	31,3 nm	Normal	2	95,45 %	63 nm	L20	20 mm
l_{x50}	50,000276 mm	35,3 nm	Normal	2	95,45 %	71 nm	L50	50 mm

Monte-Carlo

Quantity	Value	Comb. stand. uncertainty	Distribution	Coverage factor	Coverage probability	Expanded uncertainty	Basic quantity	Value
l_{x1}	1,000162 mm	27,3 nm	Triangular	1,94	95,45 %	53 nm	L1	1 mm
l_{x5}	4,999898 mm	28,7 nm	Normal	1,95	95,45 %	56 nm	L5	5 mm
l_{x10}	10,00018 mm	29,5 nm	Normal	1,96	95,45 %	58 nm	L10	10 mm
l_{x20}	20,000244 mm	31,7 nm	Normal	1,97	95,45 %	62 nm	L20	20 mm
l_{x50}	50,000276 mm	37,1 nm	Normal	1,99	95,45 %	74 nm	L50	50 mm

Page *Regression analysis*



GUF

Correlation coefficient	0,974
Equation of the fitting line	54,7 nm+0,330*L
Equation of exp. uncertainty	56,1 nm+0,330*L

Monte-Carlo

Correlation coefficient	0,997
Equation of the fitting line	53,5 nm+0,412*L
Equation of exp. uncertainty	54,2 nm+0,412*L